Section 2b:
Financial Mathematics

Maths Literacy, Workshop Series 2010

Section 2b: Financial Mathematics

1. Introduction

A financial literate person is able to manage his/her personal finances by making the best decisions on how to handle his/her money.

Why is financial literacy important? A financially literate person:

- Is able to budget correctly to meet his/her expenses;
- Is able to identify financial products or services that meets his/her needs;
- Is not likely to fall victim to abusive practices and scams.

Answer the following questions to see how well you are managing your money.

1 Do you save some of your money?	**a** Yes. I try to put some aside for new clothes. **b** Of course I save for a day in future that I might need it, as well as for my retirement. **c** Yes, if I have anything left to save.
2 Do you keep track of your money?	**a** No, I have better things to do. **b** My bank statements are enough. **c** I have a budget plan.
3 How much do you pay off on your credit card each month?	**a** The minimum amount. **b** I only borrow money when I am sure I can keep up the payments and only if I really need something. **c** I sometimes use one credit card to pay off another.
4 If you want to buy a new computer with a loan, what would you do?	**a** Look for the best deal for both the computer and the loan. **b** Take advice from a friend or family member. **c** Find a computer I like and take the first finance that I receive.
5 Will you have enough money for your retirement?	**a** I am far too young to think about this now. **b** I think I will have enough when I retire. **c** I have made sure that I've got a plan to have enough when I retire.
6 How will you react if you received a phone call offering you a chance to make big money?	**a** Take up the offer. **b** Consider it carefully by seeking qualified advice. **c** Ignore it, there is a catch. If it is so easy, why isn't everybody rich?

http://www.understandingmoney.gov.au/tools/Consumer/healthcheck/

(03-03-2009)

Check your answers at the end of the section.

Learning outcomes

At the end of this section you should be able to make appropriate decisions to manage your personal finances.

START UP ACTIVITY 2.1:
Which contract will it be?

Pair up with a class mate and complete the following activity.

Your parents have offered to buy you a mobile phone on the condition that you select a plan which costs no more than R200 per month.

1. Compare and contrast the two mobile phone deals below for the following situations, and give the best option.
   - **1.1.** Suppose you will make most of your calls during peak times to another cell phone.
   - **1.2.** Suppose you will make most of your calls during off-peak times to another cell phone.
   - **1.3.** Suppose you will make most of your standard calls during off-peak times.

2. Compare the free minutes of each contract, taking into consideration their monthly charge.
   - **2.1.** Which contract will be most expensive during peak hours?
   - **2.2.** Which contract will be the best choice for off-peak hours?

3. Recommend a particular contract and explain why this will be the best option for you.

MTN BUNDLED TARIFFS			VODACOM TALKTIME TARIFFS		
	Companion	Pulse		Weekender	Weekender Plus
Connection Fee	R90.63	R90.63	**Connection Fee**	R91.20	R91.20
Monthly Charge	R114	R148.20	**Monthly Charge**	R114	R130
Free Minutes	15	120	**Free Minutes**	100	120
Free Minutes Usage	Anytime	Weekend	**Free Minutes Usage**	Weekends	Weekends
NO SMS sent	R0.89	R0.89	**SMS sent**	R0.68	R0.68
Standard Calls Peak	R2.48	R1.37	**Peak Standard**	R2.51	R2.51
Standard Calls Off-Peak	R0.68	R0.68	**Off-Peak Standard**	R0.68	R0.68
Cell-to-cell Peak	R2.48	R2.05	**Peak cell-to-cell**	R1.37	R1.37
Cell-to-cell Off-Peak	R0.68	R0.68	**Off-Peak cell-to-cell**	R0.68	R0.68
Peak national	R2.48	R2.05	**Peak national**	R2.51	R2.51

Off-peak national	R0.68	R0.68	**Off-peak national**	R0.68	R0.68
Service Call Peak	R2.51	R2.05	**Service Calls Peak**	R1.37	R1.37
Service Call Off-Peak	R0.68	R0.68	**Service calls Off-Peak**	R0.68	R0.68
Service Call Off-Peak	R0.68	R0.68	**Service calls Off-Peak**	R0.68	R0.68
http://www.cellular.co.za/africa/south-africa/mtn/mtn-bundlerates.htm *(03-03-2009)*			*http://www.cellular.co.za/africa/south-africa/vodacom/vod-talktime.htm* *(03-03-2009)*		

Time Periods Applying:

When?	**Mon-Fri**	**Sat**
20h00-07h00	Off-Peak	Off-Peak
07h00-20h00	Peak	Off-Peak

2. Interest

Borrowing money is a necessary transaction for most people and businesses. Whenever money is borrowed or goods are bought on credit, the customer pays interest on the amount due.

- The amount of money on which interest is paid is called the **initial value** (P).
- The interest charged on the borrowed money is expressed as a percentage. This percentage is called the **interest rate** (i).
- When money is borrowed, the borrower agrees to pay back the initial value and the interest within a specified period of **time** (n).

Simple interest

Simple interest calculations are used to determine the approximate interest gained once money has been invested. Suppose you invest an amount of money at a certain bank and you are informed that you will earn simple interest on the invested amount. This means that when the interest is calculated, only the initial invested value will be considered. You will not earn interest on the interest that you have earned on the initial invested amount. It is assumed in each case that the interest rate remains constant.

Simple interest is therefore the cost of borrowing money, computed on the initial value only.

The interest due depends on 3 things:

- the initial value (P)
- the interest rate (i)
- the number of years over which the loan is paid back (n)

The relationship is expressed as follows:

Formula for simple interest:
$SI = P \times i \times n$

where SI is the total simple interest paid
P is the initial value
i is the annual interest rate written as a decimal, and
n is the time in years

The amount to be repaid at the end of the n-year period is equal to the initial value plus the interest:

$$\text{Amount to be repaid } = P + SI$$

The time period is always quoted in years, so convert months to years before applying the time in the formula. Remember that:

1 month = $\frac{1}{12}$ years

2 months = $\frac{2}{12} = \frac{1}{6}$ years

9 months = $\frac{9}{12} = \frac{3}{4}$ years

73 days = $\frac{73}{365} = \frac{1}{5}$ years

292 days = $\frac{292}{365} = \frac{4}{5}$ years

EXAMPLE 5.1

Determine the simple interest when $R200$ is invested for $1\frac{1}{2}$ years at an interest rate of $9{,}5\%$ per year.

SOLUTION

$P = R200$

$i = \frac{9{,}5}{100} = 0{,}095$ and

$n = 1{,}5$ years

$SI = P \times i \times n = 200 \times 0{,}095 \times 1{,}5 = R28{,}50$

The interest earned is $R28{,}50$.

EXAMPLE 5.2

John borrows $R5000$ for 5 years at a simple interest rate of 19% per year. How much interest must he pay? What is the total amount that must be repaid?

SOLUTION

$P = R5000$

$i = \frac{19}{100} = 0{,}19$

$n = 5$ years

$SI = P \times i \times n = 5000 \times 0{,}19 \times 5 = R4750$

John has to pay $R4750$ interest.

Amount to be repaid $= P + SI$

$= R5000 + R4750$

$= R9750$

The total amount that John must repay is $R9750$.

EXAMPLE 5.3

How much interest will $R3000$ earn in 9 months at a simple interest rate of 12% per year?

SOLUTION

$P = R3000$

$i = \frac{12}{100} = 0{,}12$

$n = 9$ months $= \frac{9}{12} = \frac{3}{4}$ years

$SI = P \times i \times n = 3000 \times 0{,}12 \times \frac{3}{4} = R270$

The interest earned is $R270$.

EXAMPLE 5.4

Your friend lends you $R600$ and states that you must pay him back $R645$ at the end of the month. What is the simple interest rate that he is charging you?

SOLUTION

$P = R600$

$n = 1$ month $= \frac{1}{12}$ years

$SI = R645 - R600 = R45$

We now have to solve for i, so we make it the object of the formula $SI = P \times i \times n$.

Then $i = \frac{SI}{P \times n}$ and we substitute the values into this formula.

$$i = \frac{45}{600 \times \frac{1}{12}} = \frac{45}{50} = \frac{9}{10} = 0{,}9 = 90\%$$

The annual interest rate that he charged you was 90%.

EXAMPLE 5.5

How long will it take an investment of $R600$ to amount to $R1300$ at a simple interest rate of $12{,}5\%$ per year.

SOLUTION

$P = R600$

$i = \frac{12{,}5}{100} = 0{,}125$

$SI = R1300 - R600 = R700$

We now have to solve for n, so we make it the object of the formula $SI = P \times i \times n$.

Then $n = \frac{SI}{P \times i}$ and we substitute our values into this formula.

$n = \frac{700}{600 \times 0{,}125} = \frac{700}{75} = 9{,}3\dot{3}$ years, or 9 years and 4 months. Explain how you get 9 years and 4 months.

EXAMPLE 5.6

An acquaintance assures you that if you invest $R4000$ today, your money will double in $2\frac{1}{2}$ years. What annual simple interest rate is he promising you?

SOLUTION

We know that the original amount of $R4000$ doubles to $R8000$.
This means that our interest earned was: $R8000 - R4000 = R4000$.

$P = R4000$

$n = 2\frac{1}{2} = 2{,}5$

$SI = R4000$

We now have to solve for i, so we make it the object of the formula $SI = P \times i \times n$.
Then $i = \frac{SI}{P \times n}$ and we can substitute our values into the new formula.

$i = \frac{4000}{4000 \times 2{,}5} = \frac{2}{5} = 0{,}4 = 40\%$.

He is assuring you of a 40% simple interest rate.

EXAMPLE 5.7

Samantha bought an artwork which has increased in value by 50% in the time that she owned it. She also figured out that her investment increased in value with 8% annually (assuming simple interest). How long ago did Samantha buy the artwork?

SOLUTION

We do not know the original value of the artwork, when she bought it. We only know that it increased by 50% while she owned it. If, for example, she bought it for $R200$ and it increased by 50%, then it would now be worth $\frac{150}{100} \times 200 = R300$. Notice that we multiply 200 with $\frac{150}{100} = \frac{3}{2} = 1{,}5$. The original cost therefore increased $1{,}5$ times and we have that:

$SI = 1{,}5P$

$i = \frac{8}{100} = 0{,}08$

We now have to solve for n, so we make it the object of the formula $SI = P \times i \times n$.
Then $n = \frac{SI}{P \times i}$ and we substitute the values into this formula.

