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UNIVERSITY OF FORT HARE

Eastern Cape Education Department

Distance Education Project

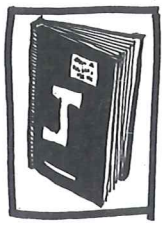
Core Learning Areas Course

Mathematical Literacy, Mathematics and Mathematical Science

Umthamo 2

Problem Solving and Investigating

(Pilot Edition) April 1999



Journal

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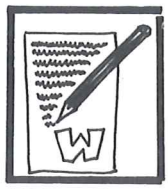
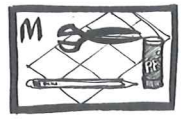


Written Report

Classroom or School

Key Activity



Making materials

Reading and Thinking

Discussion

Face-to-face umkhwezeli

Concertina File for Portfolio

Time

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Broomes, D et al. 1990. *Teaching Primary School Mathematics.* Jamaica: Ian Randle Publishers.

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Desfonges, C. 1997. *Understanding the Mathematics Teacher.* Philadelphia: Falmer Press.

Hopkins, C. 1996. *Mathematics in the Primary School.* London: David Fulton Publishers.

Mathematics is a subject which searches for studies and describes pattern. The identification and use of pattern is a fundamental aspect of the mathematising process, and occurs both consciously and unconsciously during investigations. An understanding of the concept and of the elementary patterns of mathematics, as well as learning to search for them, is basic to any primary school curriculum. Making sense of mathematics relies on an understanding of the patterns which underlie it. By seeking patterns in their own mathematical experiences, children construct knowledge for themselves. They are also learning to function in a mathematical way. The potential for experiences with pattern to develop mathematical understanding is profound. In fact, pattern can feature as a prominent and unifying characteristic of the curriculum.

Helen Pengelly. 1992. Making Patterns. Sydney: Ashton Scholastic.

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CONTENTS

Umthamo 2

Problem Solving and Investigating

Introduction

Welcome to the second umthamo of the Mathematics strand. The main focus of this umthamo is to explore alternative ways to teach and facilitate the learning of mathematics in primary classrooms.

This umthamo, we hope, will show you how you might start thinking of ways in which you can begin teaching differently. Why do we hope that it will help you begin to explore alternative approaches to your work?

Think of how a singer, or a choir, or a group has a repertoire of songs. They have choices. They can choose the right songs for a specific occasion. Or they can choose the best song to suit a specific moment. As a teacher, you need to have a repertoire of approaches for maths. We think that this umthamo will help you add to your repertoire of choices of ways to teach, and help learners learn maths.

We hope that this umthamo will help you to begin building a bridge between your classroom and the world beyond it, the world with which it is so closely connected. This is so that your classroom becomes a more dynamic and interesting place. It is so that your learners become more actively involved in their own learning.

As we write this umthamo, we are not trying to tell you what you ought to do. We are not trying to impose yet another set of 'fixed ideas' on you. We know that you are the expert when it comes to knowing your own learners. You have the experience of interacting with the learners in your class in many different learning situations. You will know which choices to make. What we hope to do is to make sure that you have a broader range of choices.

But we also know that the education system that we have in this country, as a legacy of APARTHEID, denied teachers the right to use their own *common sense* in the classroom. Teachers were not expected or even allowed to feel that they could make their own wise choices in the classroom. This was very disempowering.

Some strong teachers did use their own initiative and helped learners by developing their own unique and innovative ways of doing things. But in the past this could not be done openly. You had to teach subversively and hide what you did well.

We need to find out about these ways and write about them so that they can come out into the open to be shared.

The emphasis of this umthamo

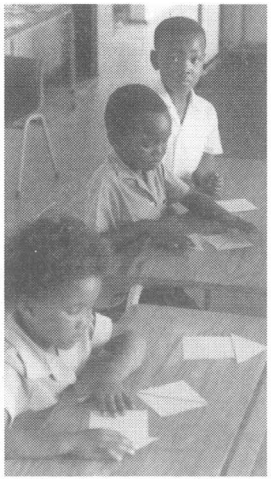
When you pick up this umthamo for the first time, and skim through it, you realise that the focus is on **problem solving** and **investigating**. In this umthamo, we want to help you to develop your expertise further as you try to understand better and make sense of the classroom occurrences that arise when you start to implement (use) a **problem solving** and **investigative** approach, as one way of dealing with maths. We also need to think about why **problem solving** and **investigating** could be considered important.

In the previous umthamo, you learned about **constructivism** in the learning and teaching of mathematics. The notion (or idea) of **constructivism** is based on the belief that learners *naturally* **construct** their own knowledge, including mathematical knowledge. We saw that learners have their own ways of thinking and working things out for themselves. Teachers need to provide opportunities for learners to reveal their personal constructed knowledge of maths.

In the case of learning mathematics in school, teachers who use a **constructivist** approach provide their learners with opportunities to construct their own mathematical knowledge. They provide opportunities for learners to interact with other learners, **constructing** ways of doing things.

Learners need to develop consensus about the advantages and disadvantages of certain ways of doing maths. At the same time, teachers need to familiarise learners with the conventions of mathematics. Even so, learners need to be encouraged to think for themselves, and to explain and discuss their viewpoints.

In this umthamo, we will try to extend our understanding of **constructivism** as we use **problem solving** and **investigation** in the teaching and learning of mathematics. We will watch to see what children do to **solve problems**. We will see what they find out for themselves when they **investigate**. And we will see how they try to make sense of what they have found out.



The Cockroft Report

Just over twenty years ago in Britain, a committee was set up to look into mathematics education. That committee made a number of recommendations. You can read them below. You will see that a repertoire of at least 7 approaches was recommended.

According to the Cockroft Report, regardless of the learners' age, it is recommended that,

Mathematics teaching should include

- **opportunities for exposition by the teacher**
- **discussion between the teacher and learners**
- **discussion between the learners themselves**
- **appropriate practical work**
- **consolidation and practice of fundamental skills**
- **problem solving, including the application of mathematics to everyday situations**
- **investigational work**

(Cockroft, 1982:71)

Let me try to explain the points in the quote from the Cockroft Report. As you probably know, there are a lot of different things that the teacher and the learner can be doing when they do mathematics.

1. First, let's think about *exposition by the teacher*. What does this mean? For example, a teacher may talk to the whole class when she gives general instructions. She may also speak to the whole class when she tells the story of something or how it works. When the teacher tells pupils things, she is relying on exposition as an approach.

2. How often do children get to tell the teacher what they know in a natural conversational way? Do children ever get a chance to ask questions of the teacher, and discuss things that concern them? Discussion involves the **sharing** of ideas, not just **receiving** ideas. In this way, children can be apprenticed into talking and thinking mathematically.

Communication is a very important tool in learning, and in problem solving, we need plenty of opportunities to talk. Children also need these opportunities to talk. It is therefore important for a teacher to plan activities carefully so that the kind of talk that is most valuable can come out of the activity.

3. Next, let's think about *discussion between learners themselves*. When learners work together in a group they tend to share their ideas and teach each other skills. This can be encouraged by giving learners in your class opportunities to explain something to each other or to a group. Children cannot learn if the teacher simply tells them what to say and write. They need to think and discuss among themselves in order to develop problem solving and thinking skills.

4. The best kind of learning comes from doing things practically. It is learning first hand. It involves a hands-on minds-on approach. It involves real, or realistic, materials. Things have to be manipulated and actually done, not just talked about theoretically. What we do practically for ourselves, is more likely to be understood.

5. When you have learned a new skill, or how to do something which you could not do before, you need to repeat, or practice that skill. We say that "practice makes perfect". By repeating activities in different settings, we become more confident in the use of the skills. We can claim that the skill is consolidated, or made stronger. Remember though, that mindless or meaningless repetition leads no-where.

6. You will learn more and think about this point as you work through lomthamo.

7. You will learn more and think about this point as you work through lomthamo.

The aspect of Maths in this umthamo

In the first Maths umthamo, the aspect of maths which we used was ***Number***. In this umthamo, we will use ***Shape*** as the basis for the activities. We will ask learners to work with, and think about, ***shapes***.

Our purpose is not to try to cover and re-teach you everything there is to know about **Shape**. In the activities, however, we will try to introduce some new ideas of what can be done with **Shape**.

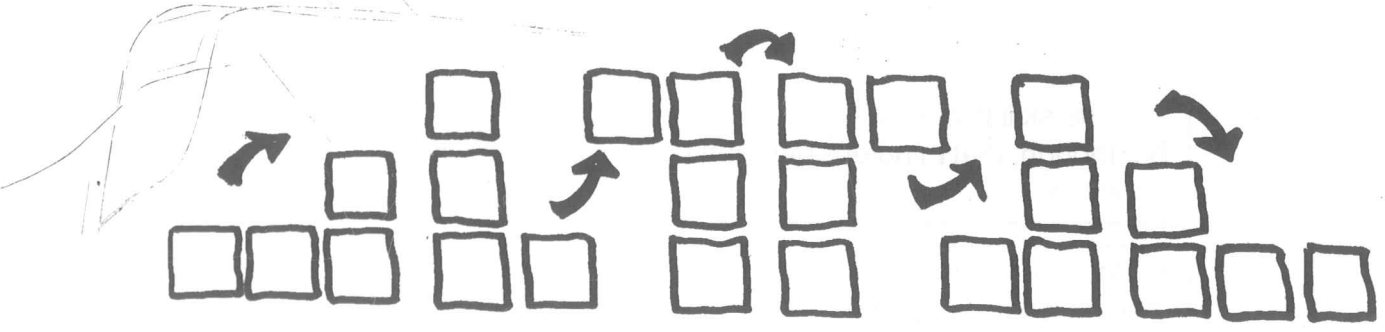
As with some of the other imithamo in the Core Learning Areas Course, we are including a Content Audit in the Appendix. In these few pages, we outline the aspects of *content knowledge* that relate to **Shape** and **Position**. It is up to you to do your own personal Content Audit. In other words, if you find that there are things that you don't know about *Shape and Position*, it is up to ***you*** to do something about it.

Remember, your Centre will have relevant resource books, and for each umthamo there is a box file with resource materials to help you. Make a point of consulting the resource box for this umthamo. If there are aspects of the content where you lack confidence, try to help yourself. You can ask colleagues, or your umkhwezeli. If you are still stuck, your Centre Co-ordinator should try to arrange things for you.

Intended Outcomes

At the end of this Umthamo you will

- have experienced yourself, and set your learners, activities which involve **problem solving**, as well as activities which involve **investigating**
- begin to see the value of a **problem-based investigative** approach as a learner-centred way to encourage the development of maths skills and knowledge.



