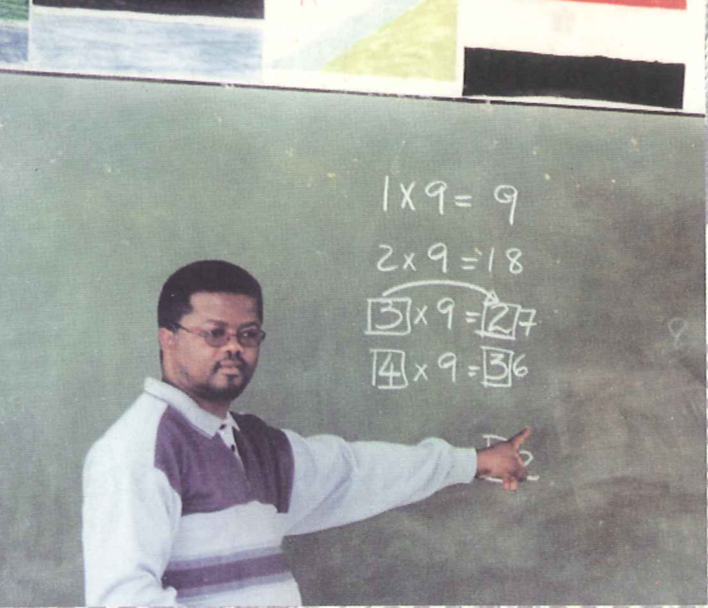

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UNIVERSITY OF FORT HARE

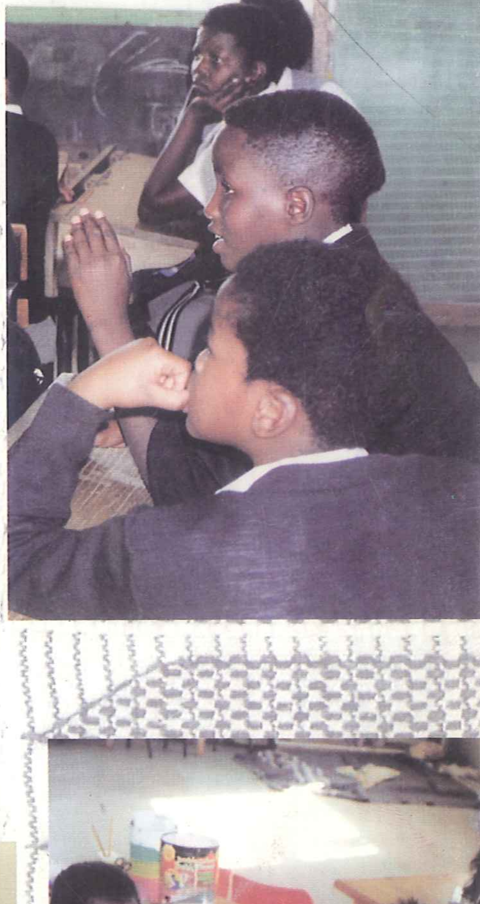
Eastern Cape Education Department

Distance Education Project

Core Learning Areas Course

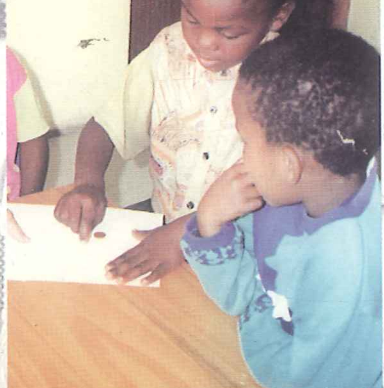
Mathematical Literacy, Mathematics and Mathematical Science

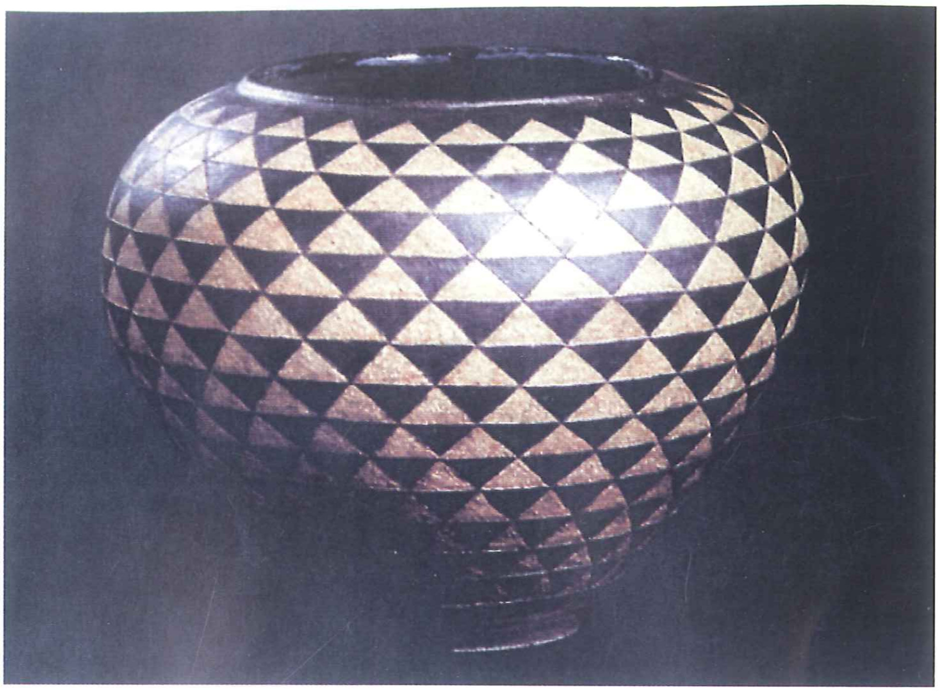
4th Umthamo



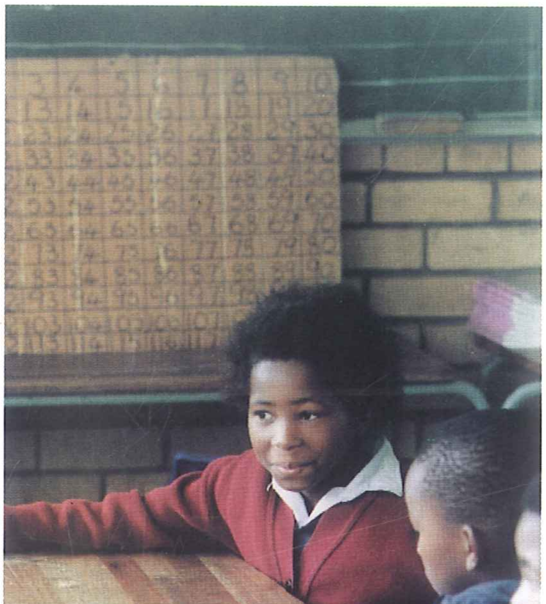
Developing Mathematical Thinking using Patterns

(Pilot Edition - June 2000)





Eric Mswane and David Veniphi of Ikhwezi Lokusa



"Instead of studying spirals and other shapes in the abstract, we can root them in the world around us - not only in nature, but also in the intricate designs of our own traditional weaving and beadwork.

Why not analyse these patterns mathematically, and write software that regenerates them using a computer, I thought? That way we could integrate mathematics with the rest of human experience, instead of putting it into a separate compartment. And we could demonstrate tangibly, how mathematics is core to everything we experience. It's a way of making Maths accessible.

Go to Hillbrow, and you'll see people taking apart multi-coloured PVC-covered copper wire, and spending many hours recycling the strands into the most exquisite baskets - often into traditional patterns that unconsciously incorporate complicated mathematical concepts of symmetry.

Chonat Getz in 'The Science of Izimbenge' in *The Mail & Guardian.* Jan 7-13 2000.

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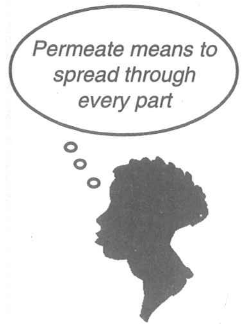
Contents

Introduction

Welcome to the fourth mthamo in the Maths strand of the B Prim Ed course. This mthamo is designed to consolidate the topics of the previous Maths mithamo. It highlights the value of **patterns** to develop mathematical thinking. **Patterns** permeate our lives, and it is useful to explore them with our learners as we study mathematics together.



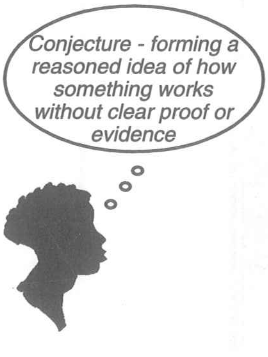
The International Mathematical Union has declared the year 2 000 to be the World Mathematical Year. This is to encourage the promotion of mathematics around the world, at all levels.

We live in a highly technological world. Computer technology is increasingly important for the 21st century (particularly in the developing world). This means that we need to get rid of the incorrect image of mathematics as "too difficult". As teachers, we need to be involved in making mathematics enjoyable, accessible, as well as popular. One way to do this is to explore mathematics in our South African plants, arts, crafts, architecture, stories, dance and games. We hope that this mthamo will prepare you to recognise and use these **patterns** with your learners. And we hope you and your learners will get excited as you make mathematical **conjectures** about them.

We wish you a great journey through this mthamo!!

What you will find in this mthamo

This mthamo demonstrates how we can use **patterns** as a mathematical resource or tool to help our learners develop mathematical thinking processes. It proposes that teachers should be aware that **patterns** are not necessarily designed for the mathematics classroom. We have to find ways of using them, so that learners are excited and challenged by mathematics. This would allow learners to manipulate geometric and numerical **patterns,** in order to develop mathematical language, and develops learners' awareness that:



- mathematics is a human activity,
- people have to make **conjectures**,
- these **conjectures** can be contested. They can be refuted, or accepted, or extended.

There are 4 Units in this mthamo. In Unit 1 we think about what we understand by the term "good practice" in Maths teaching, and what is involved in teaching Maths. This unit contains background information about Mathematics teaching. You may need to read it, and return to re-read it more carefully later.

In Unit 2 we think about what we understand by the term **pattern**. We have included a Reading from a book about using **patterns** in primary classrooms. You will find two activities for you to do as you work through that Reading.

In Unit 3 we look at how **pattern** comes into different areas of the curriculum. We consider some activities that teachers can carry out with their learners, in order to provide them with rich experiences of **pattern**.

The **Key Activity** is in Unit 4. There are 3 Options, and you need to complete at least **one**. You will need to decide which Option is most appropriate for **your** learners. But we have designed Option A for learners in Early Childhood settings, Option B for learners in the Foundation Phase, and Option C for Intermediate Phase learners.

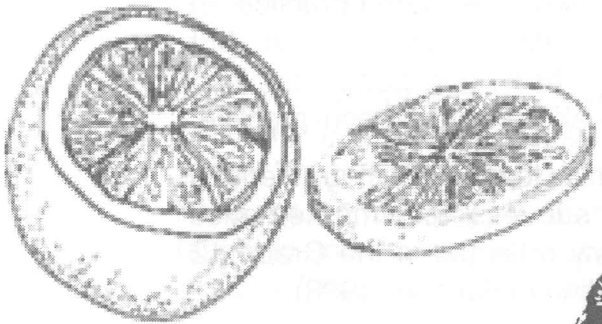
What are the intended outcomes?

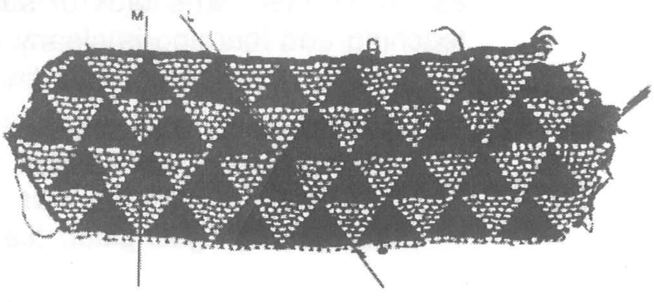
When you have worked through this mthamo, you will have

- thought about the importance of **patterns** for everyday life and the curriculum
- thought about how we can help learners develop their mathematical thinking and understanding
- planned, prepared, carried out, and reflected on an Activity which gives your learners an experience of pattern in Maths, and which encourages them to **conjecture**.

Read and think about the following questions. The writer found them useful as he developed this mthamo. You might also find them useful.

1. What **skills, values, knowledge** and **attitudes** would you like your learners to have after completing this mthamo?
2. What **mathematical concepts** would you like to have covered through using **patterns**?
3. Why do you think the use of **patterns** assists us as we teach and learn?





Unit 1 - Some thoughts around 'good practice' in Mathematics teaching

This unit contains background information about Mathematics teaching. You may need to read it, and return to re-read it more carefully later.

The changing face of South African education has generated serious debate over the past years. The needs of a country pledged and committed to reform and economic growth, has meant that the government has had to embark on a radical transformation of the education system. This is necessary in order for us to meet the demands of competing internationally.

As education tries to adapt to the needs of the economy, and the demands of the work place, there is no shortage of statements about what primary school teachers should be doing to be effective teachers of mathematics. We are told, for instance, that we should work in an *open and exploratory way*, make good use of *practical activity*, encourage *collaborative work*, develop *problem-solving skills*, foster *confidence* in using mathematics, provide *meaningful contexts* for mathematical activities, *facilitate discussion* of mathematical ideas, *build on* learners' *existing strategies*, and so on.

We may feel that we agree with these assertions, even though we may find that there is little evidence of this happening in some mathematics classrooms. You may recall, that in Umthamo 13, *Problem Solving and Investigating*, on pages 4 and 5, we explored paragraph 243 of the Cockcroft Report to see how "good practice" was defined. The Report states:

> *Mathematics teaching at all levels should include opportunities for: exposition by the teacher; discussion between teacher and pupils and between pupils themselves; appropriate practical work; consolidation and practice of fundamental skills and routines; problem-solving, including the application of mathematics to everyday situations; investigational work.* (Cockroft 1982)

It may seem relatively easy to say what "good practice" in the teaching and learning of mathematics might be. But 'easier said than done' applies here, and good intentions can be somewhat difficult to translate into classroom practice.

Often, South African mathematics teaching is characterised as "traditional". The lack of success with mathematics teaching and learning is clearly reflected in the Grade 12 results in mathematics. According to Garson (1998):

> *In 1997, less than half of all full-time Grade 12 (Standard 10) candidates were mathematics candidates. Of the 252 617 students who wrote mathematics examinations, only 22 798 passed on the higher grade.* (Less than one in ten!)