$n = \frac{1{,}5P}{P \times 0{,}08} = \frac{1{,}5}{0{,}08} = 18{,}75$ years or 18 years and 9 months

Samantha bought the artwork 18 years and 9 months ago.

EXAMPLE 5.8

Suppose your dad wants to purchase a car valued at $R45000$, in four years' time. He wants to invest his money at a simple interest rate of $8\frac{1}{3}\%$ per year to be able to afford the car. How much money should he invest now in order to achieve his aim?

SOLUTION

$$i = \frac{8\frac{1}{3}}{100} = \frac{25}{3} \times \frac{1}{100} = \frac{1}{12} = 0{,}08\dot{3}$$
$$n = 4$$
$$SI = R45000 - P$$

We now have to solve for P, so we make it the object of the formula $SI = P \times i \times n$.
Then $P = \frac{SI}{i \times n}$ and we substitute the values into this formula.

$$\begin{aligned} P &= \frac{45000 - P}{0{,}08\dot{3} \times 4} \\ P\left(0{,}08\dot{3} \times 4\right) &= 45000 - P \\ 0{,}\dot{3}P &= 45000 - P \\ P + 0{,}\dot{3}P &= 45000 \\ 1{,}\dot{3}P &= 45000 \\ P &= \frac{45000}{1{,}\dot{3}} \\ P &= R33750 \end{aligned}$$

He should invest $R33750$.

LEARNING ACTIVITY 2.2

1. Find the simple interest on R500 invested for 3 years at 8% interest per annum.

2. Suppose you borrow R2 000 for 4 years at a simple interest rate of 19%.

2.1. How much interest will be paid?

2.2. What is the total amount that must be repaid?

3. Joe deposits R700 in the bank and earns simple interest at a rate of R9 per R100 per annum. How much interest does he receive at the end of the year?

4. Determine the simple interest rate if you invested R1000 for 4 years and earned R350 interest.

5. Determine the amount invested if you earn R200 simple interest for 2 years at 5% per annum.

6. If you end up with R695 after investing R500 at 13% simple interest per year, find the period of the investment.
7. An investor wishes to have saved an amount of R3 500 in 3 years time. She can invest her money at a simple rate of 11% per annum. How much money should she invest now in order to achieve her aim?
8. A man borrowed R3 500 and a year later paid back the loan plus simple interest with a cheque for R4 200. Find the annual rate of interest, as a percentage, paid for the loan.
9. The owner of a business overdrew his bank account by R1 500. The bank charged him R50 simple interest on his overdraft at the end of a 5 month period. What interest rate did the bank charge him per annum?
10. At what annual simple interest rate should R96 be invested for 6 months so as to produce the same interest as R75 invested at 10% per annum for 1 year?

Compound interest

When the interest earned on an investment at the end of each year is not a fixed amount determined by the initial value only, but also by all other interests earned (at the end of previous years), then the interest earned is called compound interest. In this case, the interest at the end of each successive year will be more than the interest for the previous year, since the amount on which interest is calculated keeps increasing after every year.

Let us illustrate the idea below:

EXAMPLE 5.9

Find the total interest when R300 is invested for 3 years at an interest rate of 11% compounded annually.

SOLUTION

$P = 300$

$i = \frac{11}{100} = 0{,}11$

$n = 3$

1st year:
$SI = P \times i \times n = 300 \times 0{,}11 \times 1 = \text{R}33$
$R33$ interest was earned during the first year.
Amount on which interest will be earned during the second year is $R300 + R33 = R333{,}00$

2nd year:
$SI = P \times i \times n = 333 \times 0{,}11 \times 1 = \text{R}36{,}63$
$R36{,}63$ interest was earned during the second year.
Amount on which interest will be earned during the third year is $R333 + R36{,}63 = R369{,}63$

3rd year:
$SI = P \times i \times n = 369{,}63 \times 0{,}11 \times 1 = \text{R}40{,}66$
$R40{,}66$ interest was earned during the third year.
Total amount at the end of the third year is $R369{,}63 + R40{,}66 = R410{,}29$

Total interest earned over three years is $R410,29 - R300 = R110,29$

OR we can make use of the following table:

Year	Instalment	Interest earned	New instalment
1	R300	R33	R333
2	R333	R36,63	R369,63
3	R369,63	R40,66	R410,29

So the total interest earned is $R410,29 - R300 = R110,29$.

EXAMPLE 5.10

Find the interest when R1 000 is invested for 3 years at an interest rate of 10% compounded annually.

SOLUTION

$P = 1000$

$i = \frac{10}{100} = 0,1$

$n = 3$

1st Year:
$SI = P \times i \times n = 1000 \times 0,1 \times 1 = \text{R}100$
Value of investment at the end of year 1 is $R1000 + R100 = R1100$

2nd Year:
$SI = P \times i \times n = 1100 \times 0,1 \times 1 = \text{R}110$
Value of investment at the end of year 2 is $R1100 + R110 = R1210$

3rd Year:
$SI = P \times i \times n = 1210 \times 0,1 \times 1 = \text{R}121$
Value of investment at the end of year 3 is $R1210 + R121 = R1331$

Interest earned = Amount at the end of year 3 – Initial value
$= R1331 - R1000$
$= R331$

The total amount of interest earned is R331.

Calculations such as the above can become very tedious. For this reason we use the following formula to find compound interest.

Formula for compound interest:
$A = P(1+i)^n$

where A is the future value of the investment
P is the present value of the investment
i is the annual interest rate written as a decimal, and
n is the number of years

$A = P +$ interest earned

Let us look at our previous examples again by making use of this formula.

Example 5.11

Find the interest when R300 is invested for 3 years at an interest rate of 11% compounded annually.

Solution

$$P = 300$$
$$i = \frac{11}{100} = 0{,}11$$
$$n = 3$$
$$A = P(1+i)^n = 300(1+0{,}11)^3 = R410{,}29$$

Interest earned = Amount at the end of year 3 – Initial value
$= R410{,}29 - R300{,}00$
$= R110{,}28$

The total amount of interest earned is $R110{,}28$.

Example 5.12

Find the interest when R1 000 is invested for 3 years at an interest rate of 10% compounded annually.

Solution

$$P = 1000$$
$$i = \frac{10}{100} = 0{,}10$$
$$n = 3$$
$$A = P(1+i)^n = 1000(1+0{,}10)^3 = R1331$$

Interest earned = Amount at the end of year 3 – *Initial value*
$= R1331 - R1000$
$= R331$

The total amount of interest earned is $R331$.

EXAMPLE 5.13

Suppose your sister opened an account by depositing $R3500$ on 1 June 2006. If the account earned interest at a rate of $6{,}5\%$ compounded annually, how much money would be in the account on 1 March 2008?

SOLUTION

$P = 3500$

$i = \frac{6{,}5}{100} = 0{,}065$

$n = \frac{21}{12} = 1{,}75$

because from 1 June 2006 untill 1 March 2008 is a 21 month period.

$A = P(1+i)^n = 3500(1+0{,}065)^{1{,}75} = R3907{,}78$

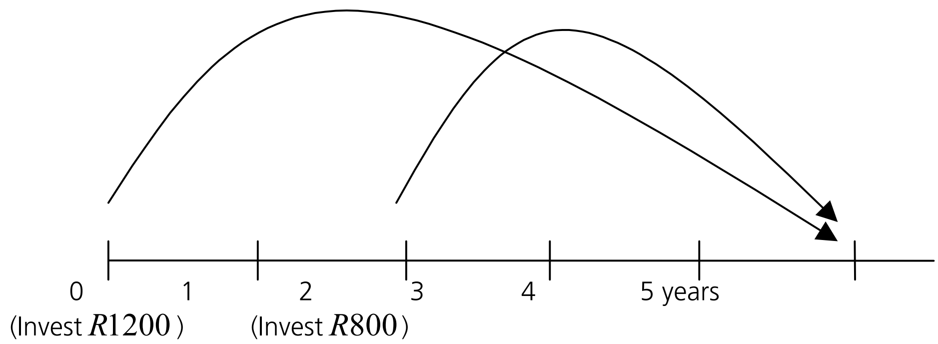
The amount in the account on 1 March 2008 would be $R3907{,}78$.

EXAMPLE 5.14

Joe made a deposit of R1 200 into his *Take Care* bank account. Two years later he made a second deposit of R800. How much is in Joe's account five years after the first deposit, if the interest rate is 9% compounded annually?

SOLUTION

In this case there are two payments that were made at different points in time. The solution is straightforward when we realise that the accumulated values of the payments can be considered separately and then simply added together.



The first payment was R1 200, which will earn interest over five years. The second payment was R800, which will earn interest over three years. We determine the interest earned on each of the amounts separately.

Future value of first payment:

$P = 1200$

$i = \frac{9}{100} = 0{,}09$

$n = 5$

$A = P(1+i)^n = 1200(1+0{,}09)^5 = R1846{,}35$

Future value of second payment:

$P = 800$

$i = \frac{9}{100} = 0{,}09$

$n = 3$

$A = P(1+i)^n = 800(1+0{,}09)^3 = R1036{,}02$

Total value of the investment after five years
= Future value of first payment + Future value of second payment
$= R1846{,}35 + R1036{,}02$
$= R2882{,}37$

EXAMPLE 5.15

In three years' time you will need R10 000 to pay for your son's studies. If your money earns 7% interest compounded annually, how much will you need to invest now so that you will have enough money to pay for his studies?

SOLUTION

$A = 10000$

$n = 3$

$i = \frac{7}{100} = 0{,}07$

We have to substitute the information into the formula $A = P(1+i)^n$ and then solve for P.

$$\begin{aligned} 10000 &= P(1+0{,}07)^3 \\ \frac{10000}{(1+0{,}07)^3} &= P \\ \frac{10000}{1{,}225043} &= P \\ P &\approx 8162{,}98 \end{aligned}$$

You need to invest $R8162{,}98$ today.

The sign $(\approx)$ means "is approximately equal to" and we use it when we want to provide a sensibly rounded answer to an exact answer. In the example above, the exact answer to the problem is actually $R8162{,}9788$. We chose, however, to round the answer off to two decimal places, since we cannot work in smaller units than cents.