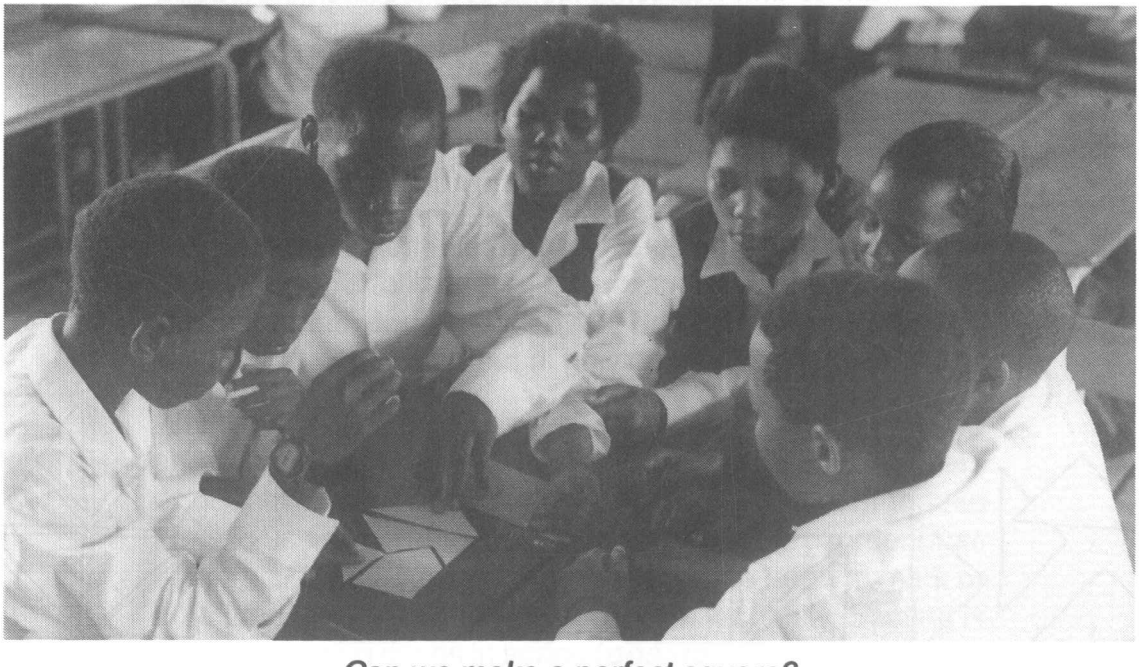
Key Activity

You will see that the **Key Activity** in this umthamo, has the same basic format as the Key Activity in the second Science umthamo (number 11). In other words, the **Key Activity** is made up of three related activities (parts).

You are asked to write a two-page report on each part (activity). Store these reports, together with carefully selected samples of your learners' work, in your Concertina File. This is a way in which you can provide evidence that you have tried the work practically. It also indicates that you have **observed** and **noted** what happened and that you have **thought** carefully about what you have **observed** and **noted**.

To finish off your report and this umthamo, you are asked to sum up what you have experienced. You need to show us that you have something personal to say about **problem solving** and **investigating** for maths work with your children. This will give evidence that you have thought carefully, and that you have constructed your own opinions, about what you have tried.

All in all you will write 7 pages. In other words, you will write 2 pages for each activity (which will be 3 x 2 pages), plus your one-page summary. You will have opportunities to share your experiences of carrying out the different parts of the Key Activity with other teacher-learners at face-to-face sessions. Then store your report safely in your Concertina File, together with the samples of your learners' work which you have selected. You may wish to include it in your Portfolio at the end of the year.



Can we make a perfect square?

Unit 1 - Starting with Problem Solving

Most teachers probably agree that it is important to help children to deal with the changing world. Many of the old ways of doing things will not suit the future. So it is not really worthwhile keeping them. No-one can predict with certainty exactly what life will be like when the children we teach today, are the mature adults of tomorrow. The most worthwhile thing that we can do is to teach our learners how to solve problems.

If learners have a positive attitude to solving new problems, then they will be much more likely to be able to cope in a changing world. If they have enjoyed the experience and challenge of finding ways to solve problems successfully, they will be motivated to tackle other problems with confidence.

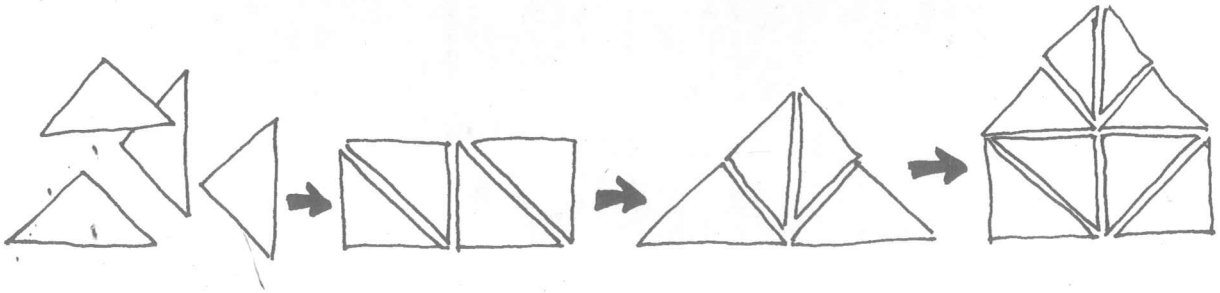
Did you get a chance to read the quote on the back of Umthamo 12? What did Carl Rogers have to say about change? You may want to go back and read the quote again. It says some very important things about the goal of education.

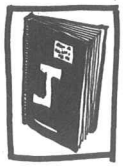
Problem solving is a part of our everyday life, whether it means making decisions on basic issues such as 'what will I have for breakfast?' or 'what clothes should I wear?', to facing up to more difficult problems of work and personal matters.

So problem solving is not only a part of mathematics. It is all around us in all aspects of our lives. That means that problem solving is the place where learning to use mathematics and real life can be linked. Some people argue strongly that mathematics is an extremely effective way to develop strategies and methods to organise our thinking to help us overcome problem situations.

If we want to link mathematics to real life then for young learners, problem solving activities should come out of everyday experience. As the learners mature, problems should involve real world settings, and should lead to maths investigations. Problem solving can also develop from, and extend, the basic mathematics that they conventionally learn.

In the first activity, we want you to think a little about what problem solving means to you personally. After you have done the activity, we want you to think about your attitude to problems and problem solving in maths.





Activity 1 - What *is* problem solving?

Think of a recent problem that you have had to face in real life, and which you have solved. Take out your Journal, write the date and time, then write down what the problem was.

Now shut your eyes for a few minutes. Think back to that problem. Try to remember when you first became aware of the problem. Try to picture what happened in your mind's eye. Then, rather like watching a movie, go over everything that you did until you solved the problem. Go over all the steps you took and the strategies you used to solve the problem.

Open your eyes, and make notes in your Journal of the steps and strategies you followed.

Lastly, spend a short time writing down what you learned about problem solving from remembering how you solved the problem which you've been remembering and writing about.

At the face-to-face session where this umthamo is monitored, you will have an opportunity to compare what you have written with what two or three of your fellow students have written. What have you written that is similar? What is different? Why do you think this is so?

You have thought of a personal example of problem solving from your own life, and you have gone over the steps and strategies that you used. Now we need to think how we can link mathematics with problem solving.

As a teacher, you are probably familiar with some problems of your learners. Think about these problems. Did you help your pupils to make sense of the problems from a mathematical point of view? The real life problems that children face at school are often to do with the need to share something. Perhaps sometimes arguments occur about who can play with a certain thing. Or maybe the problem is about finding something that is lost. And perhaps, even about who sits next to whom.

What has maths got to do with fair shares? Well, when we share, we need to think of fractions and dividing up numbers or quantities like mass and volume. If people want to take turns to use something, then there is the maths of order (*first, second, third, and so on).* There is also the need to mark or note time and to think of ways to share it equally. Seating is about arranging things in space (***position***), or rows and lines (grids). See how easy it is to relate maths to real problems?

Mathematics is all around us all the time. We are just not that aware of it. Perhaps if we encourage children to become more aware of the mathematics then they will be more confident.

As primary school teachers we need to be aware of modern trends in the teaching and learning of mathematics. These modern trends focus on the learners' needs and interests. Remember what you read from the Cockroft Report, on page 4 of the Introduction? Go back and re-read the passage again if you don't remember it clearly.

Now read carefully what Thomas E Rowan has to say about **problem solving**.

Learners active! umthamo 12

". . . . problem solving is not simply instruction about solving of problems. It is learners actively involved in constructing mathematics through problem solving, it is co-operation and questioning as learners acquire, relate and apply new mathematics knowledge. Problem solving is a setting for communicating mathematics ideas, a context for investigating relationships and connecting maths concepts and skills." (Rowan, 1980:4)

Constructing! – umthamo 5

Questioning! – that's in umthamo 2

Co-operation! that's group work in umthamo 3

Communicating! – umthamo 9

In order to make sense of mathematics, learners should be able to explain their strategies and thoughts so that they are clearly understood by others. When they solve problems, learners need to interact and co-operate so that they can share each other's knowledge and experience. Problem solving encourages learners to be critical thinkers in that it gives them responsibility and control over their own learning.

The term 'problem solving' implies that a solution or answer has to be found. In this way, it can be considered a 'closed activity'. This involves choosing between alternatives available and making a decision. The concept of problem solving can be referred to as a process of searching for solutions. It is not enough to rely on other people's solutions, or ready-made steps. You have to discover and work out your own ways to solve problems. Problem solving is very important in mathematics because it encourages learners to *actively* make sense of problems from a mathematical point of view.

To give you a taste of problem solving, you will do the following activity at the face-to-face session where this umthamo is introduced. It is a good example of an activity which requires problem solving. It also involves **shape** and **position**.

Activity 2 - The Tangram

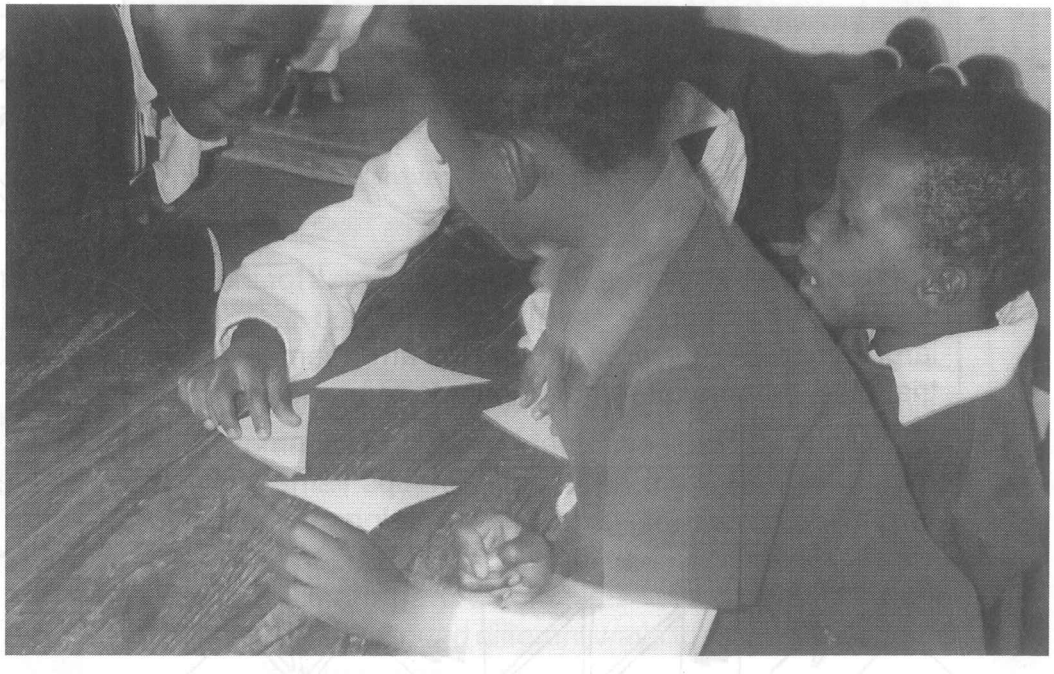
You have 5 small shapes. Work with three or four other people to solve the problem. Your problem is to fit ***all*** 5 shapes together in some way so that you form a **single** big square.