In 1997, 15 000 South African primary and high school pupils participated in the Third International Mathematics and Science Study (TIMSS). The poor results of our South African learners did little to allay fears that our current system of mathematics and science is failing our learners. The findings highlight an absence of the high level skills called for in commerce, science and engineering which are crucial for any country to succeed.

Here is a summary of the findings of the Report:

- The poor literacy rates amongst parents who cannot assist their children with homework
- The inadequate facilities such as the lack of running water, shortage of reading and writing material, and overcrowding in schools
- The inability of learners to study further due to unequal opportunities
- The lack of encouragement for girls to enter traditionally male-dominated fields, and the burden of housework
- The difference in time spent by learners on homework locally, in comparison to their international counterparts
- The language of instructions continues to be in the second or third language
- A curriculum heavily driven by content
- The perception that mathematics and science are difficult
- Poor or under-qualified teachers, which is a result of inadequate teacher training (Wedepohl).

In 1995, the Association for Mathematics Education of South Africa (AMESA) hosted a conference around the theme "*Access, Redress, and Success*". The thinking behind the association may have been that South Africa needs to be **successful** in mathematics, science and technology in order to compete in the global economy.

This is an association for all mathematics teachers in the country. The Principal of Smiling Valley Farm School is a member of this association. Perhaps you would like join, too.

AMESA believes that, in order to address the legacies of the past, we need to improve **access** to mathematics. The concern for improved access is not only peculiar to South Africa. There have been movements all over the world that have argued, and still maintain, that in order to improve the teaching and learning of mathematics, we *have* to improve **access**. That means we have to:

- **Change the mathematics**
- **Change the teaching**
- **Change the assessment**

Activity 1 – Comparing understandings of Mathematics

Mathematics is a universally recognised convention or culture. Something the human brain can do. It is a pre-cursor of science and technology. It is a passport to higher mathematics. It is empowering. (Nxawe, 2000)

Discuss with your colleague what this statement means for you. In order to help you discuss it, you may want to look at each sentence individually and ask yourself, *What does this mean for my teaching, my learners, my community, etc?*

It might be helpful to brainstorm each sentence. Is there something that you don't like about my understanding of mathematics? Why? What do you agree with? What do you disagree with? Why? Write down your understanding in your Journal.

I would like to claim that in order to improve *access* to mathematics, we also need to add **change the thinking** to the three statements above. We need to help our learners develop their mathematical thinking. But, *how* do we do this?

- ***Change the mathematics***
- ***Change the teaching***
- ***Change the assessment***
- ***Change the thinking***

Examining the definition of school mathematics

Before we look at how we can develop Maths thinking, let's examine in more detail, the definition of school mathematics offered by the Policy Document (1997).

> *Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction.*

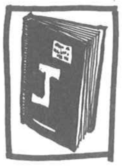
Activity 2a - Analysing the definition

Read this definition aloud to a friend or colleague who is interested in mathematics education. Please do not memorise it, as you will loose the essence of the exercise. As you read it, underline the key phrases that describe what mathematics is to you. Then write out your six key phrases or sentences.

Here are our key phrases in sentences:

- Mathematics is the construction of knowledge.
- Mathematics deals with the qualitative and quantitative relationship of space and time.

- Mathematics deals with patterns, problem solving, logical thinking, etc.
- Mathematics is a human activity.
- Mathematics is an attempt to understand the world.
- Mathematics is expressed, developed, and contested through language, symbols and social interaction.



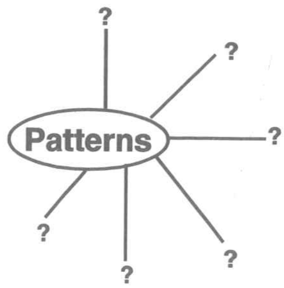
Activity 2b - Describing what a Maths teacher does

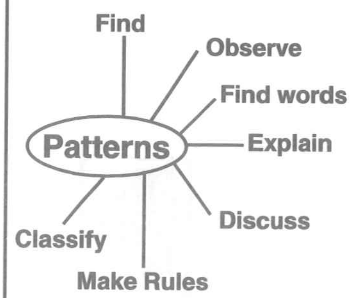
Now we would like you to design a mind-map for each of the phrases above. You need to ask yourself these two questions as you do this:

- What does this phrase or word mean?
- Can I provide an example or description of what this term means?

Next, imagine that you are meeting the parents of your learners for the first time. You need to introduce yourself to them as a teacher who also deals with mathematics. Choose one of the phrases above. Use this phrase and mind-map to help you write down what you would tell them. You want to tell them about the work you do as a mathematics teacher. But don't use the word mathematics.

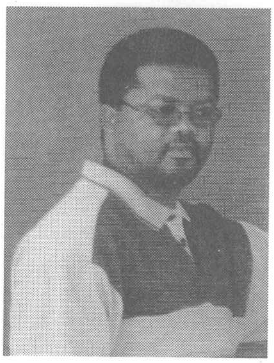
Take your Journal and try to explain what you do as someone who teaches mathematics.





This is Mthunzi's attempt to describe an aspect of what he does as a mathematics teacher.

Ladies and Gentlemen



I am glad to meet you. My name is Mthunzi. I am a teacher who allows learners to observe patterns. I encourage them to make rules about patterns, and to discuss and describe them. By doing this, learners develop a language to talk about patterns, and are able to classify different types of patterns.

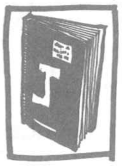
Unit 2 - Recognising Patterns

In the previous mithamo in this strand, we looked at the development of number concept, language, and problem-solving. These topics relate to:

- **understanding** ways of working with numbers;
- the use of mathematical language to **communicate** mathematical ideas, concepts and thought processes; and
- the use of various logical **processes** to test and justify **conjectures,** respectively.

This mthamo focuses on *developing mathematical thinking using patterns*.

Primary school mathematics can be categorised as consisting of four strands, **Numbers, Measurement, Shape and Space, Data Handling.** When a teacher plans work in any strand, she has to set the work in a context. And she has to make sure that her learners will be using **process skills** such as *problem solving.*



Activity 3 - What is a pattern?

Before we look at some explanation of what a **pattern** might be, we need to first look at our own interpretation of the term. In Umthamo 28 we thought about various resources for making learning possible. Now we would like you to write down examples of objects that you regard as resources for the **teaching and learning of pattern**. **Where** would you find these **patterns used**? Draw your ideas.

Now, using your examples, describe, in one sentence or more, what *you* regard as a **pattern**.

Here are some other definitions of the word **pattern**.

> *A pattern is the way something happens, is arranged, is expected to behave (for example, a decoration on carpets or, cloth or wall paper, the way people relate to one another, etc.)*
>
> *A pattern, in mathematical terms, is not just an arrangement of shapes and lines, but must have some rule governing it so that it can continue with regularity. If children are asked to analyse patterns, then an absorbing aesthetic and creative activity becomes also a logical and mathematical one, involving articulating mathematical rules and identifying shapes, positions and transformations.*(Hopkins et al. 1996:94)
>
> *The term pattern (or repeated pattern, for emphasis) is reserved for those designs which have a translation symmetry. A pattern must conceptually extend to infinity; otherwise it cannot have translation symmetry. A common name for translating something, is sliding it without turning or twisting it.*

There is some Maths in the tale which follows. As you read it look for a Maths **pattern**.

The Doubling Spoon

A story from Japan retold by Viv and Alan Kenyon

Once upon a time, in a village, there lived an old woman who could make the most wonderful dumplings. People had heard of her dumplings throughout the region. And in her village, the people liked her, not just because she made delicious dumplings, but because she was kind and was always laughing.

There are patterns in the grammar and use of language. Think of the rhymes that mothers and care-givers teach young children. Do you remember the traditional rhyme we mentioned in Umthamo 27, Nal' isele*?*

One day when this old woman was making some dumplings for dinner, one rolled off the table, into a hole in the earthen floor of her kitchen and disappeared. The old woman put her hand down into the hole to try to reach the dumpling. As she was feeling for the dumpling, the ground gave way and she found herself falling, down, down, down, down, down.

She fell down quite a long way, but she wasn't in the least bit hurt. The old woman picked herself up, and dusted her clothes. She saw that she was on a road on a hill, just like the road in front of her house. There were lots of rice fields. But there was nobody working in the fields. It seemed as though she was in quite another place. The old woman looked around to see if she could see her dumpling, but it was nowhere to be seen. She started to run down the road, looking for her dumpling. As she ran she called, "My dumpling, my dumpling! Where is my dumpling?"

You could tell this tale to your learners. We have found that children in the Intermediate Phase, are quite fascinated by the idea of a magic spoon. A teacher could do some very interesting Maths work with her learners using this as a starting point. What do you think?

After a while, she came to a wooden statue standing at the side of the road. "O Statue," said the old woman. "Have you seen my dumpling?" "Yes," answered the wooden statue. "I saw a dumpling rolling past me. It's gone that way. But let me warn you. You better not go any further. There's a wicked Oni living there who eats people." The old woman laughed and thanked the statue. Then she ran on calling her dumpling. "My dumpling, my dumpling! Where is my dumpling?"

Further down the road, the old woman came to a stone statue standing at the side of the road, and she asked, "O Stone Statue, have you seen my dumpling?" "Yes," answered the statue. "I saw a dumpling pass me. It rolled down that way. But let me warn you. You better not go any further. There's a wicked Oni living down there who eats people." The old woman laughed and thanked the second statue. Then she ran on calling her dumpling. "My dumpling, my dumpling! Where is my dumpling?"

Well, it wasn't long before she came to a third statue. This one was made of bronze. "O Bronze Statue," said the old woman. "Have you seen my dumpling?" "Don't speak about your dumpling now. Look. There's the Oni coming. He eats people. Quickly, hide behind my sleeve, and don't make a sound." The old woman slipped behind the statue and hid. As soon as she was out of sight, the Oni reached the bronze statue. He stopped and bowed to the statue, and said, "Good day, Statue, Sir." The statue politely returned the greeting.

Then the Oni lifted his head and sniffed the air two or three times. "Statue, Sir, can you smell the smell of a person?" "I think you're mistaken," replied the bronze statue, "No, no!" said the Oni. "I know this smell. It's the smell of mankind."

When the old woman heard that, she couldn't help having a little laugh. The Oni reached down his big hairy hand behind the statue, and pulled her out. She was still laughing. Then the statue said, "What are you going to do to this old woman? You mustn't hurt her." "Don't worry!" said the Oni, licking his lips. "I won't hurt her. Maybe she is the one who cooked the delicious dumpling that I just found. I just want to take her home with me to cook for us." The old woman laughed. "Very well!" said the statue. "But you must be kind to her. If you hurt her in any way, I shall be angry." "I promise I won't hurt her at all," said the Oni. "And she will only have to do a little work for us each day. Good-bye, Statue, Sir." With that, the Oni took the old woman far, far, far down the road.

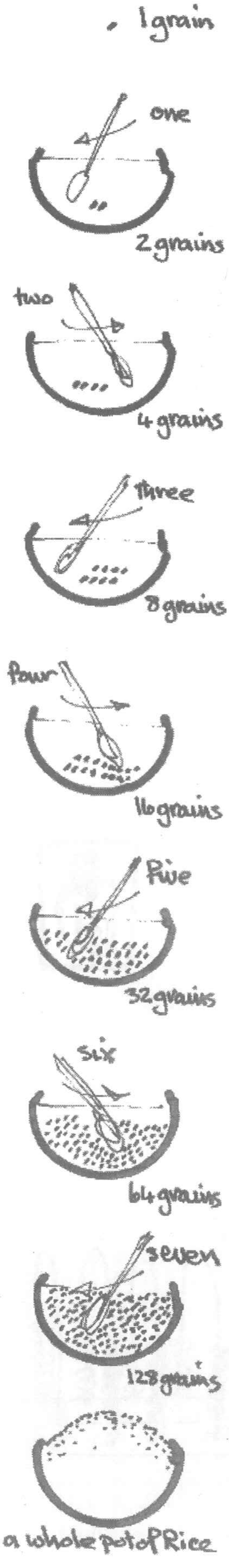
At last they came to a river that was deep and wide. There was a boat waiting. The Oni put the old woman into the boat, and rowed her across the river to his house. It was a very big house. The Oni led her into the kitchen, and told her he wanted her to cook some rice for him and the other Oni who lived with him. He took a big pot and a special wooden spoon. He gave the pot and spoon to the old woman and said, "You must never put more than one grain of rice into the pot. Then you add water. When the water starts to boil, you stir the rice with this spoon."

The old woman was very puzzled, but she did what she was told. She put just one rice-grain into the pot just as the Oni had told her. Then she added some water. When the water started to boil, she began to stir the rice with the special wooden spoon. She stirred once, and the one grain of rice became two grains. She stirred a second time, and there were four grains. The third time she stirred there were eight grains, the fourth time there were sixteen grains, and then thirty-two, and sixty-four, and so on. Every time she stirred the rice with the special spoon, it doubled the number of grains of rice. And in a very short time, the pot was full.

The old woman stayed in the house of the Oni for a long time, and each day she cooked food for them. The Oni was kind to her, and never hurt or frightened her. And she didn't have to work very hard because she only had to cook rice, and the magic spoon was a great help to her. But after some time, she began to feel quite lonely. She missed her friends and family in the village. And she wished she could go back to her home and make some dumplings.

Then one day, the Oni were all out somewhere, and she thought to herself, "Now's my chance to run away!" She took the magic spoon and slipped it under her girdle (a wide belt) and went down to the river. Nobody saw her. When she got to the river, there was the boat, so she got into it, and pushed it away from the river bank. She could row very well, and it wasn't long before she was some distance from the bank. But the river was very very wide. When she was about a quarter of the way across the river, all the Oni arrived back at the house. They found their cook had gone, and so had the magic wooden spoon. They ran down to the river and saw the old woman rowing as fast as she could across the river. The old woman was now nearly half-way across.

Maybe they could not swim. What is certain is that they didn't have a boat. So they knelt down on the bank of the river, and began to suck up all the water of the river into their mouths. The old woman kept on rowing until the water had become so shallow, that the Oni stopped sucking and began to wade across the river. Their bellies and cheeks were bulging with water. The old woman was about three-quarters of the way across the river. She dropped her oar into the boat, and took out the magic spoon and shook it at the Oni. As she shook the magic spoon, she made such funny faces, what do you think happened? The Oni burst out laughing. And when they burst out laughing, what do you think happened? Exactly! The water they were holding in their mouths poured out, and the river became full again. The Oni couldn't cross,



and so the old woman rowed safely to the other side of the river.