EXAMPLE 5.16

You heard from a friend that there is an investment opportunity that will triple your money in two years' time. What interest rate (compounded annually) does that imply?

SOLUTION

The initial amount is unknown, but we do know that in two years it will triple. Therefore:
$A = 3P$

$n = 2$

We have to substitute the information into the formula $A = P(1+i)^n$ and then solve for i.

$$\begin{aligned}
3P &= P(1+i)^4 \\
\frac{3P}{P} &= (1+i)^4 \\
3 &= (1+i)^4 \\
3^{1/4} &= \left[(1+i)^4\right]^{1/4} \\
3^{1/4} &= 1+i \\
3^{1/4} - 1 &= i \\
0{,}32 &\approx i
\end{aligned}$$

This implies an interest rate of roughly 32%.

EXAMPLE 5.17

You made a single deposit of R800 into your bank account on 6 June 2007. On 6 November 2007 your balance was R950. You cannot remember the interest rate that your bank charged, but you do remember that the lady at the bank said the interest would be compounded annually. Determine the interest rate.

SOLUTION

$P = 800$

$A = 950$

$n = \frac{5}{12}$

because from 6 June 2007 till 6 November 2007 is a 5 month period. Then:

$$\begin{aligned}
950 &= 800(1+i)^{5/12} \\
\frac{950}{800} &= (1+i)^{5/12} \\
\left(\frac{950}{800}\right)^{12/5} &= \left[(1+i)^{5/12}\right]^{12/5} \\
\left(\frac{950}{800}\right)^{12/5} &= 1+i \\
\left(\frac{950}{800}\right)^{12/5} - 1 &= i \\
0.51 &\approx i
\end{aligned}$$

We substituted the information into the formula $A = P(1+i)^n$ and solved for i.

The interest rate that the bank used was roughly 51%.

EXAMPLE 5.18

You opened an account with R2 300, earning interest at $8{,}9\%$ compounded annually. On 1 January 2006 the amount in the account had doubled. When did you open the account?

SOLUTION

We know that the account doubled over the period, so the future value is $R4600$. Then:

$P = 2300$

$A = 4600$

$i = \frac{8{,}9}{100} = 0{,}089$

We substitute the information into the formula $A = P(1+i)^n$ and solve for n.

$$\begin{aligned} 4600 &= 2300(1+0{,}089)^n \\ \frac{4600}{2300} &= (1+0{,}089)^n \\ 2 &= (1+0{,}089)^n \end{aligned}$$

This is an exponential equation of the form $a = b^n$. To solve for the exponent n, we need to apply logarithms on both sides of the equation.

$$\begin{aligned} \log 2 &= \log(1+0{,}089)^n \\ \log 2 &= n\log(1{,}089) \\ \frac{\log 2}{\log(1{,}089)} &= n \\ 8{,}13 &\approx n \end{aligned}$$

So she opened the account approximately $8{,}13$ years, or 8 years and almost 2 months, before 1 January 2006. It must have been at the beginning of November, 1997.

LEARNING ACTIVITY 2.3

1. You will need R20 000 in six years' time. If you invest R9 000 today at an interest rate of 7% compounded annually, will you have enough money in six years' time?
2. Suppose you invest R50 000 for 5 months at an interest rate of 6% compounded annually. How much interest will you have earned after 5 months?
3. Suppose you invest R10 000 for 15 days at an interest rate of 6% compounded annually. How much interest will you have earned after 15 days?
4. Ruth made a deposit of R500. Two years later she made a second deposit of R600. Three years after that she made another deposit of R900. How much money is in Ruth's account nine years after the first deposit, if the interest rate is $7{,}5\%$ compounded annually?

Enrichment: 5 &6

5. The number of professional assistants in your firm increases by 8% every year. Calculate the number of professional assistants after 10 years if you started with three assistants.

6. The cost of living increases by 13% annually during three successive years. Find the percentage increase at the end of the four years.

7. Suppose you inherit R3 000. You can choose what to do with the money:

7.1. **Choice A:** Invest your money for three years at an interest rate of $7{,}5\%$ compounded annually.

7.2. **Choice B:** Play "Lotto" and spend R5 each week. At the end of three years you hope to win the "Lotto" which would give you an amount of $R1800$.

7.3. Which is the best choice? Explain your answer.

8. If you earn R270 compound interest at 7% per year for 2 years, how much did you invest?

9. You borrowed R3 000 for 3 years and the total interest paid was R650, compounded yearly. Determine the interest rate per year.

10. If you have R1 050 in your bank account after $1{,}5$ years and you invested money at 13% compound interest annually, how much did you invest initially?

11. Helen made an investment of R2 300 that has since tripled in value. The investment earned interest of 6% compounded yearly. How long did it take Helen's investment to triple?

In the preceding section the interest was compounded annually. In many instances the interest may be compounded over other time periods such as a quarter, month, week or day. This rate, when it is expressed as an equivalent rate per annum, is known as the *nominal rate* of interest.

EXAMPLE 5.19

Find the interest when R200 is invested for $1{,}5$ years at a nominal annual rate of $9{,}5\%$, compounded half-yearly.

SOLUTION

1st 6 months:

$SI = P \times i \times n = 200 \times 0{,}095 \times 0{,}5 =$ R9,50

Amount at the end of the first six months $= R200 + R9{,}50$
$= R209{,}50$

2nd 6 months:

$SI = P \times i \times n = 209{,}50 \times 0{,}095 \times 0{,}5 =$ R9,95

Amount at the end of the second period of six months $= R209{,}50 + R9{,}95$
$= R219{,}45$

3rd 6 months:

$SI = P \times i \times n = 219{,}45 \times 0{,}095 \times 0{,}5 =$ R10,42

Amount at the end of the third period of six months $= R219{,}45 + R10{,}42$
$= R229{,}87$

Total interest earned $= R229{,}87 - R200{,}00 = R29{,}87$

OR we can make use of the following table:

Year	Instalment	Interest earned	New instalment
6	R200,00	R9,50	R209,50
12	R209,50	R9,95	R219,45
18	R219,45	R10,42	R229,87

When dealing with nominal rates of interest, we need to consider a few things before using the formula. To determine the correct interest rate (i), the nominal rate must be divided by the number of periods per year for which the interest is compounded. Furthermore, the time (n) now consists of the total number of time periods involved.

Formula for calculating compound interest:
$A = P(1+i)^n$

where A is the future value of the investment
P is the present value of the investment (initial investment)
i is the rate of interest per time period, written as a decimal
n is the number of time periods over which the interest is compounded

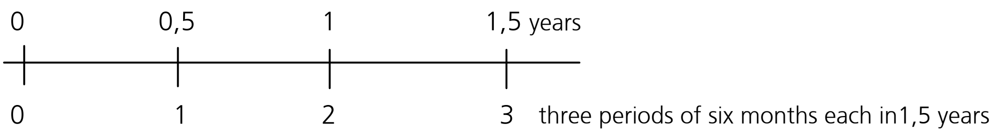
EXAMPLE 5.20

Find the interest when R200 is invested for $1{,}5$ years at a nominal annual rate of $9{,}5\%$, compounded half-yearly.

SOLUTION

The period is six months and the number of six month-periods in $1{,}5$ years (18 months) is $n = \frac{18}{6} = 3$. Since the interest is compounded half-yearly, the interest rate per six months is $\frac{9{,}5\%}{2} = 4{,}75\% = 0{,}0475$. We divide by 2, because there are two six-month periods in a year.

Consider the following illustration of the time-situation:



We want to find the future value of R200 after 3 periods of six months at a half-yearly rate of $4{,}75\% = 0{,}0475$. Let us substitute all the information into the formula and then determine the total interest earned.

$P = 200$
$i = 0,0475$
$n = 3$
$A = P(1+i)^n = 200(1+0,0475)^3 = R229,88$

Total interest earned is $R229,88 - R200,00 = R29,88$.

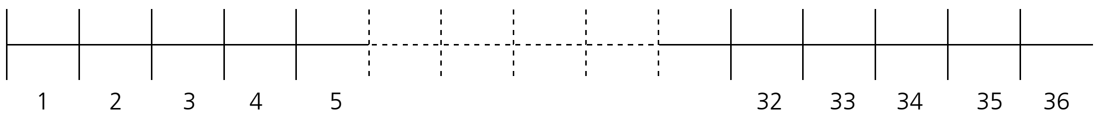
EXAMPLE 5.21

Sam wants to know how much his R2 000 will accumulate in 3 years if left in an account earning interest at a nominal annual rate of 13% compounded monthly.

SOLUTION

What does "nominal annual rate of 13% compounded monthly" mean? It means that the actual interest rate is not 13%, but $\frac{13}{12} = 1,08\dot{3}\% = 0,0108\dot{3}$ per month. In this problem a single period is a month, which gives $n = 3 \times 12 = 36$ periods (months). The interest rate is therefore $1,08\dot{3}\% = 0,0108\dot{3}$ over 36 months.

Let's draw a timeline to illustrate the situation.



36 periods – each one month in length

Notice that we are trying to find the future value of R2 000 over 36 months at a monthly rate of $1,08\dot{3}\% = 0,0108\dot{3}$. Let us substitute all the information into the formula.

$$A = P(1+i)^n$$
$$= 2000(1+0,0108\dot{3})^{36}$$
$$= R2947,77$$

So, after three years, Sam will have $R2947,77$.

The effects of compounding depend on the frequency at which the interest rate is compounded, be it yearly, half-yearly, quarterly, monthly or daily, as well as the interest rate which is used. Whenever a rate per annum is compounded over a different time period, we need to carefully determine what the rate over this time period is. This can be compared to what it would have been, had it been compounded annually.

EXAMPLE 5.22

If a rate was quoted as 20% per annum compounded half-yearly, it means that the investment actually pays 10% every six months. Does this mean 10% compounded every six months is the same as 20% per year? Let's see.

If you invest R100 at 10% per year, you will have R110 at the end of the first year.
If you invest R100 at 20% per year, you will have R120 at the end of the first year.
If you invest R100 at 21% per year, you will have R121 at the end of the first year.
If you invest R100 at 10% per half-year, receiving interest every six months, meaning you invest R100 at 20% per year compounded half-yearly, the future value will be

$$\begin{aligned} A &= P(1+i)^t \\ &= 100(1+0{,}1)^2 \\ &= 121 \end{aligned}$$

Therefore, our example illustrates that an investment at 20% per annum compounded half-yearly (which is equivalent to 10% every six months) is the same as investing at a rate of 21% per year.