As you work, think of the exact problem solving steps and strategies you are using. Be aware of each step you take, and each strategy that you use.

When you have made a single big square, spend a few minutes jotting down the steps and strategies you used.

Then share what your group has done with another group. In what ways were your strategies and steps similar. In what ways were they different?

The first part of the **Key Activity** involves a tangram problem. When you are with your own learners, set them a tangram problem to solve. If you work with learners in a Preschool class, or in Grade 1, 2, or 3, we suggest that you try Option A. If you work with learners in Grade 4, 5, 6, or even 7, we suggest that you set them the problem in Option B. Some teachers may wish to try both options with their learners. This is especially true of learners in Grades 3 and 4.



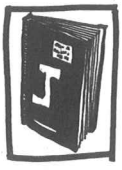
How can we make a perfect square?

Activity 3 - Solving a Problem - Building a Square

Option A

Task 1 - Making a Square

Take the set of 4 small triangles that you have been given, and make enough sets so that your children can work in small groups of 2 or 3 learners. To make the sets of triangles, you need squares of coloured paper or card. A square with sides of 10 cm is a good size. Draw 2 lines from corner to corner of the square. Cut out the 4 triangles.



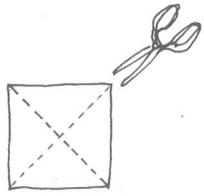
Make sure that you take your Journal to class with you. Then you can make notes of what you observe your learners doing as they carry out the task.

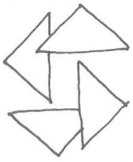
When you are with your learners, give each small group a set of 4 triangles. Tell your learners that you want them to put the triangles together to make ***one large*** square.

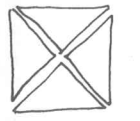
Observe your learners carefully. Notice who takes the lead in each small group. Do any of your learners seem to hold back? Why? Are they disinterested? Or are they thinking? Notice their body language. Do any of the children seem to be looking into the distance, or into space? Or are they screwing up their eyes as though they are trying to picture something in their minds? This is an ideal opportunity to make notes of different learners.

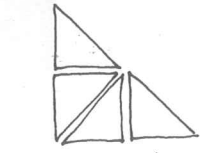
When two or three groups have managed to find a way to do this, stop your children. Show the whole class the different ways that those groups have managed to make a large square with their four triangles. Use the four large triangles that you have been given, and stick them up on the chalkboard in the way that one of the groups has done to make one large square. Draw round each triangle on the chalkboard.

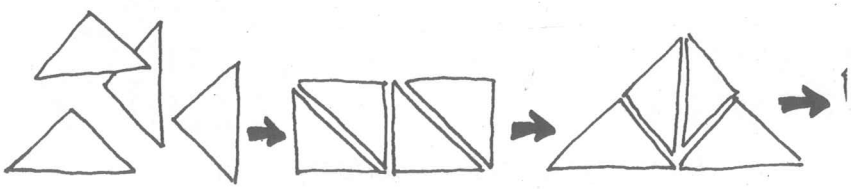
Then change the arrangement of the triangles for the next way that the triangles have been arranged to make a large square. Again, draw around the arrangement of the triangles, before you remove them to rearrange them in yet another way.

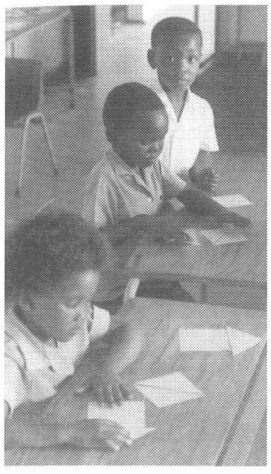




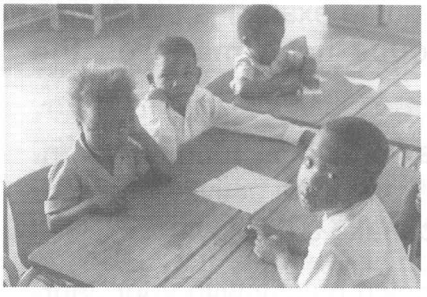


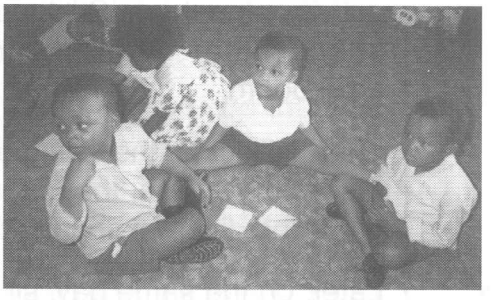






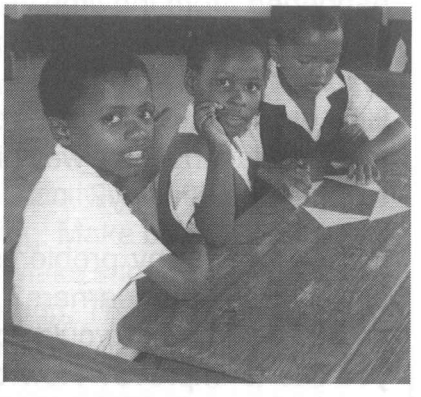
We trialled this activity at Zimele Pre-School in Mdantsane with the older group of children. The children worked in small groups of three or four learners. Some children worked very quickly. Others struggled. In some groups, one child would take the lead, while the others watched. In other groups, two children worked together to solve the problem.

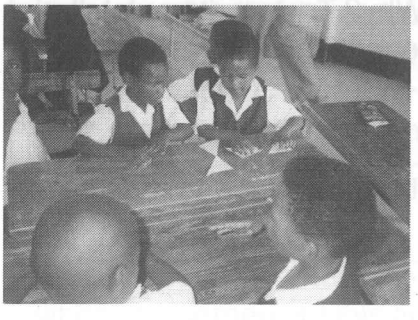


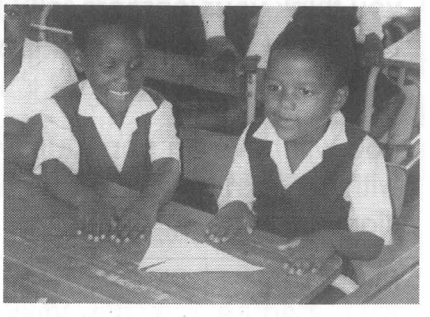


When a group had solved the problem, sometimes a child from that group joined a struggling group, and showed them the way her/his group had solved the problem.

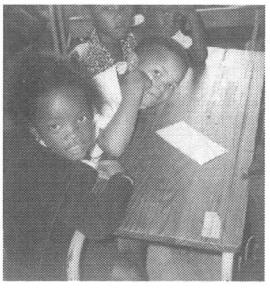
We were really interested that the children found different ways to solve the problem. They didn't all do exactly the same thing.

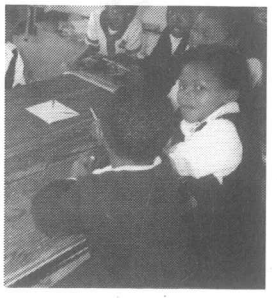






When we tried this activity with Grade 1 learners at Dumisa Junior Primary School in Mdantsane, most of the groups made two small squares first. Then they spent some time moving the triangles around to try to make just one big square. We noticed that these children took longer to solve the problem than the pre-schoolers. We also noticed that the different groups found different ways to make a big square with their four triangles.





Task 2 - What other shapes can you make?

Allow your learners some time to make any other shapes that they can with their triangles. Watch your learners carefully. Encourage them to work together.

When your learners have spent about 10 minutes on this activity, stop them. Stick up some examples of different shapes that different groups have made on the chalkboard.

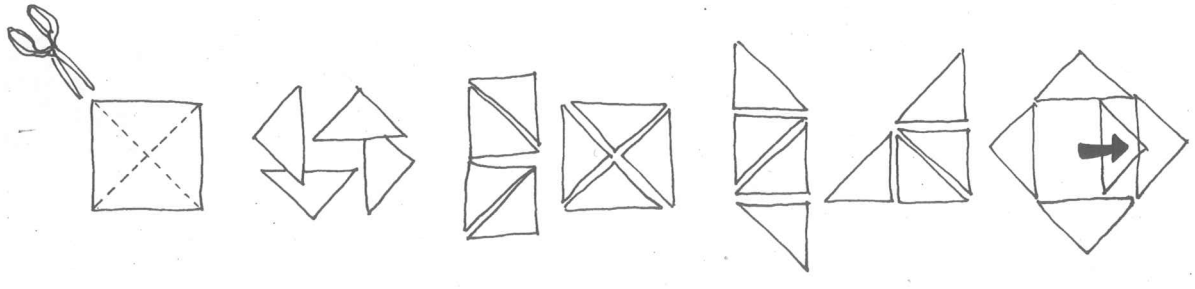
Then collect all the triangles from your groups of learners. Another time, they can repeat the activity and glue down the different shapes to make a display.

Later, on the same day, sit down quietly with your Journal. Re-read the notes and observations that you made when you carried out this activity with your learners. This will remind you of what took place. Try to picture what happened. Your notes will help to jog your memory.

Then take a clean sheet of paper, and write, *Maths umthamo 2: Activity 3 - Option A*. Describe *exactly* what happened, from the time when you introduced the activity until the learners had completed the task. Include examples of the ways different groups approached the task. How did they work to solve the problem? Was there anything similar about the way different groups worked? What surprised you? Why?

Did you have any problems when you carried out this activity with your learners? Why? If you were to do this activity again, what would you do differently? What would you like to improve? How did you feel? Why? How do you think your learners felt? Why? Try to write a detailed two page report of what happened. Then store this safely in your Concertina File.

You will have an opportunity to share your experiences with the rest of your group at the next face-to-face session.



Activity 3 - Solving a Problem - Building a Square

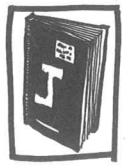
Option B

This is an activity suitable for Intermediate Phase learners. (Remember, that you may want to give your older learners a chance to try Option A first. The choice is yours.)

We have given you 10 squares with this umthamo for you to use in this activity. Cut up each square that you get into its five parts. Then put each set of five pieces into an envelope. Make sure that you take your Journal to class with you. Then you can make notes of what you observe your learners doing as they carry out the task. Watch their body language. What does it tell you? Try to notice what they do first? How do they work together?