As soon as she reached the bank on the other side, she jumped out of the boat, and ran and ran up the road as fast as she could. She ran and ran and ran until she found herself at her own home.

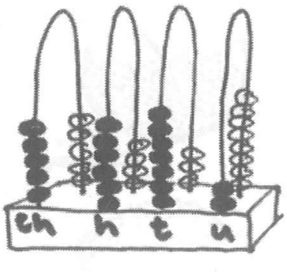
After that the old woman was very happy. She made dumplings whenever she wanted to. And she had the magic doubling spoon to make as much rice as she needed. She sold her dumplings to her neighbours and other people in the village, and so she lived comfortably, and was very content.

Now we would like you to read the following passage. It comes from a book called, *Making Patterns*, by Helen Pengelly. Helen Pengelly is an Australian. Her book focuses entirely on **pattern** and the very important role **pattern** plays in the development of mathematical thinking and understanding.

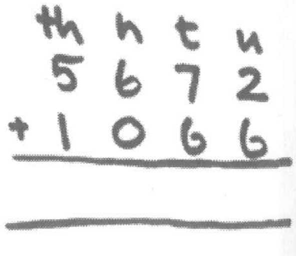
We have inserted two Activities for you to do as you read through the text. You will have an opportunity to carry out these activities at a face-to-face session.

Reading 1 - The value of pattern in children's mathematical learning

Pattern is not to be thought of as a topic to be studied, but as a **process** to frame children's experiences and guide them increasingly towards more sophisticated mathematical thought. Making patterns immerses children in an environment where they participate in mathematical activity. This process provides the personal experience from which they build mathematical knowledge. While making patterns they are interacting with, and learning about, the concepts, skills and procedures of mathematics.



Activities with patterns enable children to investigate many mathematical ideas. Our number system, for example, is an incredibly sophisticated pattern which groups numbers into tens, tens of tens, tens of tens of tens, and so on. Once understood, children use it to make predictions about how numbers are named and written.



Activity 4 - Learning to Count

In order to share similar experiences to those of your learners, I invite you to study the structure of these number names. You need to learn to count in Yoruba within 20 minutes. You need to focus on yourself as you learn to count in this language.

Furthermore, how do learners start learning to count? What do you mean by "learning to count"?

Look carefully at the structure of the number names and how they relate to the number symbols. Discuss the following:

1:Ookan **2:eeji** **3:eeta** **4:eerin**
5:aarun **6:eefa** **7:eeje** **8:eejo**
9:eesan **10:eewa** **11:Ookanla** **12:eejila**
13:eetala **14:eerinla** **15:aarundinlogun**
16:eerindinlogun **20:Ogun**

- What links do you notice among the words for one to ten?
- What structure do you think governs the formation of number words in Yoruba?
- Can you form any number word to 99?
- What more must speakers of English learn over speakers of any of the languages in order to be able to count?

Discuss these questions with a another teacher-learner. Then suggest a way to teach your learners how to count.

If you think of any English number name, there seems to be no link between numbers from one to ten. The numbers are relatively short. Now look at how your own indigenous number names are linked. Can you suggest a way of teaching counting?

Since the introduction of decimal currency and the metric measuring system, measurement, with the exclusion of time, is also based on this structure.... The same pattern transfers from one situation to another.

The multiplication tables are one way of expressing number sequences. Counting in groups (5, 10, 15, 20...), referred to as interval (skip) counting, is another. When these sequences are mapped on a 10 x 10, one to 100 numerical grid, these patterns become visual as well as numerical.

1	2	3	4	●	6	7	8	9	●
11	12	13	14	●	16	17	18	19	●
21	22	23	24	●	26	27	28	29	●
31	32	33	34	●	36	37	38	39	●
41	42	43	44	●	46	47	48	49	●
51	52	53	54	●	56	57	58	59	●
61	62	63	64	●	66	67	68	69	●
71	72	73	74	●	76	77	78	79	●
81	82	83	84	●	86	87	88	89	●
91	92	93	94	●	96	97	98	99	●
101	102	103	104	●	106	107	108	109	●
111	112	113	114	●	116	117	118	119	●

Multiples of 5

Measurement of time is complex, because the patterns governing it are in several different bases. For example, there is base 12 (hours on a clock face), 24 (hours in a day), 60 (minutes in an hour and seconds in a minute), four (seasons), seven (days in a week) and so forth. These are all repeating sequences.

The terra cotta tiles that adorn the verandas of many old homes are two-dimensional patterns. These tessellations are formed as shapes are reproduced, according to a specific spatial orientation.

Activity 5 - Tessellations

A way to experience the excitement and beauty of mathematics, is to introduce tessellations. This combines maths and art. By tessellations we mean tiling. Polygons or curved figures are used to completely cover a plane (flat surface) so that there are no gaps or overlaps, just like the tiles on a kitchen floor or bathroom wall.

Two-dimensional shapes like squares, rectangles, parallelograms, triangles and even hexagons are shapes that fit together closely (tessellate). You don't change the **size** or **shape** of the figures but you do change the **position** as you cover the flat surface. There are 3 kinds of changes (transformations) that can be made. The shape can be slid in any direction to a new position (translation). The shape can be turned (rotated) in any direction. And the shape can be flipped over (reflection) to show its opposite side. (See page 18 for examples of these three changes/transformations).

Step 1 - Tessellating Triangles

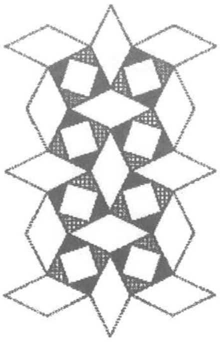
At the first face-to-face session, try to tessellate half an A4 sheet with a cardboard cut out triangle. Remember, leave no gaps and have no overlaps. Try to use the three kinds of changes. Compare and discuss your patterns. Raise questions about your patterns.

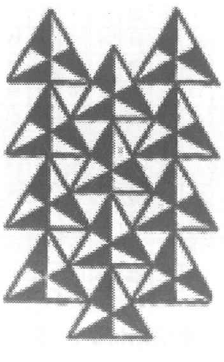
Step 2 - Demonstrating Transformations

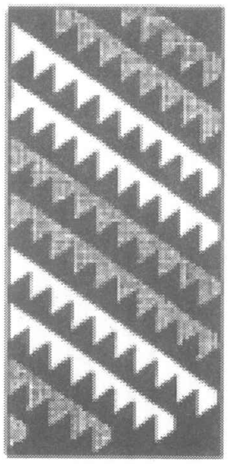
Use your cut out triangle to show where you have made each of the changes (the three transformations). Translate (slide), rotate (turn) and reflect (flip) the triangle to the position of a neighbouring triangle.

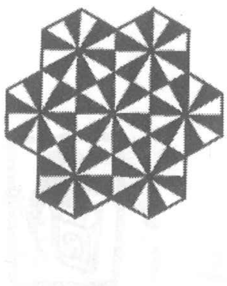
Step 3 - Thinking of other Shapes

Sometimes we use only one shape to tile a surface. (Which shapes will tessellate by themselves?). Sometimes we use two or more shapes. Can you think of or find examples?

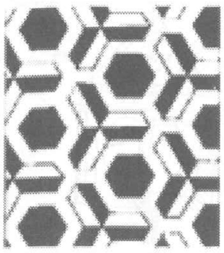


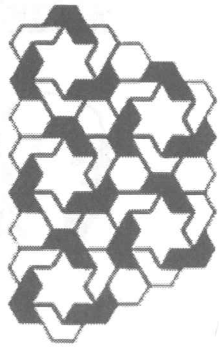






Geometry is not only the study of the properties of polygons, it can also include the study of the movement of those polygons. This is called transformation.





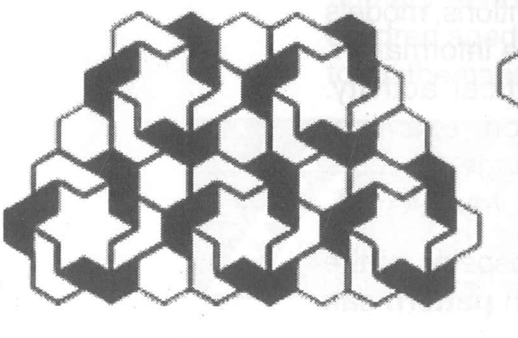
The colour arrangement of the shapes enhances the patterns. Each pattern is determined by the shape of the tiles and by the way they fit together without leaving any gaps. Different shapes offer different possible designs and these designs are repeated to cover an allocated area, such as a veranda, the lid of a trinket box, a patchwork quilt or the top of a table with shapes inlaid. These tessellating patterns are prevalent in the art and artefacts of the Middle East.

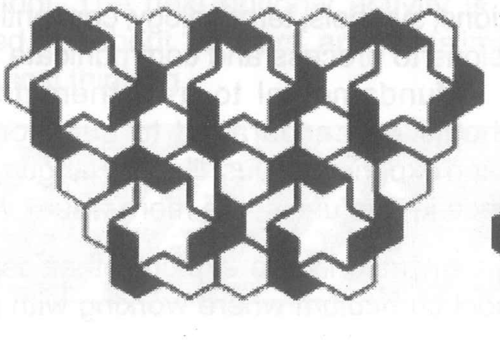
An understanding of and ability to use pattern develops over time. In the first instance, children learn what the term means by making patterns. They do this regularly, once every week or so, during their first few years at school. While children continue to find different and interesting ways of thinking about these experiences, they remain motivated to make and record patterns. It is only later, when the concept is established, that it is possible to search for patterns in mathematical information.

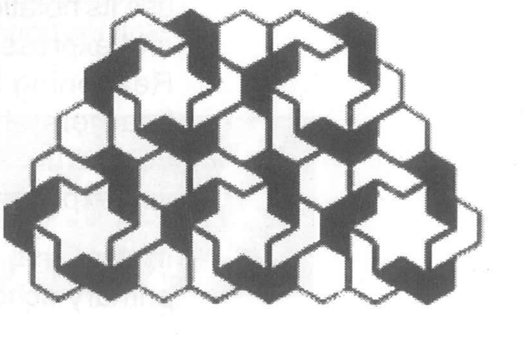
Once the concept of pattern, the focus of this book, is understood, children use it during mathematical investigations. As children progress through the primary school they have many opportunities to explore mathematical ideas, to find patterns in the information gathered during these investigations, to make generalisations and, in many instances, to look for ways of expressing these generalisations as relationships. Finding a pattern brings order to seemingly random information by establishing connections within it. As a result, the patterns of mathematics are perceived. Pattern searching becomes the means for building understanding. Identifying, describing and using pattern is a feature of learning experiences outlined in other books in this series.

Constructing meaning for oneself fosters ownership, commitment and intrigue. Children are in control of their learning. As a result they willingly engage in mathematical inquiry.

(Pengelly, H. 1992: 5-6)







Unit 3 - Pattern work for Learners across the Curriculum

Mathematics is an area of the curriculum, which searches for, studies and describes **pattern**. The identification and use of **pattern** is a fundamental aspect of the mathematising process. It occurs both consciously and unconsciously during investigations. An understanding of the concept, and of the elementary patterns of mathematics, is basic to any primary school curriculum. And it is essential that learners have opportunities to search for patterns.

Making sense of mathematics relies on an understanding of **patterns** which underlie this area of the curriculum. By seeking for **patterns** in their own mathematical experiences, children construct knowledge for themselves. They are also learning to function in a mathematical way.

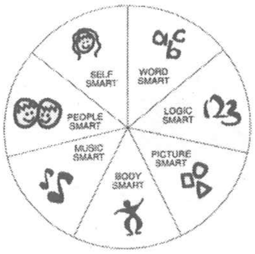
In order for learners to be able to manipulate number and geometric **patterns**, they have to be involved in observing, representing and investigating **patterns** in social and physical phenomena and within mathematical relationships. Learners have a natural interest in investigating relationships and making connections between phenomena. Mathematics, therefore, has the potential to offer a way of thinking, and of structuring, organising and making sense of the world.

Mathematics is a human activity, as the definition on page 6 suggests. Learners need to be aware of and appreciate the contribution of *all* peoples of the world to the development of mathematics. We need to challenge the view that mathematics is a European product. We need to provide opportunities for learners in Africa to understand the historical background of their communities' use of mathematics.

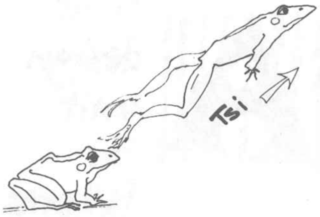
Mathematics should foster a **critical** outlook in learners so that they can engage with issues that concern their lives individually, in their communities, and beyond. It should foster critical thinking about how social inequalities, particularly those concerning race, gender and class, are created and perpetuated.

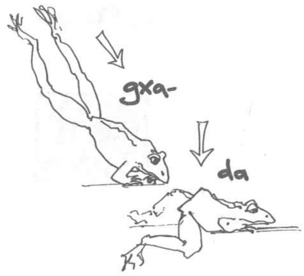
Lastly, since mathematics is a language, learners have to use its notations, symbols, terminology, conventions, models and expressions to process and communicate information. Reasoning is fundamental to a mathematical activity. Learners should be encouraged to question, examine, **conjecture** and experiment. Just like any language, the more you experience it, and use it, the more **fluent** you become.

In this Unit, we would like to explore other aspects of the primary school curriculum where working with **pattern** can



be fostered and developed. You will remember that in Umthamo 26, we thought about Howard Gardner's theory of Multiple Intelligences. Gardner believes that everybody is intelligent in different ways. And teachers who foster and value different forms of intelligence are really facilitating their learners' development.

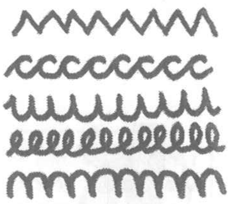




Teachers in many different parts of the world realise the value of exposing their learners to different experiences with **pattern**. Some experiences involve making and printing decorative Art and Crafts **patterns.** Others involve focusing on rhythm in music and dance. There are also **patterns** in the grammar and use of language. Think of the rhymes that mothers and care-givers teach young children. Remember the traditional rhyme we mentioned in Umthamo 27, *Nal' isele*? And, as you have seen, stories sometimes incorporate something to do with **pattern,** too.