Enrichment section starts

Interest rates must be comparable in order to be useful. Government requires that consumers be assisted with comparing the actual costs of borrowing. In finance and economics the following terminologies are common:

Periodic rate is the interest that is charged for each period. The periodic rate is defined as the annual nominal rate divided by the number of compounding periods per year.

Nominal interest rate is unadjusted for inflation, for example, a 12% annual nominal interest rate compounded monthly has a periodic (monthly) rate of 1%.

Effective annual rate is an imagined rate that would yield the same final value as the compounding plan over one year.

To compare different interest rates, we will always need to convert to an effective rate. In our example above, the 20% is called a nominal interest rate and the 21%, which is actually the rate that you will earn, is called the effective annual rate.

The effective interest rate is calculated as if compounded annually.

The effective rate r is calculated in three steps:

Step 1: First divide the quoted rate (nominal rate) by n, the number of compounding periods per year (for example, n=12 for monthly compounding).

Step 2: Add 1 to the result and raise it to the power n.

Step 3: Subtract 1.

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where r is the effective annual rate
i is the nominal rate written in decimal form
n is the number of compounding periods per year (4, if the interest is compounded every

three months, for example)

Example 5.23

For a nominal interest rate of 6% compounded monthly, find the equivalent effective interest rate.

Solution

$$i = \frac{6}{100} = 0{,}06 \text{, then}$$

$$\begin{aligned} r &= \left(1+\frac{i}{n}\right)^n - 1 \\ &= \left(1+\frac{0{,}06}{12}\right)^{12} - 1 \\ &\approx 0{,}0617 \end{aligned}$$

The effective annual rate is $6{,}17\%$.

Example 5.24

Find the effective annual rate for a loan with a 10% nominal annual rate compounding daily.

Solution

$$i = \frac{10}{100} = 0{,}1$$

$$\begin{aligned} r &= \left(1+\frac{i}{n}\right)^n - 1 \\ &= \left(1+\frac{0{,}1}{365}\right)^{365} - 1 \\ &\approx 0{,}10516 \end{aligned}$$

The effective annual rate is $10{,}52\%$.

From the previous two examples it is clear that a loan compounding daily will have a substantially higher effect on the future value than one compounding annually. For example, a loan of R100 000 at 10% interest compounded daily, will make the borrower pay R516 more than one who was charged 10% interest, compounded annually.

Enrichment section ends here.

We look at more examples on compound interest.

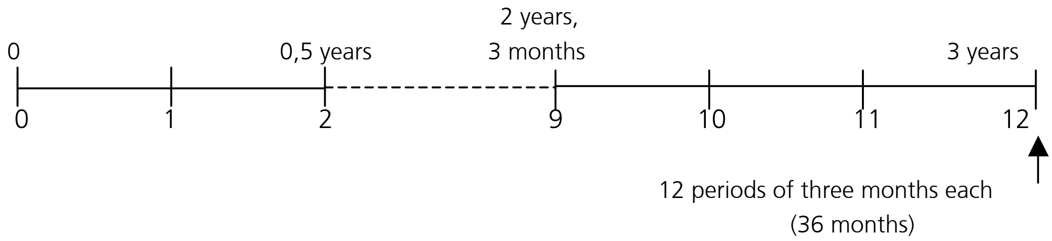
Example 5.25

An amount of R2 000 is deposited into a bank account which pays an annual interest rate of $8{,}6\%$, compounded quarterly. Find the balance after 3 years.

SOLUTION

The time period is a quarter of a year ($\frac{12}{4} = 3$ months). There are four quarters (three month periods) in a year, so the number of quarters in 3 years is $n = 3 \times 4 = 12$. Since the interest rate is compounded quarterly, the interest rate per quarter is $\frac{8{,}6\%}{4} = 2{,}15\% = 0{,}0215$.

Let's draw a timeline to illustrate the situation.



We want the future value of R2 000 over 12 periods of three months each at a quarterly rate of $2{,}15\% = 0{,}0215$. Let us substitute all our information into the formula.

$P = 2000$

$i = 0{,}0215$

$n = 12$

$A = P(1+i)^n = 2000(1+0{,}0215)^{12} = R2581{,}61$

The balance will be $R2581{,}61$ after three years.

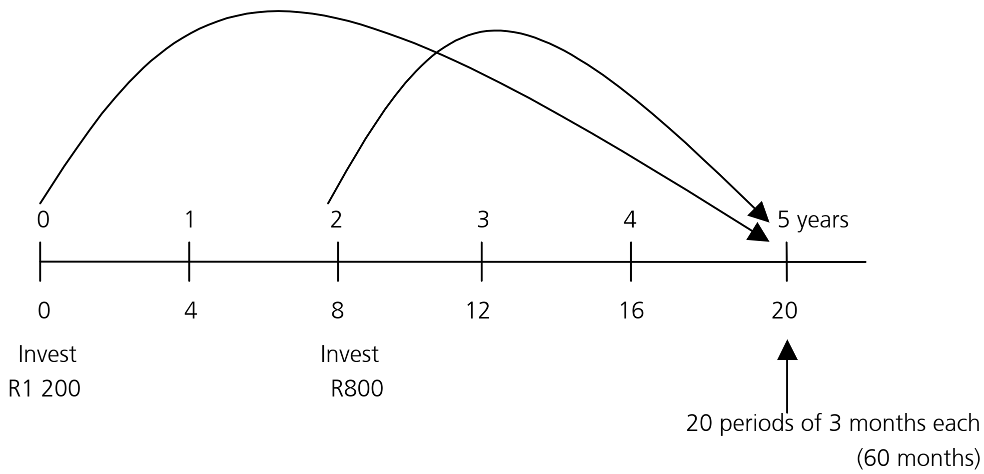
EXAMPLE 2.26

Joe made a deposit of R1 200 into his bank account. Two years later he made a second deposit of R800 into the same account. How much money is in Joe's account five years after the first deposit, if a nominal annual rate of 9% compounded quarterly, is earned?

SOLUTION

The period is a quarter of a year ($\frac{12}{4} = 3$ months). There are four quarters (three month periods) in a year, so the number of quarters in 5 years is $n = 5 \times 4 = 20$. Since the interest rate is compounded quarterly, the interest rate per quarter is $\frac{9\%}{4} = 2{,}25\% = 0{,}0225$.

In this case there are two payments at different points in time. The solution is straightforward if we realise that the accumulated values of the payments can be considered separately (as indicated on the timeline) and then simply added together.



The first payment was R1 200 and would have earned interest for five years, or 20 periods. The second payment was R800 and would have earned interest for three years, or 12 periods.

Future value of the first deposit:

$P = 1200$

$i = 0{,}0225$

$n = 20$

$A = P(1+i)^n = 1200(1+0{,}0225)^{20} = R1872{,}61$

Future value of the second deposit:

$P = 800$

$i = 0{,}0225$

$n = 12$

$A = P(1+i)^n = 800(1+0{,}0225)^{12} = R1044{,}84$

Total future value of investment
= Future value of first deposit + future value of second deposit
$= R1872{,}61 + R1044{,}84$
$= R2917{,}45$

EXAMPLE 5.27

Your dad wants to retire on his 60th birthday with R1 000 000 from an investment he is about to make at his bank on his 45th birthday. The bank pays a nominal annual rate of 5% compounded monthly. Give your dad advice on how much he must invest on his 45th birthday to have R1 000 000 when he retires?

SOLUTION

The time period is a month. There are 12 months in a year, so the number of months in 15 years is $n = 15 \times 12 = 180$. Since the interest is compounded monthly, the interest rate per month is $\dfrac{5\%}{12} = 0{,}416\% = 0{,}00416$.

We are trying to find the present value of R1 000 000 over 180 periods of one month each at a monthly rate of $0{,}416\% = 0{,}00416$. Let us substitute all our information into the formula and solve for P.

$A = 1000000$
$i = 0{,}00416$
$n = 180$

Then:

$$\begin{aligned} A &= P(1+i)^n \\ 1000000 &= P(1+0{,}00416)^{180} \\ P &= \frac{1000000}{(1+0{,}00416)^{180}} \\ P &= R473668{,}86 \end{aligned}$$

He should invest $R473668{,}86$ on his 45th birthday.

EXAMPLE 5.28

Sue made an investment 3 years ago. The investment doubled in value at a certain nominal annual rate, compounded half-yearly. What nominal annual rate was used here?

SOLUTION

The time period is half a year (six months). There are two six month periods in a year, so the number of six month periods in 3 years is $n = 3 \times 2 = 6$. We are trying to find the nominal annual rate compounded half-yearly that caused the investment to double in value. Let's substitute our information into the formula and solve for i.

If $A = 2P$ and $n = 6$, then

$$\begin{aligned} 2P &= P(1+\tfrac{i}{2})^6 \\ 2 &= (1+\tfrac{i}{2})^6 \\ 2^{1/6} &= \left[\left(1+\tfrac{i}{2}\right)^6\right]^{1/6} \\ 2^{1/6} &= 1+\tfrac{i}{2} \\ 2^{1/6}-1 &= \tfrac{i}{2} \\ i &\approx 0{,}2449 \end{aligned}$$

So the nominal annual rate was approximately $24{,}49\%$.

EXAMPLE 5.29

How long will it take R750 to grow to R1 500 if the nominal annual rate is $8\frac{1}{4}\%$, compounded quarterly?

SOLUTION

Since the interest rate is compounded quarterly, the interest rate per quarter is $\frac{8\frac{1}{4}\%}{4} = \frac{\frac{33}{4}\%}{4} = \frac{33}{4}\% \times \frac{1}{4} = 2{,}0625\% = 0{,}020625.$

We want to find the time it will take the present value of R750 to grow to R1 500 at a nominal quarterly rate of $2{,}0625\% = 0{,}020625$. Let us substitute all our information into the formula and solve for n.