Divide your learners into groups of about five or six. Give each group an envelope which has the five pieces. Ask each group to put the pieces together and make up a complete square.

The learners will have to work co-operatively in order to find a way to make a square. If each child holds a piece they can see which other pieces are likely to fit with their own piece.

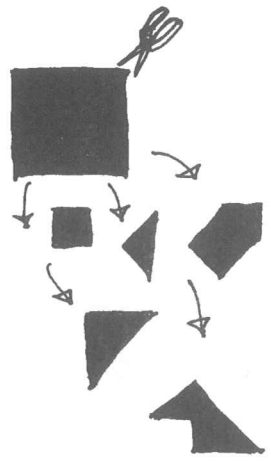


As your learners carry out this activity, unobtrusively observe how each child is working. Make brief notes in your Journal of your observations. Make sure that all the learners are participating in the group. They will need to discuss, to share their ideas and to help one another.

Judge how involved your learners are. When some groups (if not most, or all) have managed to make a square, stop all the groups. Chair a WHOLE group discussion. Get each group to describe the process that they went through in order to make a square out of the seven pieces.

Later, on the same day, sit down quietly with your Journal. Re-read the notes and observations that you made when you carried out this activity with your learners. This will remind you of what took place. Try to picture what happened. Your notes will help to jog your memory.

Then take a clean sheet of paper, and write, *Maths umthamo 2: Activity 3 - Option B.* Describe *exactly* what happened, from the time when you introduced the activity until the learners had completed the task. Include examples of the ways different groups approached the task.



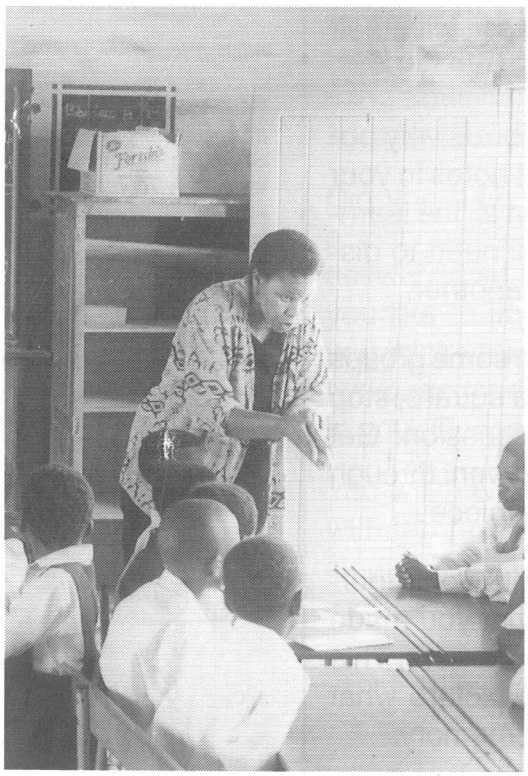
When we trialled this activity at Bulumko Higher Primary School in Mdantsane, we were really impressed at the way the groups of Grade 5 learners worked. Every child was involved. Every child concentrated. Every child wanted to put the pieces together to make a square. There was a lot of talking and very interesting body language.

How did they work to solve the problem? Was there anything similar about the way different groups worked? What surprised you? Why?

Did you have any problems when you carried out this activity with your learners? Why? If you were to do this activity again, what would you do differently? What would you like to improve? How did you feel? Why? How do you think your learners felt? Why? Try to write a detailed two page report of what happened. Then store this safely in your Concertina File.

In this activity the learners need to be able to co-operate. Each child depends on others to help supply the pieces which they need in order to complete the square. Without co-operation and collaboration, the activity would fail. Through this investigation, children need to work collaboratively by sharing ideas, and talking about shapes and angles.

You will have an opportunity to share your experiences with the rest of your group at the next face-to-face session.



Vuyelwa explains the problem carefully

The group struggles to solve the problem

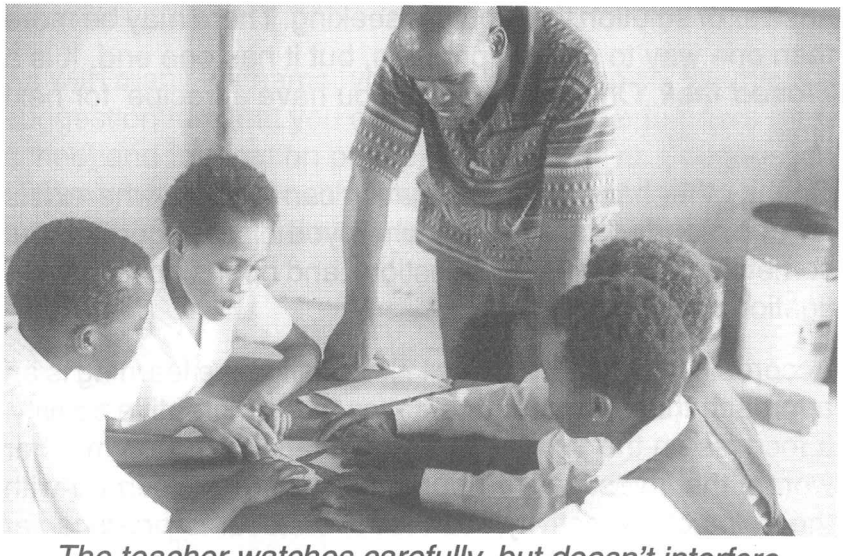
Thinking about Unit 1 - Reflection

One thing should be clear to you after having done the first part of the Key Activity. You will have seen that all learners come to the classroom with informal problem solving skills and methods. These are methods which they themselves have invented and discovered. These methods are based on their own conceptual understanding. They are powerful, personal and useful.

You should have noticed the quality and quantity of talk that is encouraged. You will have seen how easily and naturally a group interacts and works together on a problem. You should also have noticed just how much maths is dealt with informally, when learners have a problem to solve.

Pupils will have been **naming** and **identifying shapes**. They will be noticing and talking about the **properties of shapes**. They will have been manipulating the **position of shapes**. And they will have been using words to explain the **mathematical changes** they have made to **positions of shapes**. They will have **rotated shapes**, and **turned** them over. They will have **estimated** and **compared lengths**. They will have direct experience of **tessellation** (the way shapes fit together). They will experience that the **area** of a large shape is equal to the area of its parts. They may have thought about the **perimeter** of shapes.

You will have learned a great deal about the problem solving **process** and the kinds of steps and strategies learners use. When is trial and error useful? When does it pay to think logically? How do learners come to the solution? Is it a sudden "Aha!" experience? Does the solution emerge gradually? You will also have seen how easily **problem solving** leads to further **investigation**.



The teacher watches carefully, but doesn't interfere

Unit 2 - Moving into Investigation

In the previous Unit, you have seen how solving a challenging problem (**problem solving**) can easily lead to the process of further investigation (using an **investigative approach**). This can clearly be seen in Option A of Activity 2.

Solving a problem has captured the learners' interest. The learners are keen (motivated) to find out more for themselves. They have solved the problem of making a square from the 4 small triangles. Now it is easy to extend the maths work to involve investigation by asking them to see what other shapes they can make.

Let's spend a little time thinking about what the word ***investigate*** means. Investigation is a concept that is commonly used in every language. We use it in many different contexts. For example, when a car is stolen or a person is murdered, the police investigate these cases by asking people questions in connection with these crimes. They look for clues and seek information.

Another example is when you don't know the whereabouts of your child. You might ask around. Maybe you 'phone her friend. When you do this, you are actually investigating. So this concept is not new to us.

If your child's friend is unable to give you any information, you might try other sources like the police or local hospitals in your search, or investigation. So investigation is like being a detective, finding out, making guesses, trying to prove or confirm.

Problem solving and investigation are related activities. Let's think of the difference between them. When solving a problem you have a definite goal in mind. There is a specific answer or solution that you are seeking. There may be more than one way to solve a problem, but it has one end. It is a ***closed task***. Once it is solved, you have a 'recipe' for next time.

Remember closed questions versus open questions in Umthamo 12?

On the other hand, an investigation can lead anywhere. It is an **open-ended task**. Often when you discover or find out something it raises more questions and opens up the investigation further.

This links with the work we did in Umthamo 11.

According to Cockroft (1982:17), investigative learning is an approach that shows mathematics to be a creative activity. It focuses on the ***process*** more than the ***product***. In other words, the investigative approach is more concerned with the *doing* of the activity, than with whatever is *produced* at the *end* of the activity.

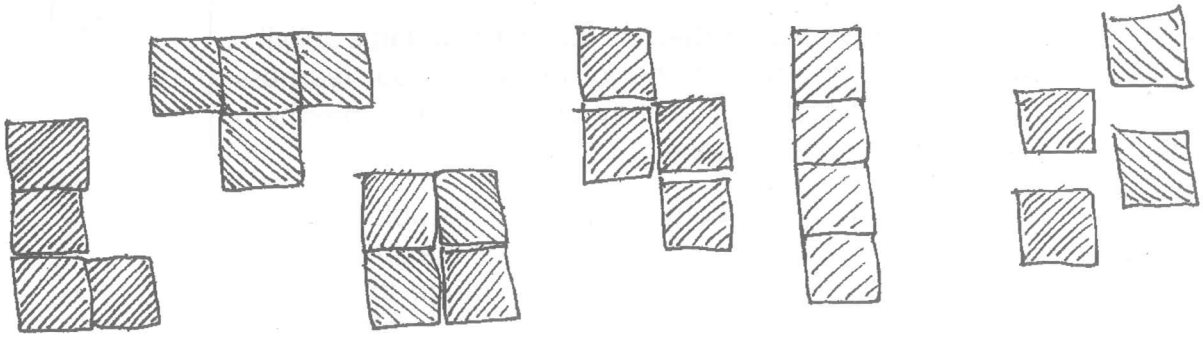
The **investigative approach** to learning aims at encouraging learners to be active participants and to develop their thinking skills. Learners have to think logically and make their own generalisations when they are investigating. This can happen, if teachers encourage their learners to develop ways of viewing and making sense of the real world. Teachers can do this by creating imaginative things for their learners to explore.

The big advantage about both these approaches, is that the learners have to ***think*** for themselves and ***do*** things for themselves. They have to be active participants. This kind of work will always be learner-centred. The learners have to make sense of things for themselves. If they discover or find ways to do things for themselves, then they will feel empowered. They will feel good about themselves.

Here is another strength of these approaches, and especially the investigative approach. They are well suited to allowing learners to work in groups. The advantage of this is that these experiences provide opportunities for them to talk about mathematics. There is co-operation that can come from a shared problem, and the encouragement of listening to and appraising the value of each other's contributions. There has to be 'give and take' in the interactions. By working in groups, the responsibility to work is shifted to members of the group. They have to decide what to investigate, and make plans and choose appropriate tools. They have to consider alternatives, and make choices. Children accept investigation as a challenge and become highly motivated.