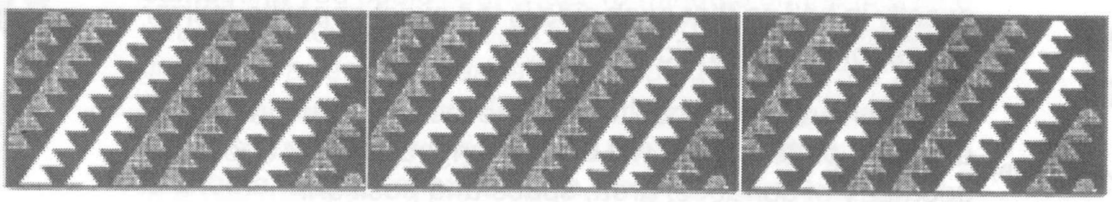
As you work through this Unit, try to think of other traditional songs, dances, rhymes and stories, that you know, or know of. This is an important part of culture which we need to foster and nurture in our classrooms.

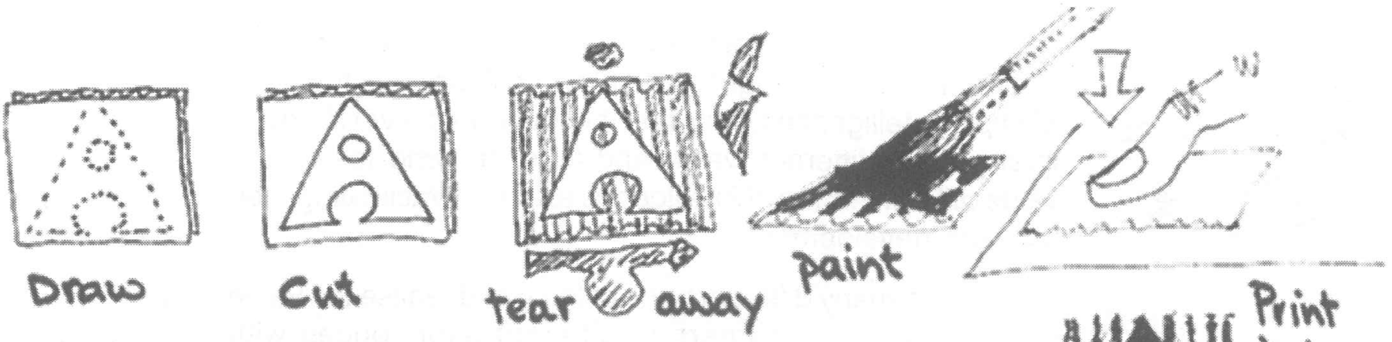
Pattern and Maths in Arts and Crafts

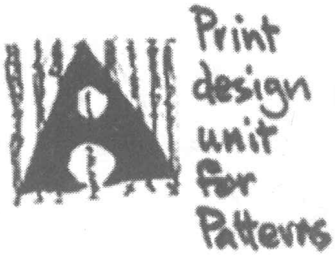


Traditionally, in school, children copied lots of repetitive patterns when they learned to write. In some schools, today, they are encouraged to decorate the margins of the worksheets that they paste into their note-books with coloured-in patterns. These activities are rather unimaginative and mindless. But in pre-schools, children have a lot of fun making all sorts of patterns. They make repetitive patterns with prints of their own hands. They do finger painting patterns by covering a page with coloured starch-paste and then dragging their fingers across the page. Sometimes they print patterns using found things like leaves, bottle-tops, corks, and keys. You will also find them setting out the building bricks or scrap boxes in long lines of a repeating pattern.

Sadly, in our schools, the above activities are not extended into more serious work. They are ignored higher up in the primary school. The next optional activity is suitable for children aged 8 to about 13 years, and it has important links to mathematical thinking.



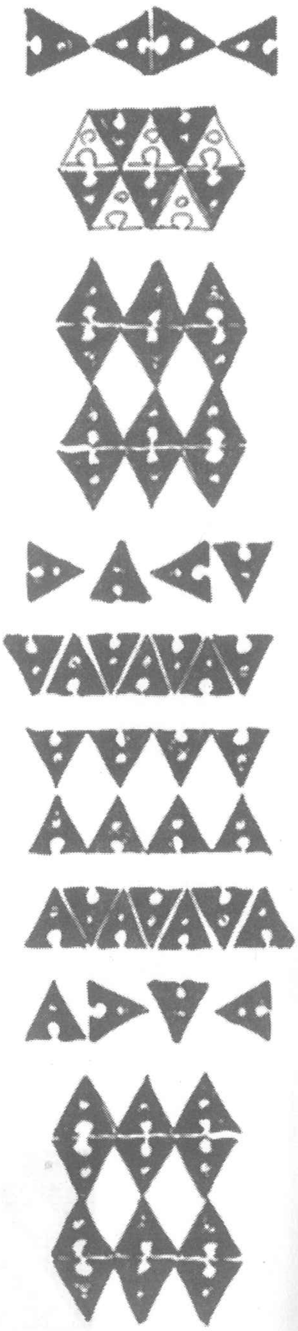




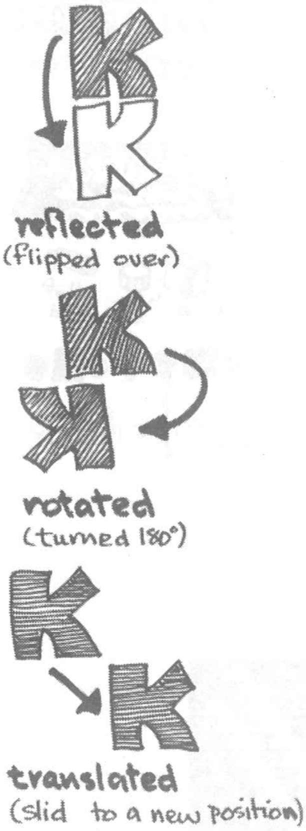
Cardboard printing

A cheap way to print patterns is to use corrugated cardboard from old boxes. Each child cuts out a square shape as a basic unit. There are a few ways that a design for printing can be done. One way is to cut and paste other cardboard shapes directly onto the cardboard square. Another way is to cut the outline of the shape into the top layer of the corrugated cardboard. Then you can tear off the cardboard around the spaces and even cut out some of the corrugated layer. But the bottom layer of cardboard should **not** be cut so that the block has a firm base for when you print. You can also glue down stuff like string to form the design.

Older learners will enjoy cutting out the initial letter of their name and seeing what patterns can be made.



To print, you need to make thick sticky paint. Starch added to powder paint is suitable. Sponge or paint the top surface of the block. Then quickly and carefully press the block, paint-side down, onto a clean sheet of paper to copy a print of the design. Repaint and repeat the process to produce a sequence of blocks in rows. Learners can investigate different ways of moving and turning the block to get interesting patterns.



Without realising it, the children will get a feel for some important mathematical concepts as they cover a specific surface area with repeated rows of units. They will also get a feel for the way shapes relate and can be changed or transformed as they are moved in different ways. The design can be slid and repeated in a new position (translated). Or the design can be turned (rotated) using different angles like 90° or 180°. Or it could be turned over onto its opposite side (reflected), so that it is symmetrical.

Some patterns or designs are symmetrical and can be divided exactly in half so that each side is the exact opposite (mirror image) of the other side.

If children are challenged to describe the way they have made a printed pattern, they will be challenged to think about the geometry of surfaces, area, space and position.

Pattern and Maths in Music

In our schools, traditionally, music has meant singing. But music is much more than songs and singing. Of course, singing and songs are important. Songs have been an important part of everyday work-life. In Africa, people have sung songs as they worked.

But an essential part of any song is its rhythm, or beat. And in primary schools in many parts of the world teachers play rhythm games with their learners to help them develop 'an ear' for patterns of rhythms, or beats. One game which we have found that primary school children enjoy, is clapping out the rhythm of words.

A good way to start is for the teacher to clap a pattern or rhythm. It doesn't have to be very long. But it's a good idea if some of the beats are long (or slow), and others are short (or quick). The learners listen to the rhythm, or pattern, and then clap back the exact same pattern or rhythm.

Many teachers in pre-school classes in fact do this as a listening activity.

The teacher can clap a few different patterns or rhythms. She may even get some of the children to clap a pattern which she and their peers can copy. But the important thing is for the learners to have a chance to clap several *different* rhythms. We have found that learners really enjoy clapping the names of some of the places in South Africa. (The idea is to clap a beat for each syllable.)

Start off with a name like Cape Town and clap it a few times, saying the name, *Cape Town, Cape Town*. (There are two beats of equal length in the name Cape Town.) A name like Maluti would be a good one to follow this. (There is a short beat for the first syllable, followed quickly by two long beats for the two syllables which follow.) As you clap this name, say the name, *Maluti, Maluti*. Now try Lusikisiki!

In parts of Africa "talking drums" are used to beat out messages, communicating over long distances.

Get your learners to think of others. Together, work out what the rhythm, or pattern of beats of each name is, and clap it. Then you could either guide your learners as they clap different learners' names. Or you could let your learners work in pairs to work out what the pattern of beats is for their first names. (Compare a name like Vuyo with Noluvuyo. Their patterns are quite different!)

You don't need to spend a long time on this activity. It's something you could spend just 15 minutes on the first time you try it. Afterwards, it could be a 5 or 10 minute 'filler' when you have a few minutes left. You could even turn it into a guessing game in which learners clap a pattern, and others have to guess the word that has been clapped. But rhythm is another aspect of our lives in which we can find pattern.

Unit 4 - Pattern and Mathematics Investigation

Part 1 - Key Activity

So far we have looked at different **patterns** in both Mathematics and in other areas of the curriculum. In this Unit we will use the concept of pattern as understood visually, as a tool to be used in mathematical investigations. These investigations will be more numerical. While this mthamo starts with making, identifying and describing simple and concrete patterns, it gradually moves towards the making, identifying and describing of more complex and abstract ones. In the end, the patterns are to be perceived by the mind rather than existing as physical entities.

We have provided three options for the **Key Activity**. They are arranged in such a way that you can make your own choices about which option you want to try with the learners you work with. You are also free to try the other options on a voluntary basis. But you **must** complete at least one, and you will need to store your work in your Concertina File.

Remember, you are not trying to teach the learners anything specific. You are simply providing them with a guided activity in which they have an opportunity to **explore** emerging **patterns** in shapes and numbers in some way. And you have an opportunity to see how they respond to the activity. Are young children ready for **conjecture**? Will they start to think about what might happen next? What happens if a shape gets **bigger** or a number **changes**?

Option A – The Patterns of Number with Shapes

This option is for those of you who work with very young children. It is an Activity that should just involve a small group of learners. It is not an Activity for the whole class. (In our experience, learners are interested in the work of others. So, even if you don't work with them, they will be aware of, or interested in, what is happening. Maybe some time in the future, you could plan to give others a chance. Or they may even ask you if they can try this activity when they feel they are ready for it.)

We have set out the planning of this activity in a different way. This way models how primary teachers in many parts of the world do to try to be systematic about their planning for an activity or learning unit. It demonstrates the way in which they will put their plans into action. They write brief notes on separate pieces of card or paper for each part (or step) of the activity. In Australia, primary school teachers call these notes, 'running notes'. In other words,

If you are working with learners in the Early Years, then choose a time when a helper or somebody is keeping the rest of the group occupied in some way, or when they are playing, drawing, painting, or working on their own. You need to pick 4 or 5 (but not more than 8) of your learners who you feel have an aptitude (an interest or feeling) for numbers and shapes.

notes to remind themselves of how to run the activity.

Read through the plans for this activity. Then list and collect the things that you will need. Next prepare yourself carefully so that you are clear about what you plan to do. You may want to make your own copies of the cards with your own modifications.

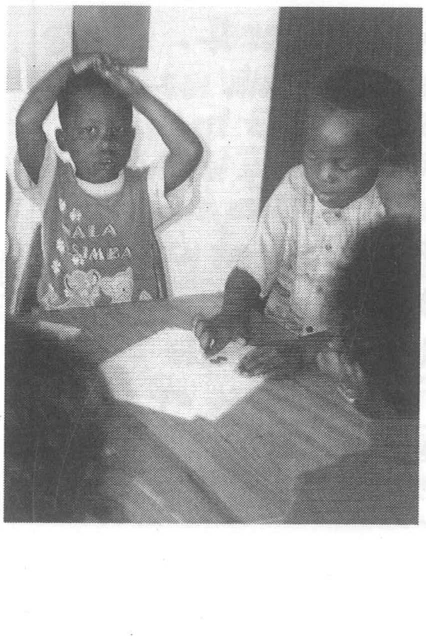
- Sit at a table with a small group

You need
- counters/bottle tops/coins
- Scrap paper
- Paper cut into 1/8 A4 size
- A crayon or Fat Koki-pen
- A large chart with 4 columns
- Prestik

SHAPES + NUMBERS

- Start by getting them to give you the names for the shape in each column
- Write down the name

- Choose learners that show an early aptitude (interest and ability) for number and shape.
- If they use a name like indlu for a rectangle, and ronte for a triangle, accept it and work with it.



Step I
- Put a single counter on a blank sheet of scrap paper in front of each learner. Ask them to trace with their finger to show what shapes can fit around the counter.
- As they show you - draw what they show.

Does a rectangle fit?

Rectangle ✗ 1 No!

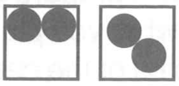
Square ✓ 1 Yes!

Circle ✓ 1 Yes!

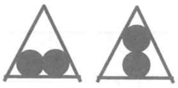
triangle ✓ 1 Yes!

- Stick up the 'Yes' papers

- If you like you can draw to show that a square doesn't work



- nor a triangle

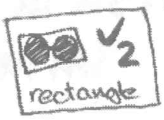


- and a circle won't work either.

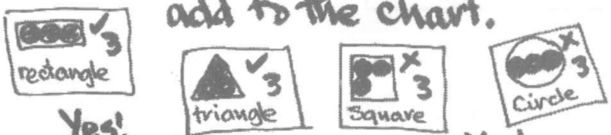
Step 2. Now put out 2 counters on each blank sheet. Ask them to show you what shapes can be made with two counters. And trace the shape

- Do a drawing to show that only a rectangle shape can be made with two counters.
- Stick it up on the chart.