$A = 1500$

$P = 750$

$i = 0{,}020625$

$$\begin{aligned} A &= P(1+i)^n \\ 1500 &= 750(1+0{,}020625)^n \\ \frac{1500}{750} &= (1{,}020625)^n \\ 2 &= (1{,}020625)^n \end{aligned}$$

This is an exponential equation of the form $a = b^n$. To solve for the exponent n, we need to apply logarithms on both sides of the equation.

$$\begin{aligned} \log 2 &= \log(1{,}020625)^n \\ \log 2 &= n\log(1{,}020625) \\ \frac{\log 2}{\log 1{,}020625} &= n \\ n &\approx 33{,}95 \end{aligned}$$

So the total time it will take for R750 to grow to R1500 is approximately 33,95 quarters = $\frac{33{,}9}{4} = 8{,}4875$ years.

ASSESSMENT ACTIVITY 2.4

1. Find the future value of R430 if it is invested for two years at a nominal annual rate of $7{,}5\%$, compounded quarterly.

1.1. Also find the equivalent effective interest rate.

2. As you walked pass the bank yesterday, you saw that the bank advertised their nominal annual rate as 11% compounded daily. If you deposit R4 000 today, how much will be in your account one year from now?

3. An investment that John made five years ago at a nominal annual rate of $11{,}5\%$, compounded half-yearly, is presently worth R4 670. How much did John invest?

3.1. Also find the equivalent effective interest rate.

4. Suppose that an amount of R2 500 grows to R3 750 in six years at a certain nominal annual interest rate, compounded monthly. What was this nominal rate?

5. What nominal annual rate of interest, compounded half-yearly, did Peter earn if his investment of R6 780 earned R780 of interest over four years?

6. How long will it take for R5 000 to grow to R10 000 if the nominal annual interest rate is $5{,}5\%$, compounded quarterly?

7. How many years will it take your investment of R2 000 to double at a nominal annual interest rate of 8%, compounded monthly?

3. Annuity

An annuity is a finite series of payments made at equal intervals of time. The payments may be all the same, or they may vary. Examples of annuity payments are mortgage payments, loan repayments, rental payments and insurance premiums.

We are going to work with an annuity called a *simple ordinary annuity*. An annuity is called simple if interest is compounded at the same time the annuity payment is made. An annuity is called ordinary if a constant amount is paid at the end of each period.

Future value of an annuity

The future value of an annuity is the amount due/saved at the end of the term.

Let us illustrate the idea with an example:

Example 5.30

If you deposit R500 at the end of each year for the next three years at a nominal annual rate of 8% compounded yearly, how much will you have in your bank account after three years?

Solution

$P = 500$

$i = \frac{8}{100} = 0{,}08$

$n = 3$

1st year: The deposit of R500 was made only at the end of the first year, so there was no interest earned during the first year.

2nd year: We started with R500.
The interest earned during the second year can be calculated as follows:
$SI = P \times i \times n = 500 \times 0{,}08 \times 1 = \text{R}40$
At the end of the second year we had the initial R500 plus the interest of R40. We also deposited another R500 at the end of the second year.

Amount at the end of year 2
= R500 + R40 + R500
= R1 040

3rd year: We started with R1 040 at the beginning of the year.
The interest earned during the third year can be calculated as follows:
$SI = P \times i \times n = 1040 \times 0{,}08 \times 1 = \text{R}83{,}20$
At the end of the third year we had R1 040 plus the interest of R83,20. We also deposited another R500 at the end of the year.

Amount at the end of year 3

= R1 040,00 + R83,20 + R500,00
= R1 623,20

OR we can make use of the following table:

Year	Balance at the beginning of the year	Interest earned	Payment at the end of the year	Total at the end of the year
1	0	0	500	500
2	500	40	500	1040
3	1040	83,20	500	1623,20

After three years you will have R1 623,20 in your bank account.

Calculations such as these can become very tedious and lengthy. We therefore introduce the following formula to find the future value of an annuity.

Formula for determining the future value of an annuity

$$F_v = R \times \frac{(1+i)^n - 1}{i}$$

where F_v is the future value of the annuity (total payments plus interest)
R is the annuity payment amount, made per time period
n is the total number of payments to be made
i is the interest rate per time period, written as a decimal number

Let us look at our previous example again, but now we use the formula.

EXAMPLE 5.31

If you deposit R500 at the end of each year for the next three years at a nominal annual rate of 8% compounded yearly, how much will you have in your bank account after three years?

SOLUTION

R = R500
n = 3
$i = \frac{8}{100} = 0{,}08$ Then $F_v = R \times \frac{(1+i)^n - 1}{i}$

$$F_v = 500 \times \frac{(1+0{,}08)^3 - 1}{0{,}08}$$

$$F_v = R1623{,}20$$

After three years you will have R1 623,20 in your bank account.

EXAMPLE 5.32

Suppose you want to be a millionaire by the time you are 65. You are able to deposit R500 at the end of every three months and you will earn interest at a nominal annual rate of 11% compounded monthly. Determine whether or not you will be a millionaire at 65 if you are now 25 years old. Give reasons for your answer.

SOLUTION

The time period is three months. There are four periods of three months in a year, so there will be $n = 40 \times 4 = 160$ periods of three months in 40 years. Since the interest is compounded monthly, the interest rate per month is $\frac{11\%}{12} = 0{,}91\dot{6}\% = 0{,}0091\dot{6}$.

$R = R500$

$n = 160$

$i = 0{,}0091\dot{6}$

$$\begin{aligned} F_v &= R \times \frac{(1+i)^n - 1}{i} \\ &= 500 \times \frac{\left(1 + 0{,}0091\dot{6}\right)^{160} - 1}{0{,}0091\dot{6}} \\ &= R180320{,}43 \end{aligned}$$

You will only have R180 320,43 by the time you are 65 years old. That is still R819 679,57 short of a million, so you will definitely not be a millionaire.

EXAMPLE 5.33

Rudi is saving to buy a new motorcycle in two years' time. The motorcycle will cost him R25 000. If he can make monthly payments into an account earning a nominal annual interest of 12% compounded quarterly, how much must he deposit each month to have R25 000 available in two years' time?

SOLUTION

The time period is a month. There are 12 months in a year, so there are $n = 2 \times 12 = 24$ months in 2 years. Since the interest is compounded quarterly, the interest rate per quarter is $\frac{12\%}{4} = 3\% = 0{,}03$.

$n = 24$

$i = 3\% = 0{,}03$

$F_v = R25000$

Let us substitute all our information into the formula and solve for R.

$$\begin{aligned}
F_v &= R \times \frac{(1+i)^n - 1}{i} \\
25000 &= R \times \frac{(1+0{,}03)^{12} - 1}{0{,}03} \\
25000 \times \frac{0{,}03}{(1+0{,}03)^{12} - 1} &= R \times \frac{(1+0{,}03)^{12} - 1}{0{,}03} \times \frac{0{,}03}{(1+0{,}03)^{12} - 1} \\
1761{,}55 &= R
\end{aligned}$$

Rudi has to deposit R1 761,55 into his account each month to have R25 000 available in two years' time.

EXAMPLE 5.34

Your dad wants to save money for your newborn sister's university studies. He estimates that R40 000 will be adequate. If he can save R5 000 every year and earn interest at a nominal annual interest rate of 13%, compounded monthly, calculate the time it will take to have R40 000 available.

SOLUTION

Since the interest is compounded monthly, the interest rate per month is

$\frac{13\%}{12} = 1{,}08\dot{3} = 0{,}0108\dot{3}$.

$R = R5000$

$i = 1{,}08\dot{3}\% = 0{,}0108\dot{3}$

$F_v = R40000$

Let us substitute all our information into the formula and solve for n.

$$\begin{aligned}
F_v &= R \times \frac{(1+i)^n - 1}{i} \\
40000 &= 5000 \times \frac{(1+0{,}0108\dot{3})^n - 1}{0{,}0108\dot{3}} \\
40000 \times \frac{0{,}0108\dot{3}}{5000} &= 5000 \times \frac{(1+0{,}0108\dot{3})^n - 1}{0{,}0108\dot{3}} \times \frac{0{,}0108\dot{3}}{5000} \\
0{,}0866664 &= (1+0{,}0108\dot{3})^n - 1 \\
1{,}0866664 &= (1{,}0108\dot{3})^n
\end{aligned}$$

This equation is of the form $a = b^n$. We therefore have to use logarithms to solve for n.

$$\begin{aligned}
\log(1{,}0866664) &= \log(1{,}0108\dot{3})^n \\
\log(1{,}0866664) &= n\log(1{,}0108\dot{3}) \\
\frac{\log(1{,}0866664)}{\log(1{,}0108\dot{3})} &= n \\
7{,}7 &\approx n
\end{aligned}$$

Your dad will have to save for 7,7 years or 7 years and 8 months, to have R40 000 available.

ASSESSMENT ACTIVITY 2.5

1. You deposit R450 at the end of every month for four years. You earn interest at a nominal annual rate of $8{,}5\%$ compounded monthly. Calculate the amount in your bank account after four years.
2. John plans to contribute R5 000 at the end of every year into a retirement account paying a nominal annual rate of $8{,}5\%$ compounded annually. If he retires in 30 years' time, how much will he have?
3. Sydney saved R300 at the end of every month for 10 years. If his money earned a nominal annual rate of $7{,}5\%$ compounded every three months, how much does he have after 10 years?
4. Your dad wants to give your youngest sister R50 000 on her 21st birthday. Suppose his savings earn a nominal annual interest rate of 9% compounded quarterly. Determine the size of the payments that he will have to make yearly for the next eight years, so that he will have R50 000 available on her 21st birthday.
5. How much should be deposited monthly into an account if the aim is to accumulate R15 000 over five years? Assume the account earns interest at a nominal annual rate of 7% compounded daily.
6. At the age of 35 John began contributing R1 650 every month to a retirement fund. If his money earns interest at a nominal annual rate of 12%, compounded monthly, how long does it take for his retirement fund to grow to R45 000?

Present value of an annuity

The present value of an annuity is the value of a future payment or series of future payments on a given date.

Formula for calculating the present value of an annuity

$$P_v = R \times \frac{1-(1+i)^{-n}}{i}$$

where P_v is the present value of the annuity (total payments plus interest paid so far)

R is the payment made per time period
n is the total number of payments
i is the interest rate per payment period written as a decimal

EXAMPLE 5.35

Stuart bought himself a two-bedroom townhouse and makes monthly payments of R4 000 over twelve years. The nominal annual rate is $12{,}5\%$ compounded monthly. He would like to know the present equivalent cash price of the house.