The activity for this Unit has more of a focus on **investigating**. In keeping with the rest of the umthamo, we are still dealing with **shape** and **position**. Learners will investigate the way shapes can be put together, or the way shapes can be split.

As with many imithamo, you have some choice. We have a suggestion for what you can do with younger learners (preschool and foundation phase). We also have a suggestion for how you can work with older learners (intermediate phase, and perhaps first years of the senior phase).



When we trialled these activities, we were surprised to find that some were interchangeable. Older children enjoyed and were challenged by the work suggested for younger children. (Perhaps this was because the practical learner-centred approach was denied them in their earlier years?)

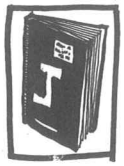
We also found that the younger children are not to be underestimated. If the problem or investigation is communicated at their level, then they can easily do things that you don't expect them to be able to do.

If you feel brave enough, you should feel free to try this yourself. Just make sure that you do not confuse the little ones, and put pressure on them to do something that they are not ready for. Be sensitive. Be guided by their questions, interest, and body language. At the same time, if you think the older learners are bored by a task, move on to something more challenging.

Activity 4 - An Investigation

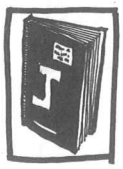
Option A - 4-square shapes (tetrominoes)

For this activity, you need sets of 4 small squares for each group. Squares with 10 cm sides are suitable. If you are not provided with squares, you will have to make your own. Make enough sets so that your children can work in small groups of 2 or 3 learners.



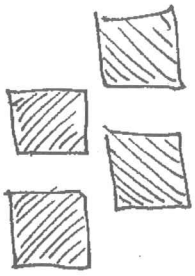
Make sure that you take your Journal to class with you. Then you can make notes of what you observe your learners doing as they carry out the task. Watch their body language. What does it tell you? Try to notice what they do first? How do they work together?

When you are with your learners, give each small group a set of 4 squares. Tell them that you want them to find as many different ways as they can to arrange their four squares. The squares must fit together, side to side.



Again observe the children as they carry out this activity. Make notes in your Journal of the ways your learners work, the strategies that they use, and how they help each other, or work alone.

As you watch your learners, you will be able to judge when to ask them to stop. When most of the groups have found at least 3 different ways to arrange their set of



squares, ask the groups to stop. Then get each group to share with the rest of the class one of the ways that they arranged their squares.

As the groups share the shapes that they have made, make the same arrangement with some squares of your own on the chalkboard. Stick each group of squares down with a small amount of prestik. This is an important part of the activity because you are taking what the children have made in front of them *flat* on their desks, and you are making a representation of that arrangement on the chalkboard, which is *vertical*.

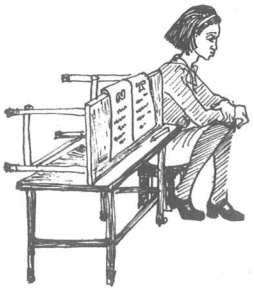
When all the different ways have been displayed, discuss with your learners whether some of the shapes are similar, or even the same. Don't force them to see what you see, or tell them what you have come to know. This is an opportunity for you to give them a chance to think, and to push their thinking a little further. It is better for them to discover something for themselves. When we find out something for ourselves, we are more likely to make it our own, and to remember it.

If you teach Grade 1 or 2 learners, and nobody comments on certain arrangements being the same, or even similar, we suggest that you may like to take two shapes that are in fact the same, but that look a little bit different. You can take 4 small squares in *one* colour, and 4 small squares in *a different* colour. Then take the one arrangement, and place it on top of the other arrangement. This will help your learners to see that some arrangements are in fact the same. The same shape is formed, but it is just in a different position (upside-down, or back-to-front, or lying on its side).

You can leave the activity at this point. At a later stage, many of your learners will try to prove for themselves whether some arrangements are the same or in fact different.

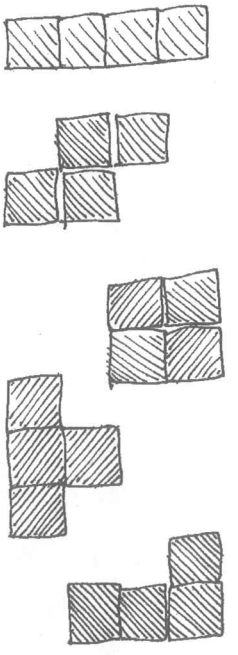
As your learners work at this task, observe what different groups do. Notice how they carry out this investigation and make notes of your observations in your Journal. Describe in detail what you would do in your Journal.

Later, on the same day, sit down quietly with your Journal. Re-read the notes and observations that you made when you carried out this activity with your learners. This will remind you of what took place. Try to picture what happened. Your notes will help to jog your memory.



When we trialled this activity at Dumisa, the Grade 1s noticed that many of the shapes that they had made were like the shapes of different letters of the alphabet. For example, the letter L and the letter T.

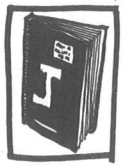
This will form a class record of what the children found out in their investigation. As a teacher, you are modelling an important aspect of investigation. You are modelling the systematic ***recording*** *of what has been found out.*

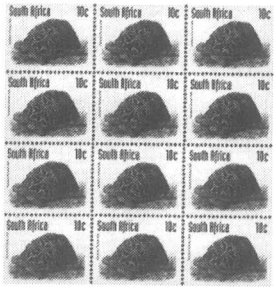


Then take a clean sheet of paper, and write, *Maths umthamo 2: Activity 4 - Option A.* Describe *exactly* what happened, from the time when you introduced the activity until the learners had completed the task. Include examples of the ways different groups approached the task. How did they work to solve the problem? Was there anything similar about the way different groups worked? What surprised you? Why?

Did you have any problems when you carried out this activity with your learners? Why? If you were to do this activity again, what would you do differently? What would you like to improve? How did you feel? Why? How do you think your learners felt? Why? Try to write a detailed two page report of what happened. Store this report safely in your Concertina File.

You will have a chance to share your experiences of this activity with other teacher-learners at the next face-to-face session.





Activity 4 - An Investigation

Option B

We have included 10 A4 size photocopies of a sheet of 10c stamps with this umthamo. There are twelve stamps on the sheet, and they are arranged in a three-by-four block. You will need a photocopy for each group of learners for this activity. If you have a big class you will have to make more copies. Each group will also need 4 old buttons, coins, beans, or pebbles for this task.

Make sure that you take your Journal to class with you. Then you can make notes of what you observe your learners doing as they carry out the task. Watch their body language. What does it tell you? Try to notice what they do first? How do they work together?

Task 1 - Ways to Separate Stamps.

First of all, hand out one copy of the twelve stamps to each group. Then tell your learners a short introductory story. You could tell them that there were two children, Nobantu and Vusumzi. Nobantu had a sheet of twelve stamps. Vusumzi needed 4 stamps, so he asked Nobantu if she would give them to him. Nobantu said she would, and she tore off 4 stamps, making sure that the remaining 8 stamps were still in one piece. She knew that if she had loose stamps, she could easily lose them.

Tell your learners you want them to find out how many ways there are to divide the stamps so that each set is in one piece (unbroken). Then hand out 4 buttons to each group and tell them that these can be used to mark off the 4 stamps for Vusumzi.

It might be a good idea to do one example on the chalkboard to make sure that all your learners are clear of the task, and how they can use the buttons to help them to solve the problem. Get one of your learners to help you.

Stick up one of the photocopies of the sheet of stamps. Then ask one of your learners to stick the four buttons on four of the stamps with prestik. The four stamps **must** be connected. And the remaining eight stamps **must not** be broken either.

Then ask your learners to find as many **different** ways as they can to divide the stamps without breaking either set. Tell them that while they are investigating, they must solve the problem of how to record the ways they have found.

As your learners work, move from group to group observing what your learners are doing. Don't feel you have to intervene straight away if a group is struggling. Let them try to work together to solve the problem.

As soon as a group has managed to find a way, give them a piece of rough paper to record the shapes of the two sets of stamps. You may need to help your learners with this step. You could do this with each group, or you could do another demonstration on the chalkboard. Encourage them to solve the problem of how to record their findings in an accurate and systematic way.

We found that when we tried this activity at Bulumko Higher Primary with Grade 5 learners, this took quite some time. It was not an easy thing to record and draw the shapes that they had made. But it is a very important part of problem solving and investigative work to record what we do, as well as our findings.

Task 2 - Extending the Activity

You can extend this activity by asking your learners how many ways they could divide the stamps, if the 12 stamps were originally in a two-by-six block. Are there more ways? Or are there less? Why is this so?

Then you could try varying the numbers. What would happen if Vusumzi needed 5 stamps?

You could either set these follow-up problems in the same lesson, or in another one. It will depend very much on your learners and what takes place when they carry out this activity.

As your learners work at this task, observe what different groups do. Notice how they carry out this investigation and make notes of your observations in your Journal.

Later, on the same day, sit down quietly with your Journal. Re-read the notes and observations that you made when you carried out this activity with your learners. This will remind you of what took place. Try to picture what happened. Your notes will help to jog your memory.

Then take a clean sheet of paper, and write, *Maths umthamo 2: Activity 4 - Option B.* Describe *exactly* what happened, from the time when you introduced the activity until the learners had completed the task. Include examples of the ways different groups approached the task. How did they work to solve the problem? Was there anything similar about the way different groups worked? What surprised you? Why?

Did you have any problems when you carried out this activity with your learners? Why? If you were to do this activity again, what would you do differently? What would you like to improve? How did you feel? Why? How do

you think your learners felt? Why? Try to write a detailed two page report of what happened. Store this safely in your Concertina File.

You will have a chance to share your experiences of this activity with other teacher-learners at the next face-to-face session.

Thinking about Unit 2 - Reflection

Since investigation is a form of discovery, you will have seen that your learners will be able to use their own methods to solve the problem. They will find out things for themselves through investigation. You will also have seen that your role is to encourage your learners to explore and explain their own strategies in order to develop mathematical thinking.

You will have encouraged them to record the results of their investigation in some way. They will have had a chance to share and compare their findings and ideas with others. Considering the work of others in comparison to your own, is essential for learners to develop into critical thinkers.

Again, you will see that a great deal of mathematical talk and thinking has happened informally. Younger learners will have been **constructing shapes** and thinking more about **similarities** and **differences**. Older learners will have been looking for shapes **imbedded** in other shapes. They will have been thinking carefully of how shapes fit together (**tessellate**). They may also have informal experience of the relationship between **shape**, **area**, and **perimeter**.

You will also have seen how a teacher of younger children can model the **recording process** in **investigation**. If you work with older learners, you will have seen how older children **solve** the **problem** of **recording**. How can they systematically note what they have found out through **investigation**?

A shortcoming of the approach in the past was that we seldom asked children to think of things for themselves. They usually copied or repeated exactly what had been explained or done first by the teacher. The emphasis was on **reproductive** work. The teacher copied (reproduced) from the textbook, and the learners copied (reproduced) from the teacher.