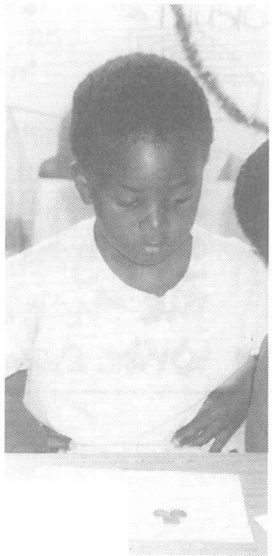
Yes!



Step 3
- Now put out 3 counters.
- What shapes can they make with 3 counters?
- let them build the shapes and trace the outline of the shape with a finger.
- Draw what they show and add to the chart.



Yes! No!

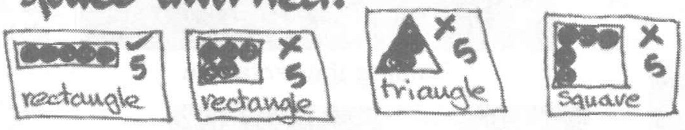


- Say, "What about a triangle?" if they don't think of it.
- Only show them as a last resort.

Step 4 Now try the number 4. What shapes do you think we can make with 4. Wait to hear what they think and then let them try. Draw the shapes they show and stick up the ones that work.

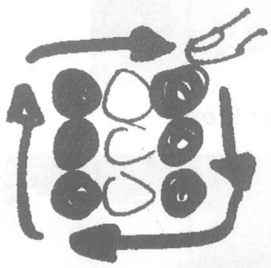
- Pay attention to what they say and listen to how they explain their thinking. Do you see evidence of Conjecture.

Step 5 Now try 5 counters
Repeat the procedure as in the other steps.
This should be interesting because they should be able to imagine a new kind of rectangle with more than one row of counters or a triangle with one space unfilled.

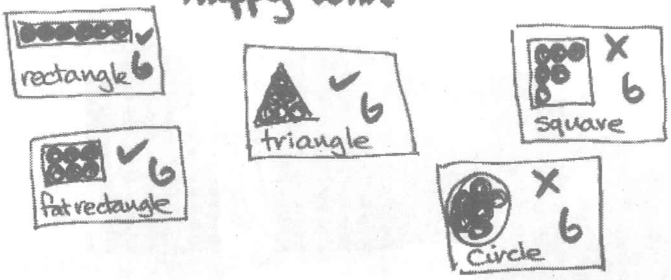


- Again, listen to see if any learners can predict what might happen with more numbers.

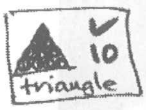
- One learner at Zanokhanyo Pre-Primary envisaged a large square when she had 6 counters. She put two vertical rows with a space in the middle for a column of 3. Then she traced the square. (She was the same child who could count herself in when we trialled Umthamo 21.)

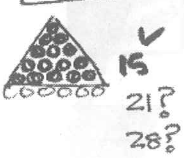


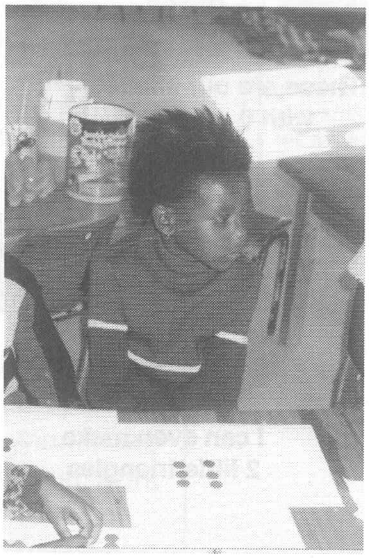
Step 6. Now 6 counters should be interesting. What do your learners do? What do they say? What shapes are they happy with



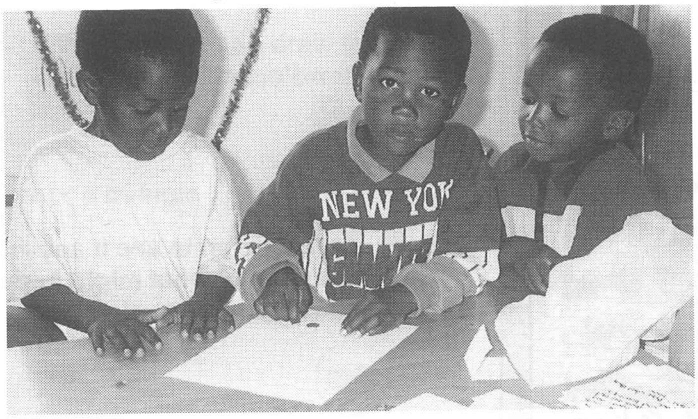
Step 7. Decide whether to continue.
What do the group feel?
Choose – to go on
– to stop
– to challenge different learners to investigate further. How many counters do you need to make a bigger triangle?
What is the next square number?

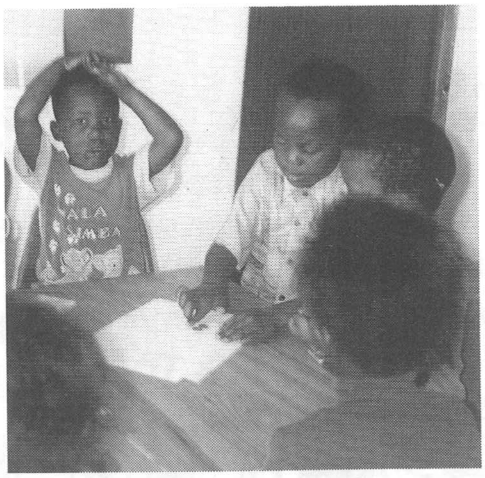




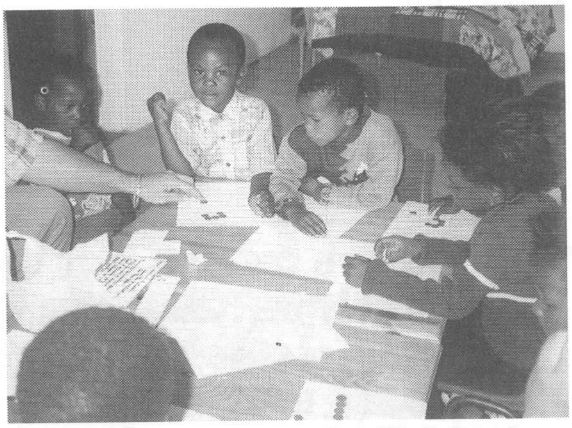


Outlining shapes around one counter

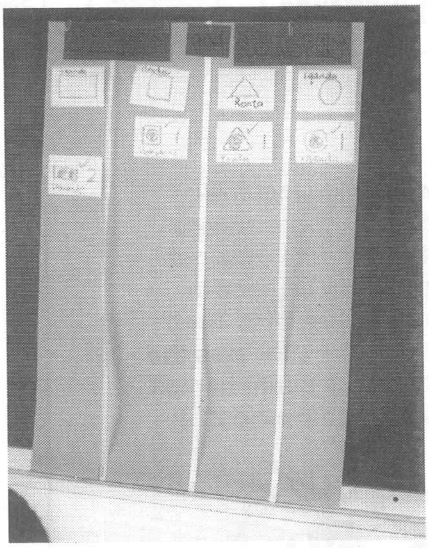




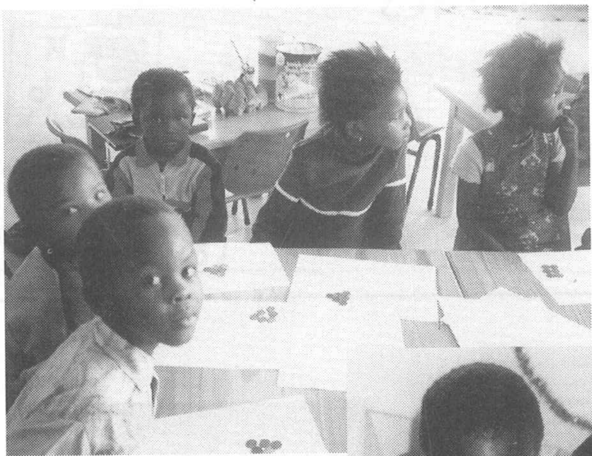
Tracing the rectangle



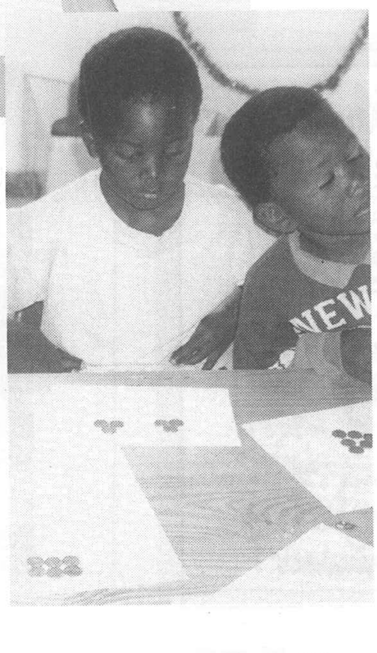
What shapes can we make with 6 counters



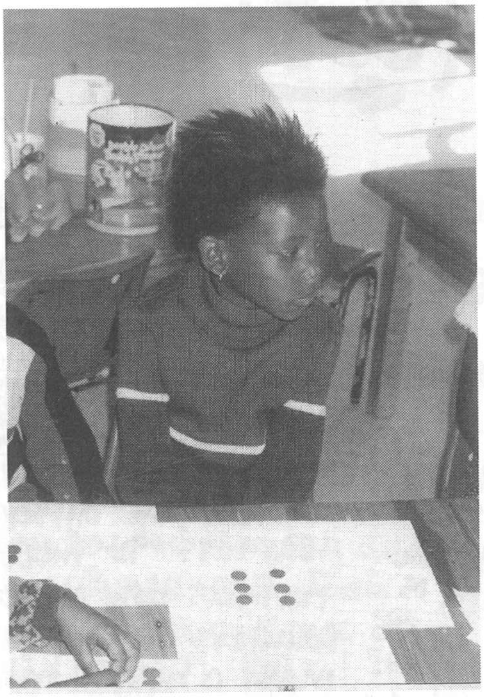
The chart after Step 2



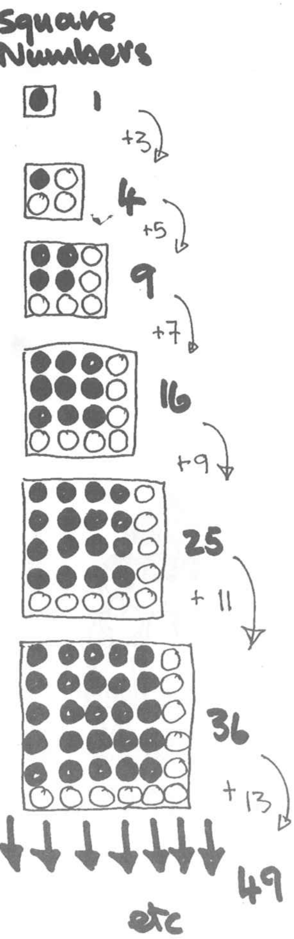
These are our shapes
with 6 conters

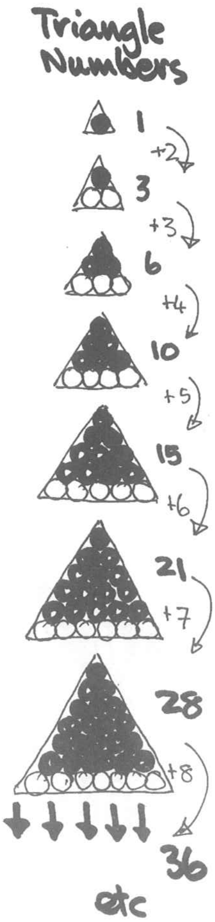


I can even make
2 little triangles



If I had 3 more counters I could make
a square of 9





Reflection – How it went

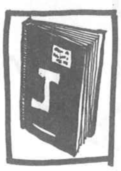
When we trialled the Activity with pre-schoolers in Hanover, it worked quite well. One mistake was to ask for a group of 8 learners, and not just to take a smaller group with a special aptitude for Maths. This meant that there were some children who were a little uncertain and felt inhibited, so the group didn't really relax. However, on reflection, it was surprising how relaxed they were with 3 relative strange adults in the room, two of whom were *'abelungu'!* That is always a problem with trialling. So we have to take account of the effect that outsiders have on what would normally happen and how children would respond.

Another thing that didn't work well, was to not accept the learners' description of one of the shapes as *'indlu'.* That was what the children were used to. And this Activity was not about names of shapes, but about what shapes can be formed by arranging different numbers of counters. Another time, we would choose to accept the names the learners agreed to use. Then at a later stage of schooling, they could be introduced to the more formal words. There is plenty of time. The more important part of the activity is that children enjoy building up shapes from counters or coins, try to trace the outline of the shapes and also begin to think a bit more about the counting numbers in a real context.

Another important thing is that the teacher has a chance to model systematic recording of what is being done or found. But we realised that this must not be emphasised and should just be incidental.

Later on in their education, when they deal with recording data, and do graphs in Maths, Science and Geography, they will have a subliminal memory of an experience to build on. (The idea that information can be organised and recorded in a systematic chart will already be there in their minds somewhere, like a foundation for this new stuff to build on.

Writing your own reflections



Later in the day, after you have completed this Activity, spend time thinking carefully about what happened. Try to reflect on what surprised you, and why. What did you learn from this experience? What will you do differently when you do this Activity again? Why? What can you do to extend your learners' experiences of pattern?