Solution

The time period is a month. There are 12 months in a year, so there will be $n = 12 \times 12 = 144$ months in 12 years. Since the interest is compounded monthly, the interest rate per month is $\frac{12{,}5\%}{12} = 1{,}041\dot{6}\% = 0{,}010416\dot{6}$.

R = R4 000
n = 144
$i = 1{,}041\dot{6}\% = 0{,}01041\dot{6}$

$$P_v = R \times \frac{1-(1+i)^{-n}}{i}$$
$$P_v = 4000 \times \frac{1-(1+0{,}01041\dot{6})^{-144}}{0{,}01041\dot{6}}$$
$$P_v = R297651{,}74$$

The equivalent cash price now (present value) of the house is R297 651,74.

Example 5.36

Your dad owes the bank R40 000 and wants to make equal monthly payments that will have the total amount paid off in two years' time. What will the monthly payments be if the bank is charging a nominal annual rate of 13% compounded monthly?

Solution

The period is a month. There are 12 months in a year, so there will be $n = 2 \times 12 = 24$ months in 2 years. Since the interest is compounded monthly, the interest rate per month is $\frac{13\%}{12} = 1{,}08\dot{3}\% = 0{,}0108\dot{3}$.

P_v = R40 000
n = 24
$i = 1{,}08\dot{3}\% = 0{,}0108\dot{3}$

Let us substitute all our information into the formula and solve for R.

$$P_v = R \times \frac{1-(1+i)^{-n}}{i}$$
$$40000 = R \times \frac{1-(1+0{,}0108\dot{3})^{-24}}{0{,}0108\dot{3}}$$
$$40000 \times \frac{0{,}0108\dot{3}}{1-(1+0{,}0108\dot{3})^{-24}} = R \times \frac{1-(1+0{,}0108\dot{3})^{-24}}{0{,}0108\dot{3}} \times \frac{0{,}0108\dot{3}}{1-(1+0{,}0108\dot{3})^{-24}}$$
$$40000 \times 0{,}0475418 = R$$
$$1901{,}67 = R$$

The monthly payments will be R1 901,67.

Example 5.37

Because of a cash-flow problem during the holiday you charge R2 000 in expenses to your credit card. You can only afford to make a minimum payment of R300 per month in paying off your debt. If the nominal annual rate is 16%, compounded daily, how long will it take you to pay off the R2 000?

Solution

Since the interest is compounded daily, the interest rate per day is
$\frac{16\%}{365} \approx 0{,}0438\% = 0{,}000438$.

P_v = R2 000
R = R300
$i = 0{,}0438\% = 0{,}000438$

Let us substitute all our information into the formula and solve for n.

$$\begin{aligned} P_v &= R \times \frac{1-(1+i)^{-n}}{i} \\ 2000 &= 300 \times \frac{1-(1+0{,}000438)^{-n}}{0{,}000438} \\ \frac{2000 \times 0{,}000438}{300} &= 1-(1{,}000438)^{-n} \\ 0{,}00292 - 1 &= -(1{,}000438)^{-n} \\ 0{,}99708 &= (1{,}000438)^{-n} \end{aligned}$$

This is an equation of the form $a = b^n$, so we have to use logarithms to solve for n.

$$\begin{aligned} \log(0{,}99708) &= \log(1{,}000438)^{-n} \\ \log(0{,}99708) &= -n\log(1{,}000438) \\ \frac{\log(0{,}99708)}{\log(1{,}000438)} &= -n \\ 6{,}678 &\approx n \end{aligned}$$

It will take you almost 7 months to pay off your debt.

ASSESSMENT ACTIVITY 2.6

1. An annuity pays R2 000 per year for four years. Find the present value of this annuity if the nominal annual interest rate is $13{,}5\%$ compounded yearly.
2. Jenny is planning a holiday to Europe and wants to be able to withdraw R6 000 from her account every month for one year. Her account earns a nominal annual rate of $8{,}5\%$ compounded monthly. How much should Jenny deposit into her account before she leaves?

3. A newly-wed couple took out a bank loan in order to purchase furniture for their home. The loan was to be repaid in monthly instalments of R950 over two years. Calculate the present value of these repayments if the nominal annual rate is 14% compounded monthly.

4. Luke bought a motorcycle for R13 000 and will pay it off in equal monthly payments at the end of each month for two years. If the nominal interest rate is 14% compounded monthly, what is the monthly payment that Luke has to make?

5. You decide to borrow R10 000 from a bank in order to pay for your studies. The bank charges a nominal annual rate of 15% compounded quarterly over six years. What monthly payments will you have to make on this loan?

6. Your dad bought a new car for R250 000. If he makes monthly repayments of R4 500, work out how long it will take him to pay off the car. Assume the nominal annual rate is 12% compounded monthly.

Amortisation

There are different ways of repaying debts. One of the methods of repayment is by amortisation. For an amortisation loan, the lender may require the borrower to repay parts of the loan per time period. Each periodic payment covers part of the loan, as well as interest on the balance of the loan. In amortisation problems we try to find the periodic payment required to pay off the debt. To do this we use the present value of an annuity.

Let us illustrate the concept by the following example:

Suppose a business takes out a four-year loan of R2 000 at a nominal rate of 9%, compounded yearly. The loan agreement calls for the borrower to pay the interest on the loan balance each year and to reduce the loan balance (principal) yearly by R500. Since the loan amount declines by R500 every year, it will be fully paid in four years' time ($R500 \times 4 = R2000$). We can calculate the total payment per year by preparing an amortisation schedule as follows:

Year	Initial balance	Interest paid	Principle paid	Total payment	Ending balance
1	R2 000	R180	R500	R680	R1 500
2	R1 500	R135	R500	R635	R1 000
3	R1 000	R90	R500	R590	R500
4	R500	R45	R500	R545	0
Total		**R450**	**R2 000**	**R2 450**	

Note that, in this example, the payment each year reduces. The reason for this is that the loan balance, on which interest is charged, becomes less.

The most common way of amortising a loan is for the borrower to make a single, fixed payment every period. Let us consider the example above. What would the amortisation schedule look like if the payments were fixed? We first need to determine the payment. To do this we can use the present value formula $P_v = R \times \frac{1-(1+i)^{-n}}{i}$.

Since the interest is compounded yearly, the interest rate is $9\% = 0{,}09$.

$P_v = R2000$
$i = 9\% = 0{,}09$
$n = 4$

Let us substitute all our information into the formula and solve for R.

$$\begin{aligned} P_v &= R \times \frac{1-(1+i)^{-n}}{i} \\ 2000 &= R \times \frac{1-(1+0{,}09)^{-4}}{0{,}09} \\ R &= 617{,}34 \end{aligned}$$

The borrower will therefore make four equal payments of R617,34. In this example the total payment per period is known. To calculate the principal portion we calculate the interest and then subtract it from the total payment.

Let us check whether this will pay off the loan.

Year	Initial balance	Principle paid	Interest paid	Total payment	Balance
1	R2 000	R437,34	R180,00	R617,34	R1 562,66
2	R1 562,66	R476,70	R140,64	R617,34	R1 085,96
3	R1 085,96	R519,60	R97,74	R617,34	R566,36
4	R566,36	R566,37	R50,97	R617,34	0
Total		**R2 000,01**	**R469,35**	**R2 469,36**	

If you compare the two loan amortisations you will see that the total interest paid is greater in the amortisation option where equal payments are made. The reason for this is because the initial two repayments of the equal-payment amortisation are less and therefore more interest is charged.

EXAMPLE 5.38

Your family takes a R600 000 mortgage from the bank to buy a new home. The bank charges interest at an annual rate of 13% compounded monthly over 20 years. Find the monthly payments that your family has to make on this loan.

SOLUTION

The time period is a month and the number of months in 20 years is $n = 20 \times 12 = 240$. Since the interest is compounded monthly, the interest rate per month is $\frac{13\%}{12} = 1{,}08\dot{3}\% = 0{,}0108\dot{3}$.

$P_v = R600000$
$n = 240$
$i = 13\% = 0{,}0108\dot{3}$

Let us substitute all our information into the formula and solve for R.

$$\begin{aligned} P_v &= R \times \frac{1-(1+i)^{-n}}{i} \\ 600000 &= R \times \frac{1-\left(1+0{,}0108\dot{3}\right)^{-240}}{0{,}0108\dot{3}} \\ R &= 7029{,}44 \end{aligned}$$

The family must repay the loan with monthly instalments of R7 029,44.

Example 5.39

In the previous example, find the outstanding balance for the loan at the end of the twelfth year.

Solution

At the end of the twelfth year there are still eight years of payments left on the loan. The time period per instalment is a month. There are 12 months in a year, so there are $n = 8 \times 12 = 96$ months in 8 years. Since the interest is compounded monthly, the interest rate per month is $\frac{13\%}{12} = 1{,}08\dot{3}\% = 0{,}0108\dot{3}$.

$R = 7029{,}44$

$n = 96$

$i = 13\% = 0{,}0108\dot{3}$

Let us substitute all our information into the formula and solve for P_v.

$$P_v = R \times \frac{1-(1+i)^{-n}}{i}$$

$$P_v = 7029{,}44 \times \frac{1-\left(1+0{,}0108\dot{3}\right)^{-96}}{0{,}0108\dot{3}}$$

$$P_v = 418238{,}99$$

At the end of the twelfth year the family still owes R418 238,99.

Example 5.40

Suppose your family bought a house 12 years ago and financed it at an interest rate higher than what is available today. They would now like to refinance their loan at a lower interest rate. If they borrowed R450 000 and got a 20-year mortgage at an annual rate of $14{,}25\%$ compounded monthly, how much do they still owe on their mortgage?

Solution

First we need to find the monthly payments. The instalment-period is a month and there are $n = 20 \times 12 = 240$ months in 20 years. Since the interest is compounded monthly, the interest rate per month is $\frac{14{,}25\%}{12} = 1{,}1875\% = 0{,}011875$.

$P_v = R450000$
$n = 240$
$i = 14,25\% = 0,011875$

Let us substitute all our information into the formula and solve for R.

$$\begin{aligned} P_v &= R \times \frac{1-(1+i)^{-n}}{i} \\ 450000 &= R \times \frac{1-(1+0,011875)^{-240}}{0,011875} \\ R &= 5677,74 \end{aligned}$$

We want to know how much your family owes on the house. The outstanding amount after twelve years is the actual present value of the future payments over the next eight years.