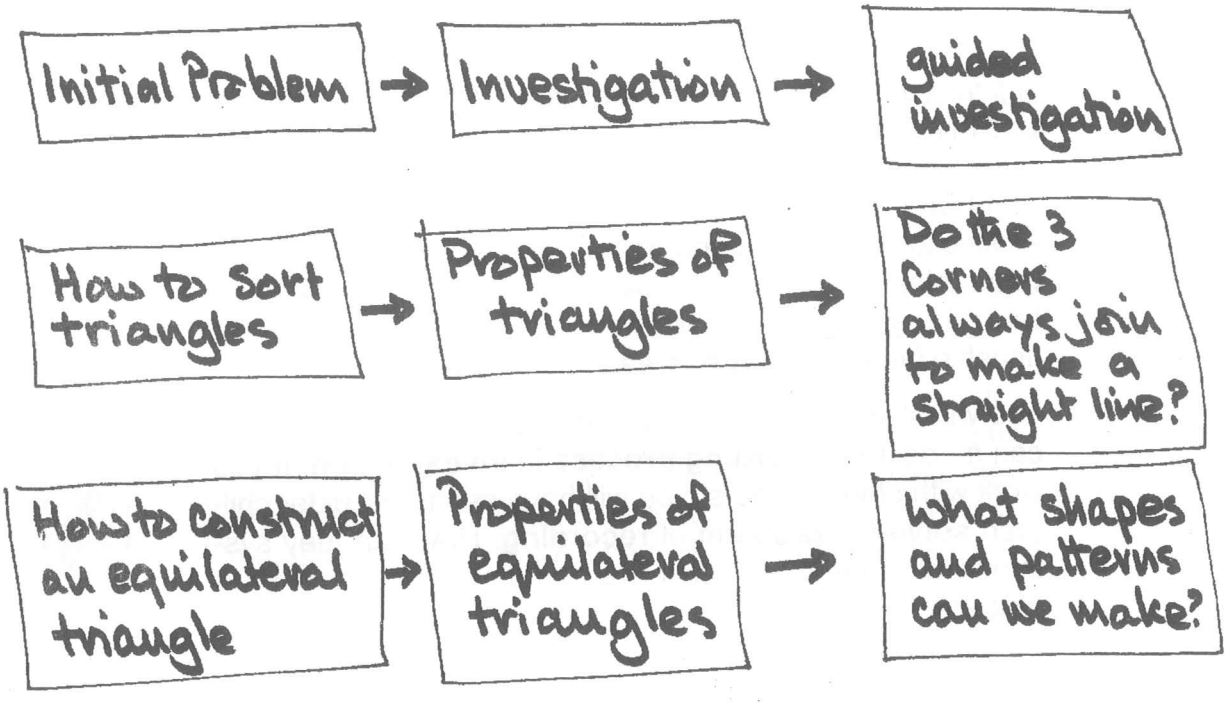
If children are asked and trusted to work things out for themselves, and they show what they can do, then the emphasis shifts to **productive** work. When learners are challenged to reveal or show what they are capable of doing, then we have a true **outcomes based approach**.

Unit 3 - Using and Applying Problem Solving and Investigation

In the Introduction, you read about different approaches to maths work recommended in the Cockroft Report. The fourth point referred to **appropriate practical work**. The Cockroft Report points out that all learners need frequent opportunities to do *appropriate practical work*. We need to provide activities for our learners, which require them to solve problems, and carry out investigations ***practically***. You can't solve problems and do investigations without doing things practically. So the three approaches harmonise.

In the activity which follows for this Unit, we will give you guidance for a task where your learners have **an initial specific problem** to solve. This then leads or extends into **an investigation**. We will also show you ways to guide a **further investigation** that helps or leads learners to find out specific things.

A version of the investigative approach would be a guided investigation where the teacher asks certain key questions, or suggests a certain approach which leads the investigation to a specific result.



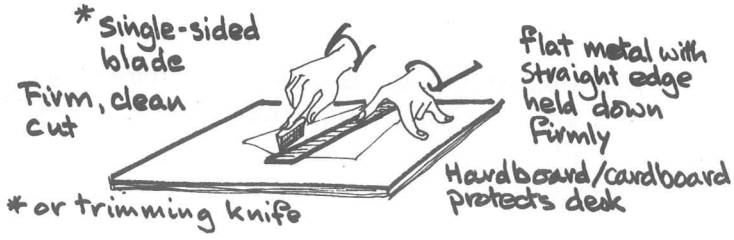
The suggested activity for the younger learners is an investigation of triangles in general. In the writers' experience, when young children are at school, they are only asked, or expected to be able to identify and name a triangle. This is not very challenging. Many two- or three-year-olds can do this long before pre-school or Grade 1. Perhaps they can find out more about triangles if we give them the chance to investigate triangles practically.

Introduction to Activity 5 - Option A

Before you start

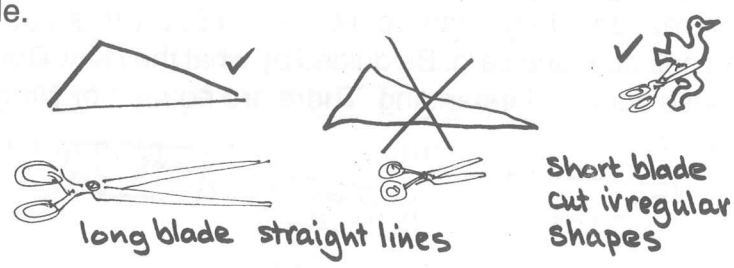
This activity will take a little preparation. You will have to carefully make sets of different triangles for each group in your class. You can use scrap paper, or old magazines. Some of the triangles can be cut from used card, or old stiff plastic. But, in fact, it is better if some of the triangles are made from scrap paper. Then your children will free to fold and even tear triangles as they investigate.

If you want to be able to improvise and make your own material for practical maths work, you need certain essential equipment. You need to be able to cut paper or card so that the shapes you cut are neat and precise.



The best kind of ruler to use to cut a straight line, is a metal ruler. (You could also us a flat piece of metal with a straight edge.) Then you need a cutting knife (a Stanley knife is good and costs about R34-00 at a stationery shop or a hardware shop), or else a one-edged blade (which costs about R1-50 at a chemist). A snap blade cutter costs about R10-00. Replacement blades cost R4-50.

You also need a flat piece of cardboard or scrap hardboard as a surface to cut on so that you don't cut into your desk or table.



Using scissors is the next best way. But long blade scissors are better for cutting a straight line in a precise way. Short blade scissors don't cut a good straight line. Short blade scissors are good for cutting out irregular shapes.

Format for making sets of triangles

For each group you will need one A4 sized sheet of paper or card, to cut out the assortment of triangles. Your umkhwezeli will show you how to do this at the face-to-face session where this umthamo is monitored. Otherwise, follow the set of instructions on the last page of the Appendix (page 40).

You also need to make a set of small self-standing label cards for each group. You need 4 or 5 small rectangles of stiff card for each group. If you fold the rectangle in half, length-wise, it will stand easily and you can write a label on one side.

Activity 5 - Finding out about Triangles

Option A

Task 1 - A Problem - Sorting Triangles

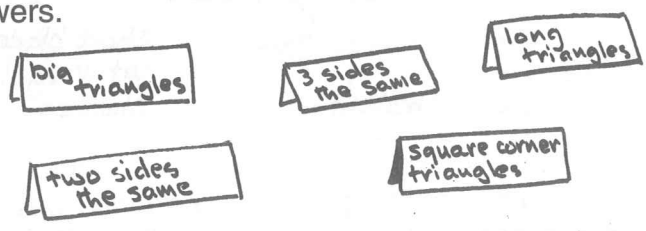
Group your children. Explain that each group will be given a set of triangles. Their first task will be to **sort** the triangles carefully into separate groups. They must have reasons for the way they sort the triangles. Hand out the sets of triangles, and leave the children to get on with the sorting.

This is your opportunity to observe your children as they work in groups. You can then assess them informally as a way to get to know them better. You can assess their participation and interaction with others. You can assess their initiative and their mathematical skills. Or, you can also assess their communication skills.

If you decide to note down and record some of your observations, then that is the first stage in formalising continuous assessment.

Don't interfere, or interact with any of the groups until they have completed the task. If they are not clear, or if they have a problem, they can invite you to help them.

As a group finishes, go to them and ask them to explain the way they have sorted their triangles. When they give their reasons, help them to label each set with a name on a self-standing card. Be guided by what they say. Don't impose your understanding. There are no right or wrong answers.



When a group has given you their reasons, tell the group that they can investigate to see what shapes can be made with the different sets of triangles that they have. This will give them something to do as you move on to work with other groups.

When all have finished, you can either go on to the next task, or you can conduct a brief discussion of the different ways triangles can be sorted.

For Grade 2 or 3, the learners could draw the sets of triangles in their maths books, as a record of their practical work.

Task 2 - Investigating Triangles

(This task can follow on from the previous activity, or you can set it for your learners on another day.)

Tell your learners that their next task is to look at all the triangles again to see what they can find out or discover for themselves about triangles.

- What things are the same for all triangles?
- What things are special about some kinds of triangles?

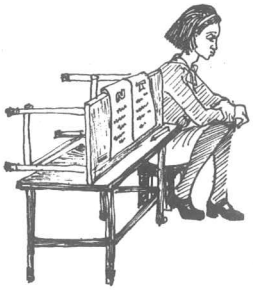
Encourage them to share and discuss what they see.

Give them ten minutes. If any groups seem stuck, or quiet, you may want to give them some guiding points. For example,

- Can all triangles fold in half?
- Can they find any triangles with the same shape, but a different size?
- Are there any triangles with all sides equal?

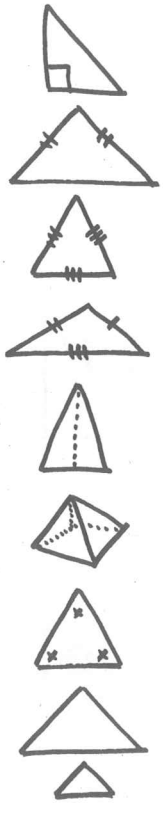
After about ten minutes, conduct a whole class discussion. As the learners share what they have noticed and found, record what they say about triangles on a large of sheet of paper, or on the chalkboard. Sort their observations into two columns.

If you want to, you can finish off by letting your learners come up and use prestik to stick an example next to each point for the '*Some triangles*' column. You could also stick large triangles next to the other column to mark the 3 sides (1, 2, and 3), or the 3 angles (A, B, and C).



Investigating Triangles

What we found out about Triangles	
All triangles	**Some triangles**
3 sides	fit a square
3 points/angles	2 sides the same
flat shapes	3 sides the same
(and so on)	no sides the same
	fold in half
	join to make a pyramid
	3 points the same size
	same shape, different size
	(and so on)



Task 3 - Guided Investigation

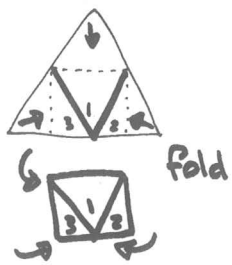
This can also be done on another day. It depends on the time you have available to you, and how 'fresh' and interested your class is. You may also decide to do this with a group of learners on its own, while others in the class do other work.