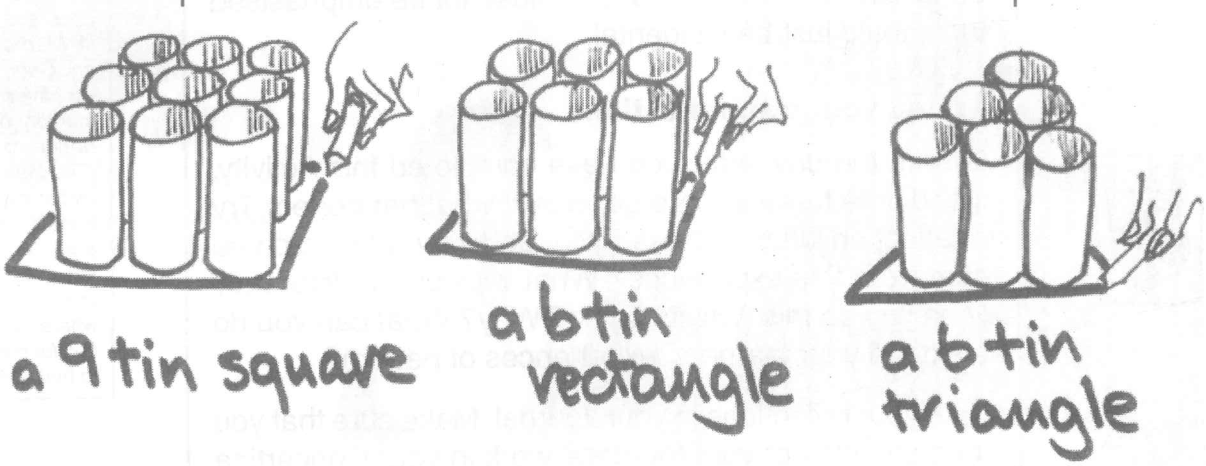
Write your reflections in your Journal. Make sure that you store samples of your learners' work in your Concertina File, together with any special notes you have made on what they did. Be prepared to share this work on pattern at your Portfolio Presentation at the end of the year. Make sure that you also record and reflect on any further work that comes from the ideas in this mthamo.

How this work could be taken further

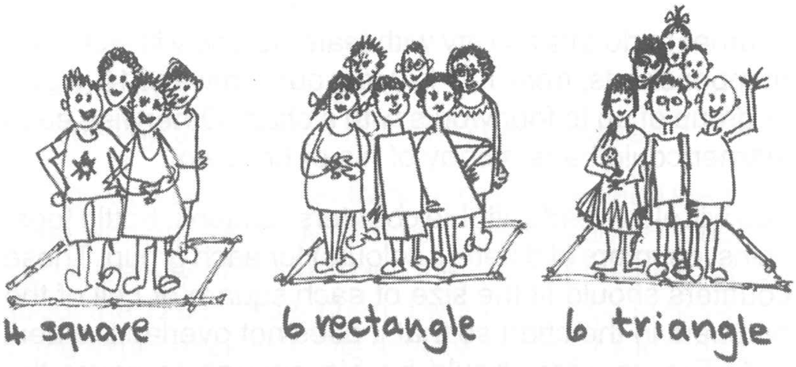
We actually went on to Bulelani Primary School to try the Activity with Grade 3's. We ended up with nearly the whole class, instead of a small group. But it still worked well. Towards the end of the Activity, older children came in. We went onto bigger and bigger numbers building up triangular and square shapes in our imagination looking for the pattern or rules.

Although there were about 50 children in the classroom at the end, only 4 or 5 really got excited by the mathematical ideas behind triangular and square numbers. We could see from their body language and expressions that this was a key learning experience for them. We could also see that perhaps their attitudes to Maths would have been influenced by their experience. Others looked puzzled although they stayed interested. We realised that you can't expect everybody to gain equally. But it is important to provide activities that are challenging, even if only a few 'catch on' at first. With more experience and further activities, others will follow and more and more learners might get excited by Maths and Maths thinking.

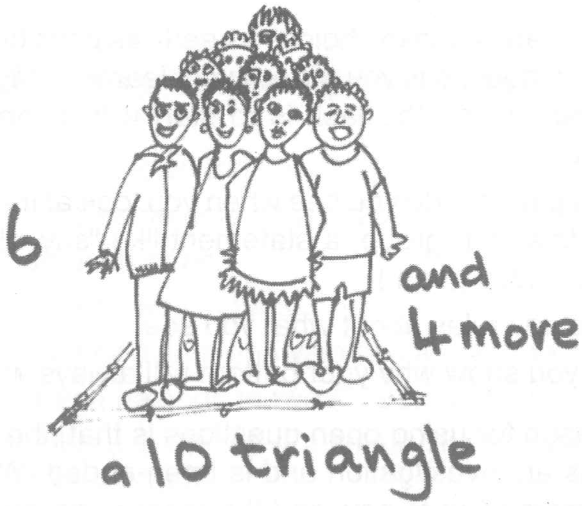
With the pre-schoolers we thought that we could do some really nice work using bigger objects like empty coke cans to arrange in closely-packed triangles, squares and rectangles. If this was done outside the shape formed could be traced in the sand. In the classroom, the outline of the shape could be drawn in chalk on the floor.



It would also be fun to actually do it in a physical way, by getting children to stand to form shapes. How many different shapes can a group of six make if they stand in rows close together?

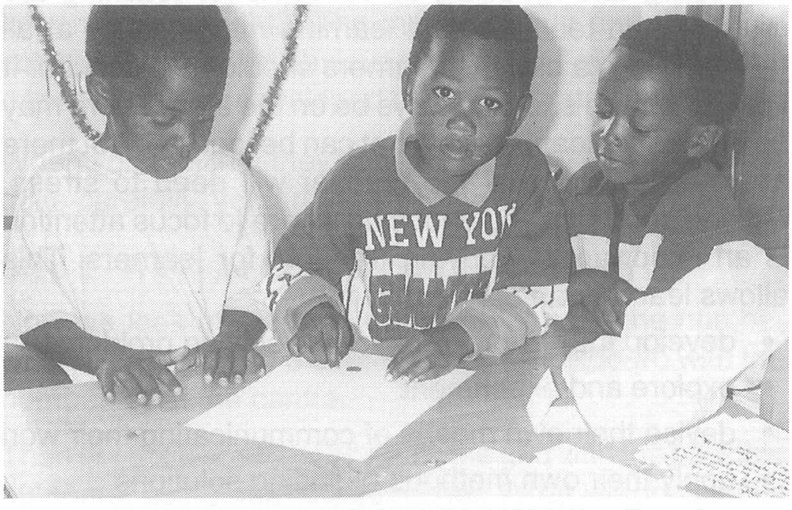


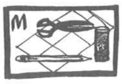
The shapes could be marked off by scratching in the sand with a stick or by laying down straight bamboo sticks or dry reeds.



Then you could ask questions like, How many more children must we add to make a bigger triangle? Yes, a row of 4 more will fit to make a bigger triangle.

Now, how many must we add to make a bigger triangle? Can you see the emerging pattern of numbers?





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

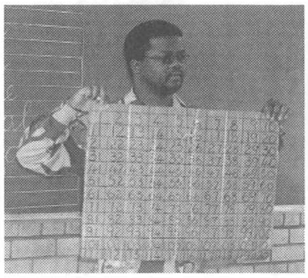
120 Number Chart

Option B - Patterns in a Number Chart

In order to do this activity with learners you will first need number charts, from 1 to 120, set out in rows of 10. A pair or group of up to four works with a chart. Otherwise each learner could have a copy of his or her own.

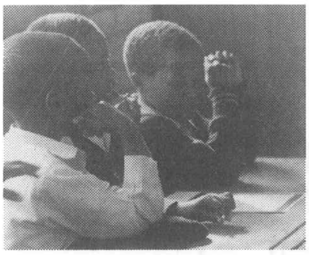
You will also need suitable counters (buttons, bottle tops, coins, counters of different colours) for each group. These counters should fit the size of each square or cell of the numbers in the chart so that it does not overlap the next cell. The counter should be big enough to cover the number. We used 1 cent coins as they fit well with our cells.

Step 1 - Exploring the 120 Number Chart



It is important to make choices as early as possible about the kind of questions you will ask your learners. My choice when introducing the task is to start with open-ended questions like:

- What patterns do you see when you look at the chart? (For this year group, a statement like "say what you see", is also good.)
- Make any rules about what you see.
- Can you show why your pattern will always work?



The reason for using open questions is that, the activity is set as an investigation and is open-ended. We don't tell learners what to say, and the teacher doesn't direct learners' attention to patterns. This is in line with the section of "good practice" we dealt with earlier on in this mthamo in Unit 1. The idea is to first work with what the learners have noticed, explored and found out. Later the teacher stresses some directions and ignores others.

Stressing and **ignoring** as concepts for teaching are crucial when teaching and learning mathematics at all levels. This is a skill that learners should also acquire. It means that we should always be on the alert. There may be things that learners say that can be ignored. But there are other things that the teacher will need to stress. Making decisions and choosing where to focus attention is an important underlying freedom for learners. This allows learners an opportunity to:

- develop their own strategies for solving problems
- explore and experiment
- devise their own means of communicating their work
- apply their own methods of finding solutions

- organise, share and discuss their own thoughts and ideas
- work with an investigation
- develop and defend their own **conjectures**.

When I was doing this activity with teachers and learners while working for the Mathematics Education Project, at the University of Cape Town, they taught me new things about the 120 number chart. I would like you to have the same experience with this activity. During the lesson and after it, jot down everything your learners have taught **you** about mathematics.

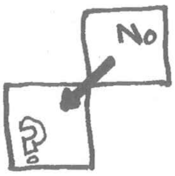
*You also need to encourage your learners to write their own journals about what they have learnt. For example, ask them to write down everything they do **not** want to forget about the lesson. They could also write something they would like to tell someone else. Or they could write something they would like to use again.*

Also in working with patterns, a question like, "*What changes and what stays the same?*" has been a useful question for me.

What sort of things can you expect learners to come up with?

- The ones go down the column.
- 10 is in the same column as 120.
- Rows go up in ones.
- Columns increase in tens.
- All the numbers that end with two are in one column.
- This column belongs to two (2, 22, 52, etc).
- The numbers at the top are small. The numbers near the bottom are big.

You may find **conjectures** such as, "*The units digit in each column is always the same for all numbers in that column.*"

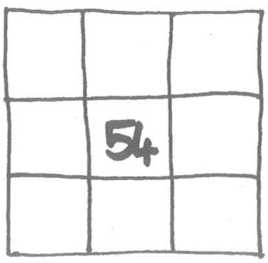


After a while, I invite them to look at what happens to the numbers, as you go down diagonally to the left, to the right? What happens when you go up diagonally to the left and to the right? (The movement up and down the column may not be obvious as yet. You may find that it depends on how developed their place value concept is.)

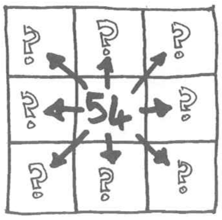
When you feel that they have explored the number chart quite carefully, you are ready to go on.

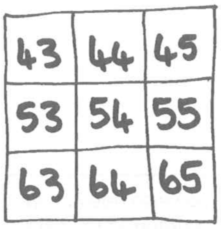
Step 2 - A Closer Look at part of the Chart

Now we look at a 3 by 3 square with only one number written. Draw a 3 by 3 square on the chalkboard with the number 54 at the centre.

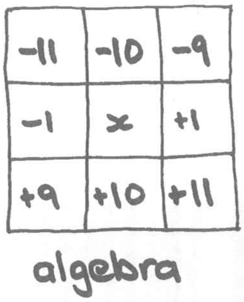


Then get the class to help you fill the grid with the possible numbers, using their **conjectures**. What we do is focus





This can be explained using algebra. If you have "any number", let's say "x" in the middle cell. In order to find its neighbour on the right you add 1, to get "x + 1". To get the one on the left you subtract 1, to get "x – 1".



our attention on a number and its relationship with its neighbours. We are trying to work out the rules for movements from one square to the next in the **8** possible directions. We do this by asking the difference between the neighbouring numbers.

You need to decide if you want to repeat this with another number. You can ask learners to move from the number 34 to the number 39, and outline the route, and count the number of moves. When most of the class are confident about what they have found out here, you can move onto the next step.

Step 3 - Looking at Bigger Movements

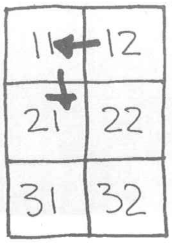
Learners should have lots of experiences in these movements until they can see that going across the row 10 steps, can be replaced by going down the column once. For example, 34 + 9 could be seen as "move down the column one step, and move across to the left one step".

We then explore more movements using the grid. For example,

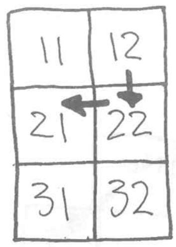
53 +   = 78   *How do we get from 53 to 78?*

45 –   = 29   *How can we get from 45 to 29?*

Learners must be encouraged to suggest their own movements. They also need to include going up the grid.



12-1+10=21



12+10-1=21

A few will start noticing that the movement from 12 to 21 can be either, "Start at 12. Move one step down and one step to the left," which means 12 + 10 - 1 = 21. Another way is to say, "From 12, one move to the left, and then one move down the column," which means 12 - 1 + 10 = 21. I have found it important not to pressurise them into looking for the shortest route straight away. It is better for them to make their own choices about which routes they want to follow. They will see the shorter routes when they are ready.

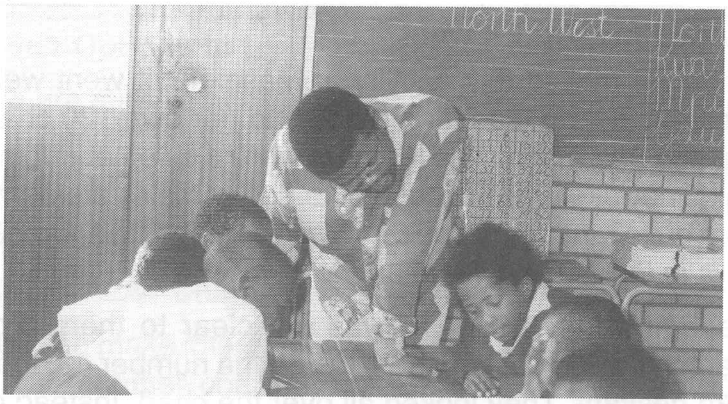
Get your learners to use the chart to find answers to the following:

34 + 25 =

58 – 23 =

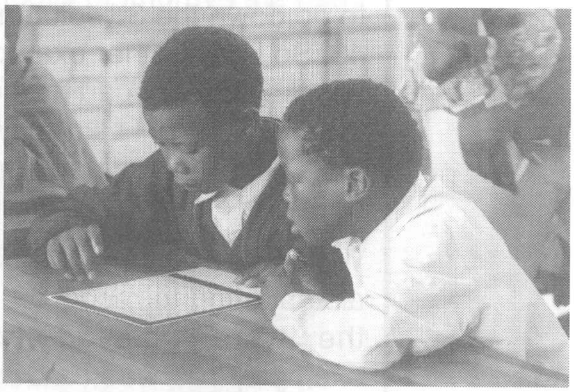
76 –   = 39

Then learners can design their own problems for their partners, or for their opposite pair to solve.