The period is a month and there are $n = 8 \times 12 = 96$ months in the remaining 8 years. Since the interest is compounded monthly, the interest rate per month is $\frac{14,25\%}{12} = 1,1875\% = 0,011875$.

$R = 5677,74$
$n = 96$
$i = 14,25\% = 0,011875$

Let us substitute all our information into the formula and solve for P_v.

$$P_v = R \times \frac{1-(1+i)^{-n}}{i}$$
$$P_v = 5677,74 \times \frac{1-(1+0,011875)^{-96}}{0,011875}$$
$$P_v = 324181,48$$

They still owe R324 181,48 after twelve years.

ASSESSMENT ACTIVITY 2.7

1. Bonnie has just received an amount of money unexpectedly and would like to pay off her house. She pays R3 200 at the end of every month at a nominal rate of 11% compounded monthly in order to accomplish this. How much money is outstanding on Bonnie's house if she has 108 payments left?
2. Suppose John borrowed R500 000 over 20 years to buy a house at a nominal rate of $13,25\%$ compounded monthly. Five years later the interest rate on the loan dropped to 10%.
   - **2.1.** Find the monthly repayment that John originally made on the loan.
   - **2.2.** Find the new instalment that John has to pay each month after the drop in the interest rate.

Make use of excel to answer the following questions:

3. Prepare an amortisation schedule for a ten-year loan of R100 000. The nominal rate is 13% compounded yearly and the loan calls for equal annual payments.

3.1. Find the instalment per year.

3.2. Make use of an Excel spread sheet to type the following information into the specified cells. Type the instalment that you found in 3.1 into B4.

	A	B	C	D	E	F
1	**Loan amount**	100 000				
2	**Interest rate**	13% = 0.13				
3	**Loan term**	10				
4	**Loan payment**					
5						
6	**Year**	**Beginning**	**Total**	**Interest**	**Principle**	**Ending**
7		**Balance**	**Payment**	**Paid**	**Paid**	**Balance**
8	1					
9	2					
10	3					
11	4					
12	5					
13	6					
14	7					
15	8					
16	9					
17	10					
18	**Totals**					
19						
20						

a Complete the amortisation table. Make use of formulas only. No values should be type in.

b Provide totals in C18, D18 and E18.

c Make a printout of your answer page, containing the values. Also print out a separate page that shows the formulas that you used.

To show the formulas in each cell, follow these 3 steps:
Step 1: On the Tools menu, click Options, and then click the View tab.
Step 2: To display formulas in cells, select the Formulas check box.
Step 3: To display the formula results, clear the check box.

GROUP ACTIVITY 2.8

Use the given web sites to answer the questions:

1. http://www.nationwidehomeloans.co.za/tools.htm

(09-03-2009)

Suppose you want to buy a new house for R650 000 at a nominal rate of 15% with a loan term of 20 years. (Monthly payments)

1.1. Give the total transfer cost on the house.
1.2. Explain in your own words what transfer cost means.
1.3. Determine the amount that you must apply for so that you will have enough money to buy the house. Assume that you have no money of your own.
1.4. Use this home loan amount and determine your monthly payment.
1.5. Find the total amount of interest paid over 20 years.
1.6. Find the total amount paid over 20 years.

2. http://www.mortgageworld.co.za/index.php?cont=calculatorplus

(09-03-2009)

2.1. Choose the option: How much do I have to earn?

Not sure how much money you'll have to earn to afford your house payment and accompanying expenses?

2.1.1. Suppose you want a home mortgage loan of R450 000 at a nominal rate of $12{,}5\%$ for a loan term of 20 years.
2.1.2. Determine your required annual salary that you have to earn.
2.1.3. Determine your required monthly salary that you have to earn.

2.2. Choose the option: Mortgage calculator

Want to know how much your monthly payment is for your mortgage?

2.2.1. Suppose you want a home loan of R600 000 at a nominal rate of 15% for a loan term of 20 years.
2.2.2. Find the monthly payments that you will have to make.
2.2.3. Use the amortisation schedule and determine the total interest paid in year nine.
2.2.4. Use the amortisation schedule and determine the total principal paid in year fifteen.
2.2.5. Use the amortisation schedule to find the year for which the difference in interest paid and principal paid is a minimum.

2.3. Choose the option: Mortgage calculator - With graphs, monthly and annual amortisation tables

Want to know how much your monthly payment is for your mortgage?

2.3.1. Suppose you want a loan of R500 000 at a nominal rate of $13{,}5\%$ for a loan term of 20 years.
2.3.2. Determine the monthly payments that you will have to make.
2.3.3. Find the total amount of interest paid over 20 years.

2.3.4. Find the total amount of payments made over 20 years.
2.3.5. After how many years will the amount of interest paid for that year be less than the principal amount paid?
2.3.6. Determine the interest, the principal and the balance of the specific month, after 36 months have passed.

4. Depreciation

The concept of depreciation is very simple. Let's say you purchase a computer for your business. The computer starts losing value the minute you buy it. Each year that you own the computer, it loses some value, until it has no value to your business any longer. An asset's loss in value is known as depreciation. In simple words we can say that depreciation is the reduction in value of an asset due to usage, wear and tear, technological outdating or other factors.

Most possessions such as cars, caravans, electrical equipment or computers depreciate in value as time passes. At the end of each year the value of a car, for example, will be less than its value at the beginning of the year. Most assets lose their value over time (in other words, they depreciate) and must be replaced once the end of their useful life is reached. There are several accounting methods that can be used to write off an asset's depreciation cost over the period of its useful life.

The write-off periods for a selection of such assets for the 2006 tax year are shown in the table below.

Wear and Tear Allowances | South Africa Tax Guide 2006

Item	**Period of write-off** *(no. of years)*	**Item**	**Period of write-off** *(no. of years)*
Air-conditioners: window type	6	Fax machines	3
Bicycles	4	Fitted carpets	6
Calculators	3	Furniture and fittings	6
Cellular telephones	3	Gymnasium equipment	10
Computers (personal)	3	TV sets, video machines and decoders	6
Motorcycles	4	Telephone equipment	5
Typewriters	6	Trailers	5
Refrigerators	6	X-ray equipment	5

Depreciation is the process of spreading the acquiring costs of an asset over its useful economic life; that is, over the period during which it is used to produce income for the business.

In running a business, a number of fixed assets are purchased to assist in producing income in future years. Examples of such assets are real estate, buildings, vehicles, office equipment, machines, etc. The purchase price of an asset is called the original cost and is used as the basis for determining depreciation. The *salvage value* is the estimated value of the asset at the end of its useful life. The *useful life* of an asset is the period of time that the business feels the asset will be of use to them. At the end of its useful life the asset may be sold or disposed of. Salvage value also means scrap value.

There are two main methods of determining depreciation:

- The straight-line method
- The reducing balance method

Straight-line depreciation

Straight-line depreciation is the more commonly used method. According to this method the company estimates the salvage value of the asset at the end of the period during which it was used to generate income. The value of the asset depreciates by a fixed amount every year over the lifespan of the asset.

$$\text{Annual depreciation} = \frac{\text{original cost price - salvage value}}{\text{expected life}}$$

Book value after k years = original cost – (k x annual depreciation).

EXAMPLE 5.41

A typewriter bought for R2 000 is expected to be worthless to a firm and without any scrap value (i.e., zero scrap value) after five years.

1 Calculate the annual depreciation according to the straight-line method.

2 Construct a depreciation schedule for the typewriter over five years.

SOLUTION

1 Original cost is R2 000
Expected life is five years
Scrap value is zero
The typewriter will depreciate at:

$$\text{Annual depreciation} = \frac{\text{original cost price - salvage value}}{\text{expected life}}$$

$$\text{Annual depreciation} = \frac{2000-0}{5} = R400 \text{ per year}$$

2

Year	Book value at the beginning of the year	Annual depreciation	Accumulated depreciation	Book value at the end of the year
1	R2 000 *(Original Cost)*	R400	R400	R1 600
2	R1 600	R400	R800	R1 200
3	R1 200	R400	R1 200	R800
4	R800	R400	R1 600	R400
5	R400	R400	R2 000	R0 *(Written off)*

After five years we would say that the typewriter has been "written off". According to the straight-line method the net book value of an asset is reduced by the same amount each period. Basically it means that you take the total value of the asset and divide it by the number of periods. This amount is then subtracted from the balance at the end of each period.

In real life things are a little more complicated. Unlike the example it would not be fair to assume that the typewriter would suddenly stop working or be absolutely worthless after five years. It could be possible to sell the typewriter even it has been completely written off.

EXAMPLE 5.42

A vehicle that depreciates over 5 years, is purchased at a cost of R400 000 and will have a salvage value of R130 000.

1 Calculate the annual depreciation according to the straight-line method.
2 Construct a depreciation schedule for the vehicle over five years.

SOLUTION

1. Original cost is R400 000
Expected life is five years
Salvage value is R130 00
The vehicle will depreciate at:

$$\text{Annual depreciation} = \frac{\text{original cost price - salvage value}}{\text{expected life}}$$

$$\text{Annual depreciation} = \frac{400000 - 130000}{5} = R54000 \text{ per year}$$

Year	Book value at the beginning of the year	Annual depreciation	Accumulated depreciation	Book value at the end of the year
1	R400 000 *(Original Cost)*	R54 000	R54 000	R346 000
2	R346 000	R54 000	R108 000	R292 000
3	R292 000	R54 000	R162 000	R238 000
4	R238 000	R54 000	R216 000	R184 000
5	R184 000	R54 000	R270 000	R130 000 *(Scrap Value)*

EXAMPLE 5.43

The value of machinery bought for R20 000 decreases every year by the fixed amount, which is 5% of R20 000. Calculate the machinery's value after eight years.

SOLUTION

The original cost is R20 000 and the value decreases every year by 5% of R20 000.

Annual depreciation $= \frac{5}{100} \times 20000 = R1000$ per year

Depreciation over eight years = annual depreciation x 8
$= 1000 \times 8$
= R8 000

Book value = original cost – (k x annual depreciation)
= R20 000 – R8000
= R12 000

The machinery's value after eight years will be R12 000.