Ask the learners to find the triangles with all sides of the same length. Ask them, "If the sides are the same, are the angles also the same?" Then let them investigate. By placing the angles on top of each other, they should discover that no matter what size these special triangles are, all angles are exactly the same size. They fit on top of each other.

Now ask the learners to take one triangle, and tear off the three corners or angles. Tell them to see what they find, if they fit the three angles together. What is formed?

A straight line is formed. The three angles make a straight line.

There is another way that this can be done without tearing. You can take a triangle and fold down the top corner (the apex) so that it just touches the base. Then you can fold in each of the other corners to meet at the same point. The three angles join neatly on the straight line of the base side.



Now ask the children to investigate other kinds of triangles to see if the same thing happens, or not. They can tear or fold (or even trace around the angle).

Finish off by telling your learners that the only important thing is that they should be able to recognise and name triangles. The other things that they have found about triangles, they will use later in school.

Later, on the same day, sit down quietly and think about what happened in this activity. Then take a clean sheet of paper and write, *Maths umthamo 2: Activity 5 - Option A.* Describe exactly what happened, just as you did for Activity 3 and Activity 4. Store this report safely with the other two reports in your Concertinal File. Make sure that you include samples of your learners' work.

At the next face-to-face session, you will have an opportunity to compare what happened with your learners, and what happened in your colleagues' classes.

Introduction to Activity 5 - Option B

At some stage in school maths, learners are taught how to **construct**, or draw, specific shapes. The skill or ability to **construct** or draw accurate shapes and diagrams is an important one. It is needed in professions like architecture, draughtsmanship, cartography, engineering, design, technology, dress-making, surveying, carpentry, building, and so on.

Usually learners are taught step-by-step in a systematic way when they are older. But when you learn like this, it is very mechanical and the purpose for doing something is not always clear. Children do things because they are told to by the teacher. But they do not know why. The task doesn't always make sense. The passive learners do well, and the independent learners are frustrated. Learners lose interest and "vote with their minds". They switch off, fail, or drop out.

Think of the Language, Literacy & Communication (LLC) umthamo, Umthamo number 9. It dealt with a *Whole Language Approach*. Perhaps a *whole maths* approach is also possible. Perhaps if we give learners a construction problem to solve, they will be challenged. They will try to work out what is required. The may find out things for themselves. The task will certainly be purposeful and challenging. There is less chance that learners will lose interest.

In this activity with older learners, we will give them an opportunity to try and work out for themselves how to draw a special type of triangle - an equilateral triangle. Their **problem** will be to find an accurate way to draw such a triangle. Then they can **investigate** and compare different ways to **construct** an equilateral triangle. Finally, they will **investigate** the properties of equilateral triangles with you guiding certain parts of the investigation.

Reporting what we found out

Activity 5 - Working with Triangles
Option B

Task 1 - Problem - How to draw an Equilateral Triangle

If your pupils are in the Intermediate or Senior Phase, see that each group has a ruler, scissors, sellotape, pens and pencils, an eraser (rubber), paper and thin cardboard. If geometry sets are available, they can make use of them. Let them work out for themselves how to use the set squares, protractor and compass or dividers at this stage.

Tell your learners that you want them to construct (make or draw) an equilateral triangle. Encourage the children to talk and to discuss different ways that they could do this, before they try to solve the problem practically. Get them to jot down the steps or method they plan to use.

When a group is ready to start, remind them that they must note all their steps so that they can explain clearly to others how they **solved the problem**. Don't interfere with their ideas. Give them the freedom to make their own mistakes. We **learn** from our mistakes.

When they have made their equilateral triangles, tell the children to make a list of all the ways they used maths when they made their triangle. This is a very challenging task, and you may need to allow your learners some time for this.

Then get children from at least three different groups to describe to the rest of the class what they did. Tell your learners that you want them to describe ***how*** they made their equilateral triangle, and all the ways that they used maths. It might be a good idea to record these reports on a cassette tape on your tape-recorder. It may even be possible to ask each group to do this privately during Break.

As they give the ways that they used maths, make a mind-map on the board which organises and shows all the ways that they have used maths. They may talk about measuring length, measuring angles, counting, and so on.

Task 2 - To investigate the properties of equilateral triangles

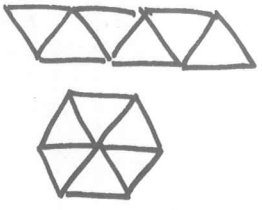
After the previous task, or on another day, give your learners the task of making 6 identical equilateral triangles. They now know how to draw them. Let them use any method to construct and cut out the triangles.

Now let the groups investigate the properties of equilateral triangles. They may have found out some properties when they did the first task. (For example they may have found that the sides are equal, the angles are equal, the angles are always 60°, and that the sum of the three angles is 180°.) Now tell them to investigate to see if they can find out any more properties (facts or things that can be said about equilateral triangles).

In the same way that you made a mind-map for Task 1, let the learners help you make a class mind map of the properties of equilateral triangles.

Task 3 - Guided investigation - Tessellation

It is unlikely that the learners will have thought about the ***shape*** we get when we fit equilateral triangles together. This means that the teacher will need to guide further investigation by asking specific questions. Ask your learners to see what shapes they can make by joining their triangles together. Ask what pattern they see if they join the triangles in a line.



Did your learners make a hexagon? Did they notice the zig-zag pattern?

As an open task, ask learners to find examples of equilateral triangles in real life. Tell them to look everywhere to see where they notice equilateral triangles.

Later, on the same day, sit down quietly and think about what happened in this activity. Then take a clean sheet of paper and write, *Maths umthamo 2: Activity 5 - Option A.* Describe exactly what happened, just as you did for Activity 3 and Activity 4. Store this report safely with the other two reports in your Concertinal File. Make sure that you include samples of your learners' work.

At the next face-to-face session, you will have an opportunity to compare what happened with your learners, and what happened in your colleagues' classes.

Thinking about Unit 3 - Reflection

Investigative learning and problem solving encourage learners to participate actively when they do practical work. They participate in the classroom by taking responsibility for their own learning. They construct and tackle their own questions by *organising*, *sharing* and *discussing* their own thoughts and their own search for solutions. This promotes independent thinking and self-reliance as well as willingness to tackle real life problems.

Investigative learning is fundamental to both the study of mathematics, and to the understanding of ways in which mathematics can be used to extend knowledge and solve problems. An important point to make is that investigational work lies not in ***getting the answer***, but in ***the process*** that is gone through in order to arrive at a conclusion.

The approach to investigative learning is a form of discovery. Learners are able to define their problems, set down procedures, and try to solve them. This involves thinking about the problem, making predictions and planning to test them. All this enables the learner to recognise mathematical patterns and eventually to show why something works out as it does. That is, being able to prove something.

When we set our pupils problems to solve and tasks to provoke their thinking, the way we organise the activity or task is very important. If we are not properly organised, and if our instructions are not clear, it is more than likely that the experience will be frustrating. We also need to be prepared to 'push' our learner's thinking. We need to persuade them not to be satisfied with just one answer, or one way to solve a problem, but to look for others.

Our record of our group's investigation

Conclusion

To complete your work on this umthamo, you need to write the last page of your report. We want you to write a brief, one-page summary to explain what you feel you have gained or learned from working through this umthamo.

Activity 6 - What I have gained from this umthamo

One page gives you space for three or four paragraphs. You need to think what those paragraphs could be about. You could plan this page with a mind-map. Put the question, 'What do I want to say?' in the middle. Then think about *topic sentences* for paragraphs.

- Do you want to tell us about your relationship with your learners?
- What are your feelings about problem solving and investigation in mathematics learning and teaching?
- Do you want to write about your own attitude to Maths?
- Did this umthamo give you any special problems that you would like to share?
- If you were to repeat this activity with another group of learners, what would you do differently? Why?

Add any other questions that you can think of. Narrow down the list to three or four questions that are most important to you. The answer to any of these questions can then be the *topic sentence* for a paragraph that you write in the final page of your report. Once you have a topic sentence, it is easy to add other sentences to support the main idea in the topic sentence.

Finally, you need to group your paragraphs in a way that makes logical sense. Then you might need to think of linking sentences. An example of a linking sentence might be.....*Although my learners seem to have gained confidence, I still don't feel very sure of myself when it comes to Maths.*

Then put this one-page summary together with the other three reports that you have written on your experiences carrying out the activities with your learners. Store this work safely in your Concertina File.

Make sure that you attach some examples of your learners' work. Include examples of their investigations and problem solving, as well as a record of some of the things that they said in their discussions. You may want to include some of your rough notes and observations.

At the face-to-face session where this umthamo is concluded, you will have an opportunity to share your experiences with other teacher-learners. You will be able to compare what has happened in your classes, and what you have learned.

Throughout this umthamo, we have made use of practical work on **shape** and **position**. We have done this because we believe practical work is one of the most important aspects in the learning and teaching of mathematics.

We have encouraged our learners to be **problem solvers**, and **investigators**. In this way, they are able to demonstrate the relationship between what they (as learners) already know and can do (skills), with what is being dealt with in the maths classroom. Remember what you read on pages 7 and 8 of Umthamo 10. There, children showed that they had skills and maths knowledge in real life. But they could not use those skills and that knowledge in the classroom setting. The classroom setting did not relate to real life. Those Brazilian children could not transfer their skills from life to the Mathematics classroom. What can a teacher do to make this transfer possible?

Mathematics should be enjoyed and understood. It should be a means of logical and clear communication of ideas. It should be a preparation for the maths that children do naturally in their daily lives. Maths at school should translate easily into the maths needed for later life. In life we need to solve problems, and we need to find things out. Maths should build children's confidence to face and solve problems and help them to be resourceful in finding out what they need to know.

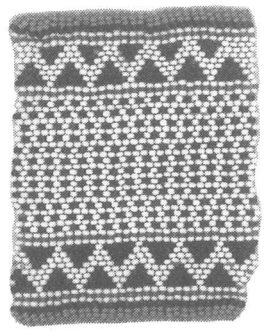
Reflecting on the role of the teacher

When maths work is **practical**, and it involves **problem solving** and **investigation**, then the teacher has a special role and special responsibilities.

By mathematisation of their lives, we mean that children will be alert to the mathematics in their world. Their eyes and minds will be open to maths in the world. They won't be blind and ignorant of the maths around them.

Children learning maths, need continuing **encouragement**, **support** and **guidance** from their teachers. As teachers, we need to encourage our learners to stop and think about what they are learning and its relevance to them. *We need to sow the seeds that can help the "mathematisation" of children's lives.*

What approach do you think we should use as teachers? Perhaps we need to start from the ideas that children have and ask, "*How can we help the children to develop their own ideas and their own thinking?*"



This means that we have to find out what their ideas are. Then we need to show them different ways of looking at their own world. We need to give them opportunities to discover different ways of ***patterning their experiences***. That means making sense of, or finding meaning in what they do. From a mathematics point of view, one way to make sense of their world, is to discover different ways of 'patterning in experiences' using number and shape. We need to provide opportunities for them to use different aspects of their intelligence thoughtfully.