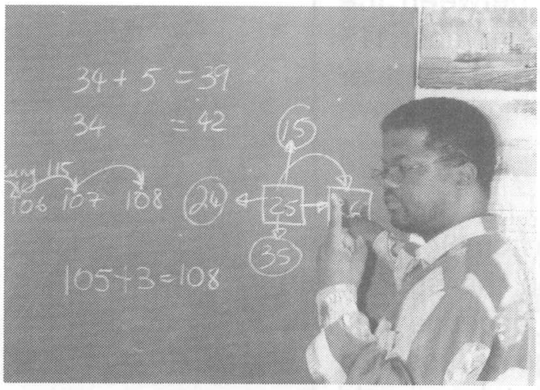


What patterns do you see?

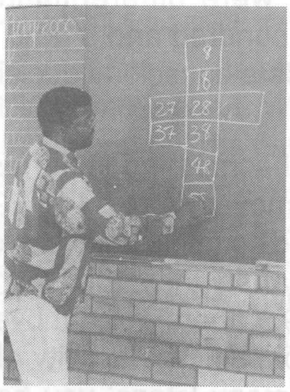
What stays the same?



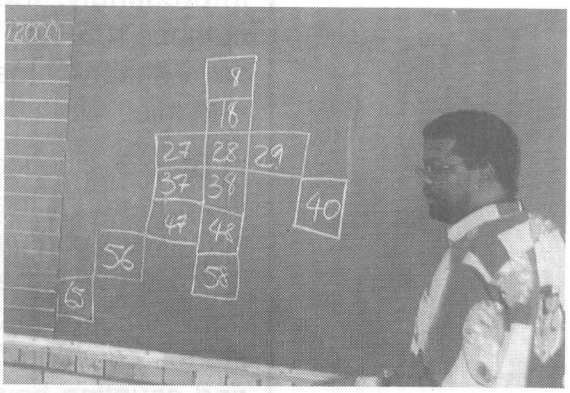
What changes?



What do we find when we move one step to the right?



So the number below 48 must be ......



Who wants to come and draw the next empty block?

This is my conjecture. If ...... then ......

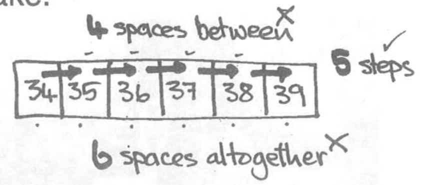
Reflection – How it went

When we trialled this Activity at Masakhe, it went well. Interestingly the teacher already had her own 120 chart that she had made. The learners were excited about the task. They all wanted to contribute. At first their statements were quite random. "*I see 63*", "*I see 12*", and "*the number 15 is in the same column as the number 25*".

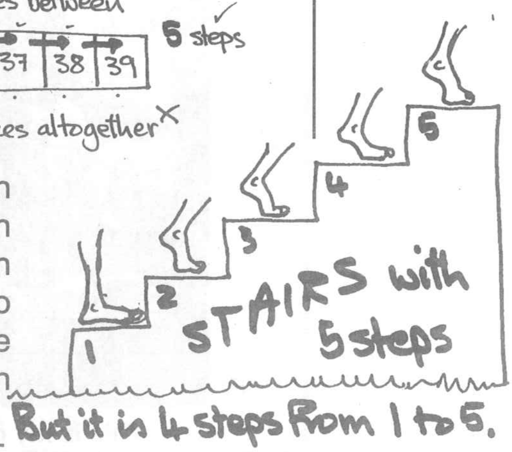
It seemed that the chart was not clear to them. For example, when I asked them to find the number 43, they were hesitant. They looked all over the chart, instead of looking in the 5th row, (the row of 40s). They didn't know to go down the 3rd column with the threes till they got to 43. So I gave other examples like 4**5**, stressing the **5** as I spoke. They began to see that they could go straight to the 6th column if I said sixty-**six**. On reflection, I could see that this was how fluency with number work could start to develop.

What was quite encouraging for me was that, they were also starting to recognise numbers more than 100. But they were confused about how you say them. For example, 105 was read as 150. This is because of the irregularity of the structure of the English language. Interestingly, they were differentiating between the number 103, as "1 hundred and 3" and the number 115 as "1 fifteen". I made a conscious decision not to bother with this.

In Step 3, I noticed something important when I asked them to count the moves from 34 to 39. I realised that some were confused counting the steps on from 34. They got 4 moves, because they focused on the spaces *between* 34 and 39. Others got 6 moves because they started counting *from* 34. The activity of going 'upstairs' and counting how many 'steps' you have taken, could help here. You don't count the step you are on. You count the **actual steps** you take.



This is quite a common error in counting on (adding) especially when children rely on their fingers. When we trialled the activities in Umthamo 5 at Mpongo Primary School, some children had the same difficulty when counting on using their fingers.

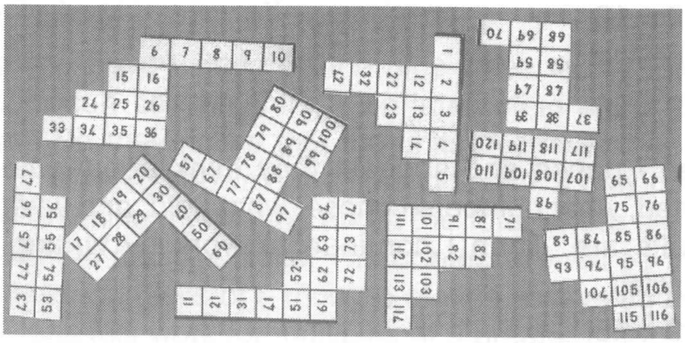


Other Options to try

We are sure you can think of many interesting activities to extend this work. This will enable children to become really fluent with numbers, as they use **conjectures** that they have become familiar with.

120 Number Chart Jigsaw

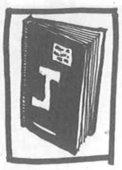
You can make interesting puzzles, if you copy and cut up a 120 number chart into different shaped-units. Store these in separate envelopes. Working on their puzzles can help learners to use and modify or improve the rules they have developed.



Patterns of Counting

Get your learners to count in 10s and put a counter on every 10 in the 120 number chart. What pattern do they get? Now count in 5s, and put the counters on every 5th number. What pattern do they get? What pattern do they think they will get when they count in 2s? Get them to put counters on every second number. Do they get the pattern they predicted? Now let them try with 3s. If you want, they can record the patterns they find.

Writing your own reflections



Later in the day, after you have completed this Activity, spend time thinking carefully about what happened. Try to reflect on what surprised you, and why. What did you learn from this experience? What will you do differently when you do this Activity again? Why? What can you do to extend your learners' experiences of pattern?

Write your reflections in your Journal. Make sure that you store samples of your learners' work in your Concertina File, together with any special notes you have made on what they did. Be prepared to share this work on pattern at your Portfolio Presentation at the end of the year. Make sure that you also record and reflect on any further work that comes from the ideas in this mthamo.

Option C - Patterns and Conjectures for Multiples

For this option, I would like to relate what I do in my classroom. You can use the two steps suggested here, or you can include some ideas suggested under Other Options.

Step 1 - Giving them something to think about

I stand by the blackboard and ask the learners to be totally silent. I ask them to work inside their own heads. I ask them to watch what I am doing and try to work out what it is. I write the following on the chalkboard.

1 x 9 = 9

2 x 9 = 18

3 x 9 = 27

4 x 9 = 36

Then I pause and suggest they think about where I started, what numbers I chose, what I did, and what the results are. Then I continue to work by writing the following.

5 x 9 = 45

6 x 9 = 54

7 x 9 = 63

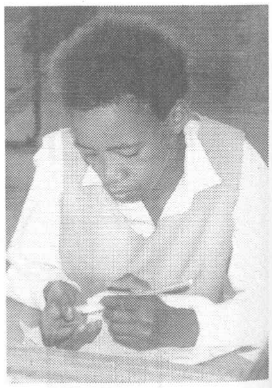
8 x 9 = 72

I remind them not to shout out. I suggest that if they think that they know what is happening, they should construct some similar examples in their heads, or think of ways to describe what they see. Meanwhile, I tell them I am going to write more and they must continue to watch.

9 x 9 = 81

10 x 9 = 90

The event is rigidly controlled *on the surface*. There is a social element to the control. It is a quiet, settled start to a lesson. If everyone is peaceful, I am happy and likely to behave like a friendly and attentive teacher. An observer might say that I am "controlling the class". At this point, I encourage them to share what they see with one another. I prefer that they work first in pairs, and then come up with ideas to share with the whole group.



I then take responses and write them down (with their names) as **conjectures** for everyone to see. I accept every response without commenting or asking questions. When they have shared their ideas, I carry on writing, without testing their **conjectures**.

11 x 9 = 99

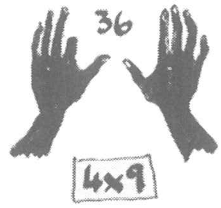
12 x 9 = 108

13 x 9 = 117

I then skip a couple and write

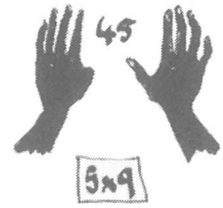
17 x 9 = 153

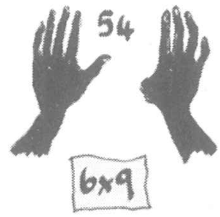
I watch carefully how they are trying to make sense of what is going on. I still don't interfere, but let them wonder how could I be so fluent about multiplying by 9. (You can read more about the idea of fluency in the Notes at the end of this activity.)



Step 2 - Looking for other patterns

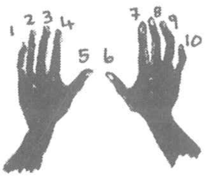
Once you have written up the nine-times table, ask if learners can see anything else interesting about the pattern. Do they notice that the units decrease by one each time you go down? The tens increase. If they don't notice anything interesting about the total of the digits, ask them to add the digits. The **conjecture** will be that the combined digits of a multiple of nine always add up to 9. Does this work for multiples of 9 greater than 10 x 9? Here is a good opportunity for some **investigation**.





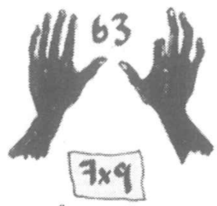
The nine-times table can also be learnt using your fingers.

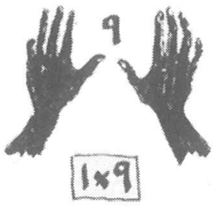
Step 3 - Finger Maths



The Pattern of the nine-times table

Get the learners to hold up their hands with palms facing away from themselves. Show them how to count from 1 – 10, bending each finger, starting from the left small finger to the right small finger.





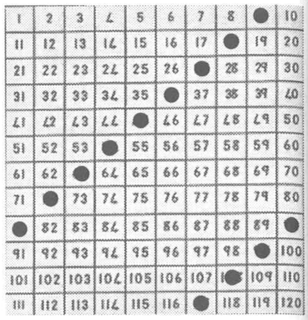
Multiplying by 9

Suppose you want to multiply 4 by 9. Hold up both your hands with the palms facing forward. Hold down the fourth finger of the left hand because you are multiplying 4 by 9. Count the number of fingers on the left of the fourth finger - this will be the tens in the answer. Count the number of fingers after the fourth finger - these will be the units. Try some more examples with your learners. Each time, hold down the finger that corresponds with the number being multiplied by 9. So for 6 multiplied by 9, you hold down the sixth finger.

Give the learners a chance to investigate to see if it works for all the multiples of nine, up to 10 x 9.

Patterns on the 120 number chart

Hand out the 120 number charts provided and ask the learners to predict what pattern will form if they cover all the multiples of 9 with a counter. Then hand out counters and let them do the task practically to see if what they envisaged (visualised or predicted) was correct. Ask them if they can discuss and give a reason for the pattern. (If you add 9 on the number chart, the rule is that you move one space down, diagonally to the left.)



Multiples of 9

Note 1

I deliberately used the term **"fluent"** at the end of Step 1 for a particular reason. I chose not to say learners 'understand' something. "Fluency" is often used as a product of having completely integrated some sequence of actions or operations where you no longer have to be necessarily conscious. For example, I no longer have to be conscious about adding 2 and 2, multiplying 5 by 5, multiplying by 10, counting backwards from any number, etc. I am fluent. Which means, I have already achieved a high level of automation. Movements can be fluent, as can spoken language or reading aloud. Computations in Maths, too, can be performed fluently.

Fluent operation might be seen as one central goal of mathematics education at any level. So a central question is how to work on gaining fluency. Understanding, on the other hand could be regarded as being familiar with something, being enlightened by it, having seen what it means. So, we can see the 'Bus-Stop' dance, recognise it, but not necessarily be fluent in performing it.

Going back to the notion of **control,** there is more to it than meets the eye. For example, I have no control over what goes on inside their heads. They can choose to ignore what is being done on the board and pretend merely to watch. They may be misreading what is there or thinking about the mathematics in what, to me, is an unexpected way. They may have such a blockage about multiplication that they cannot begin to work with it. They may already be bored and day-dreaming. Or, they may be doing comparable calculations with 9 as I have asked.

I have given them the **freedom** to engage with the task or not. I have also given them the freedom to work with it as they choose. The first of these freedoms exercises me most at the start of a new piece of work. How can I make it more likely that they choose to get involved? The other is that it is not in my power to give a freedom which they already possess, namely the way they think.

You will find more rules of divisibility on page 44

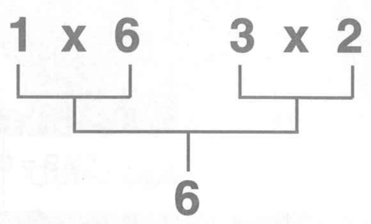
My **intention** as a teacher, however, is to offer them different ways of working with mathematics, help them appreciate the value of different approaches and enable them to make sensible choices about methods. I also have to maintain a balance between the freedom to think and construct meanings for themselves, and the externally imposed agenda of the school, society, government or the subject itself. By now, they may be asking what about multiplying by 2, 3, 4, 5, 6, 7, 8, etc?

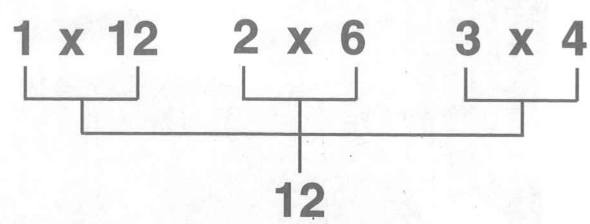
Note 2

Learners should be able to recognise the simple **multiples,** and be able to analyse the development of these tables by looking at their relevant attributes.

They need to distinguish between numbers that are divisible by 2 from those divisible by 5. They may not be able to see at this stage that there is a relationship between those that end in 0 and those that are even. For analysis, they must be able to say that the 5 times tables ends in either 5 or 0.

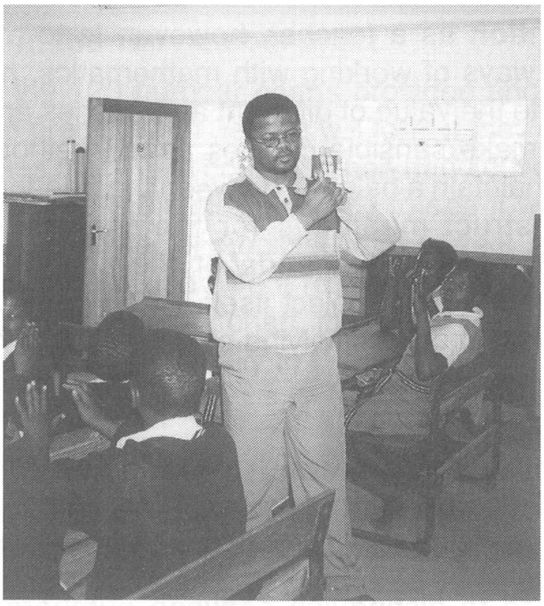
The teacher should also encourage them to recognise the properties that are common to some numbers. For example, all multiples of 4 are multiples of 2, but not all multiples of 2 are multiples of 4. They may be encouraged to form family trees of multiples. 6 has 4 members; 6, 3, 2 and 1. 5 has 2 members, 1 and 5.



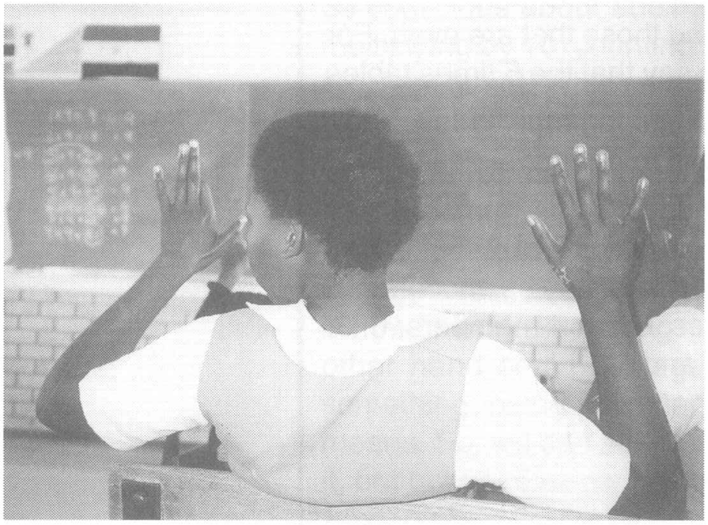


The other important skill is for them to reason deductively, where they develop conjectures. For example, a number is said to be a multiple of 10 if it ends in 0.

These conjectures will have to be justified for all cases. For example, they should be able to deduce the divisibility rules for all the multiples from 2 to 20 (see Content Audit in the Appendix).

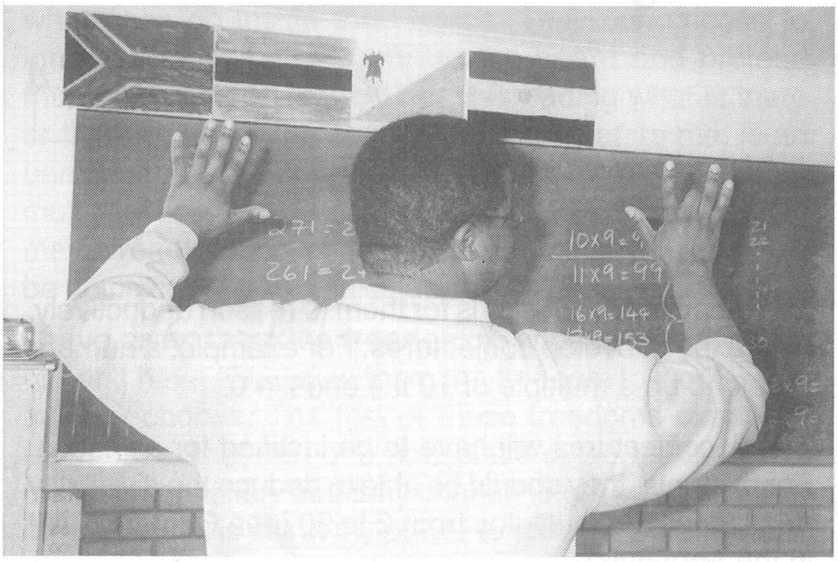


This finger stands for 4



4 x 9 = 36

7 x 9 = 63



8 x 9 = 72

Reflections

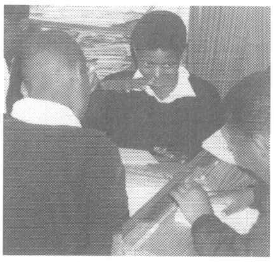
When I completed the task, I felt that I coud have done it differently. I think it would have been useful for the learners to see the patterns in groups of 10. For example, I should have written the nine-times table from 1 up to 10, and then 11-20, etc, so that they could see how the patterns develop. They came up with a number of interesting conjectures. For example, in order to get the number in the tens (eg 5 x 9) you take away 5 from 9 to get 4, and then 9 - 4 gives you the unit.

For me this was profound, and complicated. What I usually use is 5 x 9 = 45. You say, 5 - 1 to get 4, then 9 - 4 = 5 in order to get 45. So, in order to get the answer for multiplying by a number less and equal to 10, first you subtract 1 from the number you are multiplying by 9. Then, when you subtract *this* number from 9, it gives you the second digit.

Another possible conjecture is, the units go down by one as the tens increase by one. The digits of the product add up to 9. This was further extended to numbers more than 20. For example,

27 x 9 = **24**3 (27 is in the third group, so 27 - 3 = 24 --> 2 + 4 = 6 and 9 - 6 = 3.)

What was exciting here was to see how challenged the learners were about the maths they were developing. At the beginning of the Activity, one group of boys seemed rather reluctant to really engage seriously in what we were doing. But by the time we stopped, we had the feeling that they would have been very happy to carry on.



Other Options

Reverse Digits and Subtract

Take a 2 digit number, 27
Reverse its order. 72
Subtract the smallest from biggest. 72 – 27 = 45
Take another example.
What do you notice? Will this always be the case?

Take a 3-digit number, reverse its order.
Subtract the smallest from the biggest.
Try with other numbers.
Will you always get the same answer? Explain.

Try a four digit number.
What do you observe?
Try with other numbers.
Make rules about what you see. Explain your rules.

Looking for primes

Prime numbers are numbers such as 2, 3, 5, 7, 11, that cannot be broken up into factors except for themselves and 1. Is 151 prime? The search for prime numbers and methods to identify them began with the Greeks and still continues to this day. Mathematicians were and still are interested in the actual distribution of the prime numbers and in finding formulae from which they can be obtained. Attempts to find a formula from which all prime numbers can be obtained have failed. They are infinitely numerous, and they occur scattered through the orderly scale of numbers, with an irregularity that at once teases and captivates the mathematician.

The Sieve of Eratosthenes

A question often asked is: *How often, or how rarely, do prime numbers occur on the average?* Or, to put it in another way, *What is the chance that a specified number is prime?*

One of the earliest known methods for finding prime numbers is that known as the Sieve of Eratosthenes (275-194 B.C), arranged in an n by 6 grid (see below). Your learners might be interested to see if there is a pattern of prime numbers.

1 is not a prime number, so cross it. Circle 2 and cross off all its multiples. Circle 3 and cross off with a different sign all its multiples. Since 4 is already crossed, go to 5 and do the same using a different sign. Continue doing this until all the non-prime numbers (compound) are crossed. Write down everything you notice and share it with the whole group.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120

Goldbach's conjecture

One famous conjecture is that of the 18th century mathematician, Christian Goldbach, who said:

Every even number is the sum of two prime numbers, and every odd number is either prime or the sum of three prime numbers.

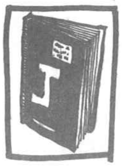
Thus $6 = 3 + 3$

$8 = 3 + 5$

but $10 = 5 + 5$

and $10 = 3 + 7$

Can you find other numbers that can be written in one way, two ways, three ways, etc.? What do you notice?



Writing your own reflections

Later in the day, after you have completed this Activity, spend time thinking carefully about what happened. Try to reflect on what surprised you, and why. What did you learn from this experience? What will you do differently when you do this Activity again? Why? What can you do to extend your learners' experiences of pattern?

Write your reflections in your Journal. Make sure that you store samples of your learners' work in your Concertina File, together with any special notes you have made on what they did. Be prepared to share this work on pattern at your Portfolio Presentation at the end of the year. Make sure that you also record and reflect on any further work that comes from the ideas in this mthamo.

4:00

Part 2 - Reading 2

Understanding *by Michelle Selinger*

Now you have completed the **Key Activity** with your learners, we would like you to read a chapter on helping learners develop their **understanding** of Maths. This Reading is separate from the umthamo.

In this Reading, Michelle Selinger examines some tasks that teachers can introduce into their activities to try and increase pupil autonomy; to encourage their learners to think about **what** they are learning, **how** they are learning it, and whether they **understand** what they are learning. These tasks are not necessarily restricted to mathematics classrooms, and indeed they have been used as strategies in all areas of the curriculum. But here Michelle Selinger has attempted to demonstrate the possible effects on learning mathematics.

This Reading is important. When you have finished reading it, you will see how Michelle Selinger believes the strategies she discusses have offered an opportunity for learners to learn and understand mathematics **relationally**. We would like you to take what she has to say in the Summary very seriously.

In the Reading, Michelle Selinger writes about concept maps. Before reading her chapter, spend some time reflecting on *your* understanding of your learners' learning. Make a mind-map (concept map) of their learning.

After reading the chapter, try to reformulate your concept map of their learning, in the light of what you have read. Record your reflections in your journal.

Conclusion

In this mthamo we have outlined the context for preparing learners to have first hand experiences with patterns. These experiences are developed where learners describe the pattern or arrange their own patterns in terms of shape, sizes, colour, texture, position, or number.

As learners experience these patterns by completing them, describing them, and making them, they become used to the concept. Eventually, this leads to finding number patterns, generating conjectures, formulae. This becomes an introduction to algebra, particularly graphs. Although patterns are an entry to abstract mathematics, pattern recognition could also assist in the teacher-student relationships, and as a medium of communication. The activities in this workbook are designed in such a way that the teacher's role is transformed from that of a bearer of information, where the emphasis is on manipulating symbols, calculations of big numbers or algebraic expressions to that of a systematiser, a facilitator, a modeller, a co-explorer, a promoter of explorations, an instigator, and a helper. The emphasis is on developing mathematical processes, giving students more responsibility, so as to be able to make conjectures and test them, formulate problems, evaluate and validate their conjectures, and be able to design independent investigations through extending their findings.

Happy, and healthy teaching!!

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Appendix - Content Audit

Rules of Divisibility

A number is divisible by

2 if it is even

3 if sum of the digits is a multiple of 3

4 if last 2 digits are a multiple of 4

5 ends in 5 or 0

6 even, and sum of digits is a multiple of 3

7 drop the last digit and subtract the double of that last digit from the remaining part. If the result is a multiple of 7, then the original was a multiple of 7

8 last three digits are a multiple of 8

9 sum of the digits is a multiple of 9

10 ends in 0

11 drop the last digit. Now subtract the last digit from the rest. If the result is a multiple of 11, then the original number was a multiple of 11.

12 satisfies rules for both 4 and 3

13 it works like the rule for 7, except, instead of multiplying the last digit by 2, we must multiply it by 9, before subtracting from the remaining part.

14 satisfies rules for both 2 and 7

15 satisfies rules for both 3 and 5

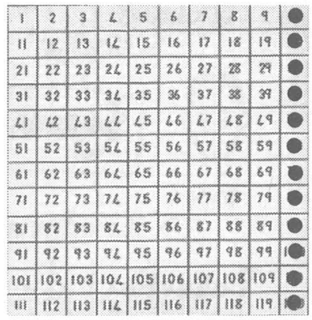
16 last four digits are a multiple of 16 (not easy to check)

17 like the rules for 7 and 13, except the last digit is multiplied by 5, before being subtracted from the remaining part.

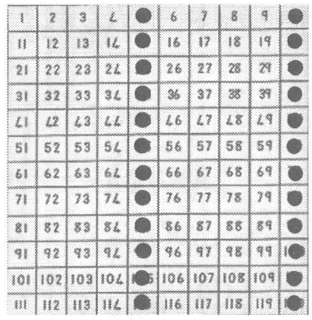
18 satisfies rules for 2 and 9

19 (this is pretty bad!) like the rules for 7, 13, and 17, except the last digit is multiplied by 17 before being subtracted from the remaining part.

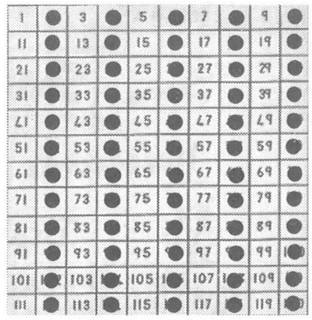
20 ends in 0, and the next to last digit [0] is even.



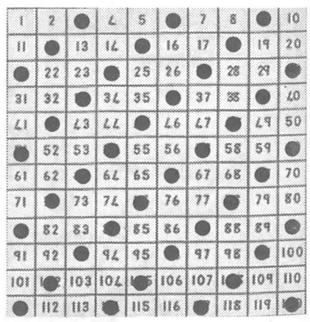
Multiples of 10



Multiples of 5



Multiples of 2



Multiples of 3

UNIVERSITY OF FORT HARE

DISTANCE EDUCATION PROJECT

CORE LEARNING AREAS COURSE

Mathematical Literacy, Mathematics and Mathematical Science

4th Umthamo - Developing Mathematical Thinking Using Patterns

First Pilot Edition - 2000

Mthunzi Nxawe, with the help of
Alan and Viv Kenyon

Co-ordinated, illustrated and edited by
Alan and Viv Kenyon

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