We need to organise activities in which learners do much of the ***talking*** and ***doing***, often in small groups without the teacher. We should watch our learners' actions, and listen carefully to their discussions. We need to pay attention to their arguments and explanations. Then we will begin to understand what sense they are making of the activities that they are involved in, and the problems that they are solving.

The main feature of our maths classrooms is the focus on children's thinking. Maths classrooms should emphasise the exchange of ideas between two or more persons. All children in class, need to use their different experiences in constructing their ideas. And, they need opportunities to find and invent patterns that explain these experiences. Each classroom should enable learners to use many different ways of representing ideas so that good flexible maths thinking can emerge.

Activity 7 - Investigative and Problem Solving Approaches

You will carry out this activity at the face-to-face session where this umthamo is concluded.

In a small group, discuss what you have learned about investigative and problem solving approaches to learning of mathematics. Compare this with the way you have taught maths in the past.

Now open your Journal. Write the date and time. Write down some of your thoughts and feelings about this umthamo. Is there anything you would like to change about the umthamo? Why? Make some suggestions of how we could improve it. You can pass these suggestions on to your umkhwezeli or your Centre Co-ordinator. Then s/he can pass it on to us.

Appendix

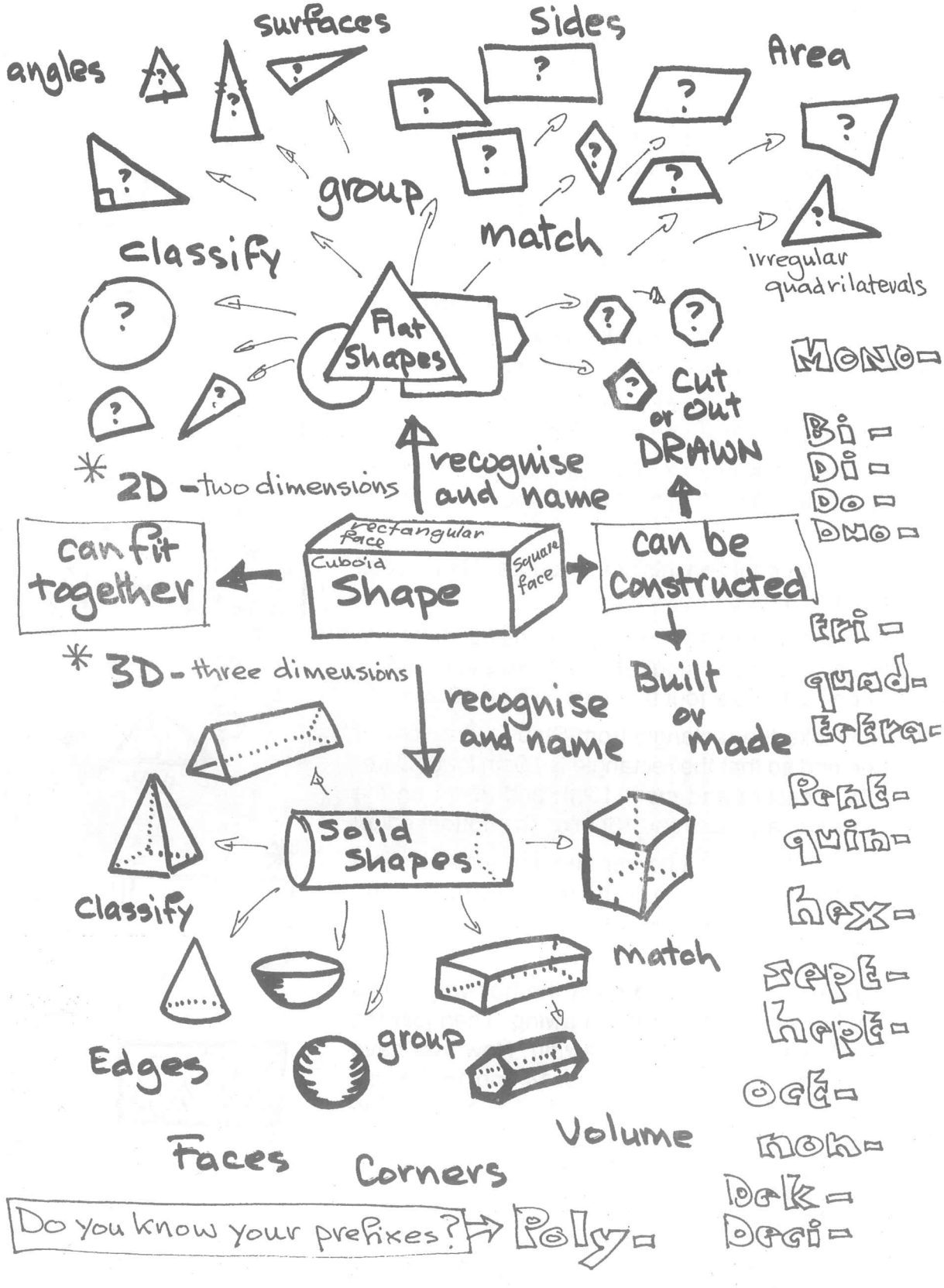
Content Audit

The topics in this umthamo are related and fall under Geometry in maths. Other aspects like symmetry, size and measurement (length, area, volume, angles, and so on) will come into other imithamo.

We have been using mind-maps to help you do your own audit to check your content knowledge. Go carefully through this mind-map. Does it make sense to you? Mark things that you don't know about, or that you are not so sure of. Then it is up to you to try and find out about these things.

Remember, there are photocopies of articles and related material, as well as books in a Resource Box for this umthamo at your Centre. So, you can help yourself.

Remember, two heads are better than one. Work with other teacher-learners to make sense of this content. In the end, 'the dumb learn nothing'. Speak up. Ask for help, if you are still stuck.

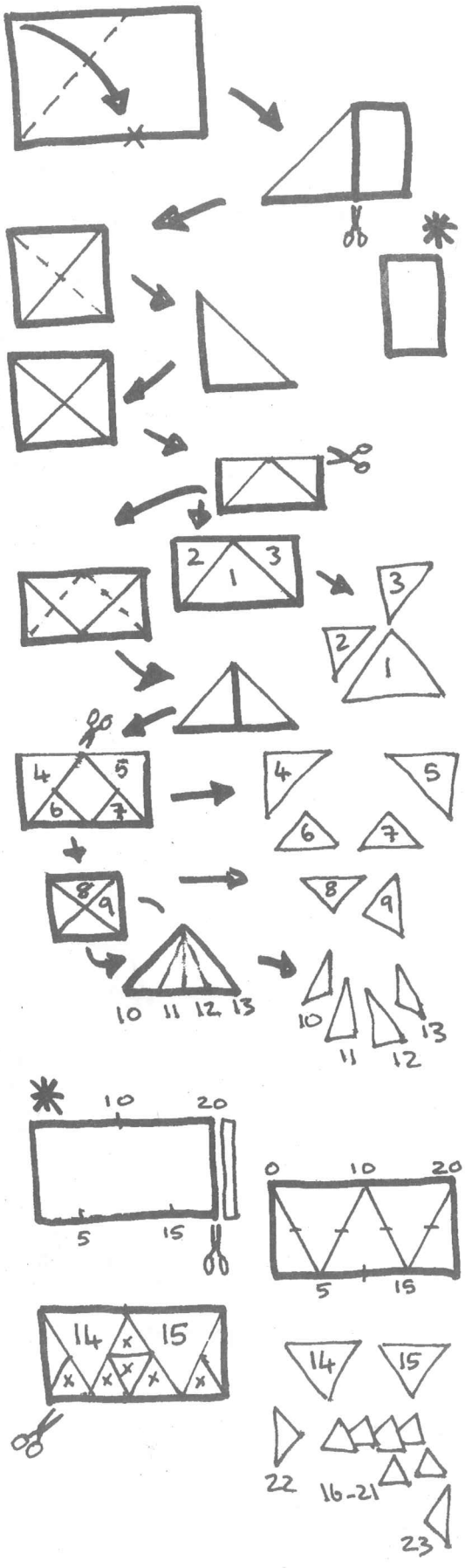

angles
surfaces
Sides
Area
group
match
classify
irregular quadrilaterals
Flat Shapes
cut or out
DRAWN
recognise and name
* 2D - two dimensions
MONO-
Bi -
Di -
Do -
DUO -
can fit together
rectangular face
Cuboid
Shape
square face
can be Constructed
tri -
quad -
Built or made
extra -
* 3D - three dimensions
recognise and name
Penta -
quin -
Solid Shapes
hex -
Classify
match
rept -
hept -
oct -
Edges
group
non -
Volume
Faces
Corners
Dek -
Deci -
Do you know your prefixes?
Poly -

Instructions for making materials for Activity 5 - Option A

Start with an A4 sheet of paper or card.

1. Fold the top left corner down and across towards you. You get a right-angled triangle, and a rectangle.
2. Carefully cut off the rectangle and keep it separate.
3. Open up the triangle to see a square with a diagonal crease. Fold the square the other way so that you make a second diagonal crease.
4. Open up the triangle to see a square with two diagonal creases that cross in the middle. Now fold the square in half to form a rectangle.
5. Open up the rectangle. Cut off the bottom half, and then cut it up to get triangles 1, 2, and 3.
6. Take the top rectangle, and fold both of the top corners down to meet at the centre of the bottom. This makes the shape of a triangle.
7. Open up the triangle. Cut off triangles 4 and 5, and then 6 and 7. This leaves you with a small square.
8. Draw diagonals on this small square. Then cut off triangles 8 and 9.
9. With the remaining two joined triangles, draw one line across each triangle from the base to the apex. Then cut the last four triangles, 10, 11, 12, and 13.
10. Now take the rectangle from Step 2. Cut 1cm off one end so that the rectangle is 20cm long. Carefully measure and mark 10cm and 20cm on the top edge, and 5cm and 15cm on the bottom edge.
11. Start at the top left corner and use a ruler to join the points 0cm to 5cm, to 10cm, to 15cm, to 20cm. You get a zig-zag line and see 3 big equilateral triangles.
12. Carefully measure and mark the half-way points on the lines shown in the drawing. Then join the points as shown in the drawing. Now you have two large equilateral triangles, 14 and 15, and six smaller equilateral triangles, 16 - 21. At each end you have an isosceles triangle (22 and 23) Cut out these triangles.

You now have a set of assorted triangles for sorting. Store them safely in an envelope.



UNIVERSITY OF FORT HARE
DISTANCE EDUCATION PROJECT

CORE LEARNING AREAS CORE COURSE

Mathematical Literacy, Mathematics and Mathematical Science

Umthamo 2 - Problem Solving and Investigating

First Pilot Edition - 1999

Vuyelwa Tokwe
Blanche Nomfanelo Bolosha

Co-ordinated, illustrated
and andedited by
Alan & Viv Kenyon

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