EXAMPLE 5.44

A company purchases a computer for R5 500. Its useful life is estimated to be five years. Use the straight-line method to find the annual depreciation if, at the end of five years, the computer is assumed to have:

1 no salvage value
2 a salvage value of R750.

SOLUTION

The original cost is R7 500 and the expected life is five years.

1 Salvage value is zero.

Annual depreciation $= \frac{\text{original cost price - salvage value}}{\text{expected life}}$

$= \frac{5500 - 0}{5}$

$= R1100$

The annual depreciation is R1 100.

2 Salvage value is R750.

Annual depreciation $= \frac{\text{original cost price - salvage value}}{\text{expected life}}$

$= \frac{5500 - 750}{5}$

$= R950$

The annual depreciation is R950.

ASSESSMENT ACTIVITY 2.9

1. A company purchases a trailer for R6 000. Its useful life is estimated to be five years. Use the straight-line method to find the annual depreciation if at the end of five years the trailer is assumed to have:
- **1.1.** no salvage value.
- **1.2.** a salvage value of R800.

2. A restaurateur decided to purchase an air conditioning system for his restaurant at R8 000. The value of the air conditioning system will decrease every year by 10% of its original value and it is expected to be worthless to the restaurateur and without any scrap value in the end.
- **2.1.** Calculate the annual depreciation according to the straight-line method.
- **2.2.** How long will it take before the air conditioning system is worthless?
- **2.3.** Construct a depreciation schedule for the air conditioning system until it is worthless.
- **2.4.** Determine the value of the air conditioning system after eight years.

3. John paid R156 000 for a new car at the end of 2003. He paid a deposit of R30 000 and arranged finance for the balance at a nominal rate of 20% compounded monthly for one year.
- **3.1.** What was the outstanding amount that had to be financed?
- **3.2.** Calculate John's monthly payments that he had to make.
- **3.3.** Eventually the car is expected to be worthless to John, without any scrap value.
  - **3.3.1.** Calculate the annual depreciation according to the straight-line method over 15 years.
  - **3.3.2.** The car depreciates by 15% of its original cost every year. Find the value of the car at the end of 2008.
  - **3.3.3.** How long will it take before the car is completely depreciated?

Reducing balance method

This method calculates the depreciation at the end of every period, on the balance of the asset at the end of the last period. You therefore reduce the value of the asset by a fixed percentage (and not a fixed amount) at the end of every period.

Depreciation is calculated as a percentage of the reducing balance. Each period uses the previous period's balance to work out the new balance. The reducing balance method of depreciation provides a high annual depreciation charge in the early years of an asset's life, but it will reduce progressively as the asset ages.

EXAMPLE 5.45

On 1 August 1988 Sue bought a motorcycle for R9 000. If the motorcycle depreciated at a rate of 20% per annum, how much was it worth when she traded it in for another motorcycle on 1 August 1991?

SOLUTION

1st year: Depreciation $= \frac{20 \times 9000}{100} = \text{R}1800$

2nd year: The new value $= R9000 - R1800 = R7200$

Depreciation $= \frac{20 \times 7200}{100} = R1440$

3rd year: The new value $= R7200 - R1440 = R5760$

Depreciation $= \frac{20 \times 5760}{100} = R1152$

The new value equals $R5760 - R1152 = R4608$.

So the value of the motorcycle on 1 August 1991 was R4608.

Let us look at the typewriter-example when explaining the straight-line method.

EXAMPLE 5.46

A company bought a typewriter for R2 000. Construct a depreciation schedule over the useful life of the typewriter (salvage value is zero), if the typewriter depreciates at a rate of 20% per annum. How long will it take for the typewriter to be worthless?

SOLUTION

The original cost is R2 000 and the typewriter depreciates at a rate of 20% per annum. The depreciation schedule for the typewriter over three years will be as follows:

Year	Book value at the beginning of the year	Annual depreciation	Accumulated depreciation	Book value at the end of the year
1	R2 000 *(Original Cost)*	$\frac{20}{100} \times 2000 = R400$	R400	R1 600
2	R1 600	$\frac{20}{100} \times 1600 = R320$	R720	R1 280
3	R1 280	$\frac{20}{100} \times 1280 = R256$	R976	R1 024

A more complete depreciation schedule prepared on an Excel sheet is shown below:

	A	B	C	D	E
1	**Year**	**Beginning**	**Annual**	**Accumulated**	**Ending**
2		**Balance**	**Depreciation**	**Depreciation**	**Balance**
3	1	2000.00	400.00	400.00	1600.00
4	2	1600.00	320.00	720.00	1280.00
5	3	1280.00	256.00	976.00	1024.00
6	4	1024.00	204.80	1180.80	819.20
7	5	819.20	163.84	1344.64	655.36
8	6	655.36	131.07	1475.71	524.29
9	7	524.29	104.86	1580.57	419.43
10	8	419.43	83.89	1664.46	335.54
11	9	335.54	67.11	1731.56	268.44
12	10	268.44	53.69	1785.25	214.75
13	11	214.75	42.95	1828.20	171.80
14	12	171.80	34.36	1862.56	137.44
15	13	137.44	27.49	1890.05	109.95
16	14	109.95	21.99	1912.04	87.96
17	15	87.96	17.59	1929.63	70.37
18	16	70.37	14.07	1943.71	56.29
19	17	56.29	11.26	1954.96	45.04
20	18	45.04	9.01	1963.97	36.03
21	19	36.03	7.21	1971.18	28.82
22	20	28.82	5.76	1976.94	23.06
23	21	23.06	4.61	1981.55	18.45
24	22	18.45	3.69	1985.24	14.76
25	23	14.76	2.95	1988.19	11.81
26	24	11.81	2.36	1990.56	9.44
27	25	9.44	1.89	1992.44	7.56
28	26	7.56	1.51	1993.96	6.04
29	27	6.04	1.21	1995.16	4.84
30	28	4.84	0.97	1996.13	3.87
31	29	3.87	0.77	1996.91	3.09
32	30	3.09	0.62	1997.52	2.48
33	31	2.48	0.50	1998.02	1.98
34	32	1.98	0.40	1998.42	1.58
35	33	1.58	0.32	1998.73	1.27
36	34	1.27	0.25	1998.99	1.01
37	35	1.01	0.20	1999.19	0.81
38	36	0.81	0.16	1999.35	0.65

With the straight-line method the accumulated depreciation is R2 000. With the reducing balance method the accumulated depreciation (after 38 years) is R1 999,35. It is almost the same, because the total value of the typewriter had to be depreciated. It will, however, take more than 38 years before the typewriter will be deemed worthless (of value zero) if we use the reducing balance method, compared to the five years that it will take if we use the straight-line method.

Calculations such as these can become very tedious. For this reason we can use the following formula to determine the new value after depreciation.

Formula for determining the new value after depreciation:

The book value at the end of k years $= \text{Initial value} \times (1 - \text{depreciation rate})^k$

The depreciation rate must be written in decimal form.

Let us look at our first example again, now by using the formula.

EXAMPLE 5.47

On 1 August 1988 Sue bought a motorcycle for R9 000. If the motorcycle depreciated at a rate of 20% per annum, how much was it worth when she traded it in for another motorcycle on 1 August 1991?

SOLUTION

The book value at the end of k years $= \text{Initial value} \times (1 - \text{depreciation rate})^k$

$$= R9000 \times (1 - 0,2)^3$$

$$= R4608$$

The value of the motorcycle after three years was R4 608, as before.

ASSESSMENT ACTIVITY 2.10

1. A motorbike is bought second hand for R12 000. Its price depreciates at 15% per year. For how much could it be sold in five years' time? (Use the reducing balance method)
2. A firm purchased machinery on 1 January 2008. The machinery cost R12 500 and was depreciated using the reducing balance method at a rate of 33%. Show the depreciation account for the first four years of the asset's life.
3. A restaurateur decided to purchase an air conditioning system for his restaurant. The value of the air conditioning system will decrease every year by 20% under the reducing balance method. At the end of the fifth year the value will be R2 457,60.
   - **3.1.** Determine the original cost of the air conditioning system.
   - **3.2.** Determine the air conditioning system's value after ten years.
4. A computer has a book value of R5 000 after three years. Depreciation was calculated under the reducing balance method at a rate of 15%. Find the original cost of the computer.

5. A motorcycle that was purchased for R45 000 is expected to have a trade-in value of R25 000 four years later. Use the reducing balance method to calculate the rate of depreciation.

5. Personal Income Tax

Personal Income Tax is a direct tax levied on the income of a person. Individuals who receive income are liable to pay personal income tax.

1 When is an individual liable for income tax?

Every individual who receives income in excess of a specific amount (also known as the "threshold" amount) in a year is liable to pay income tax. The threshold amount varies from year to year.

2 Different kinds of income that an individual can be taxed on:

- **a** Income from employment such as salaries, wages, bonuses and overtime
- **b** Investment income such as interest and rental income
- **c** Annuities
- **d** Pensions

3 What kind of tax deductions are there? (Parts of income that are "tax free")

- **a** Interest Exemption
- **b** Dividends Exemption
- **c** Medical Deductions
- **d** Retirement Annuities
- **e** Donations Exemption

4 What is employee's tax?

Employee's tax is the tax that employers must deduct from the income of their employees (salaries, wages and bonuses) and pay directly to the South African Revenue Service (SARS) on a monthly basis.

5 What is a year of assessment for an individual?

A year of assessment for an individual consists of twelve months and starts on 1 March and ends at the end of February the following year.

6 To whom is the income tax payable?

The income tax is payable to SARS and they are responsible for collecting taxes from taxpayers on behalf of the Government.

7 When is income tax payable?

The final income tax payable by an individual can only be calculated once the total taxable income earned by the individual for the full year of assessment has been determined.

8 What is SITE?

SITE stands for Standard Income Tax on Employees. It is deducted from the income of all full-time employees who earn below the specific threshold for a specific year of assessment (which was R60 000 for the 2008 year of assessment).

9 What is PAYE?

PAYE stands for Pay-As-You-Earn. PAYE is deducted by the employer from the income of all full-time employees who earn in excess of the SITE threshold for a specific year of assessment (which was R60 000 for the 2008 year of assessment).

10 What proof does an employee have that tax was deducted from his/her earnings?

An employer must issue the employee with a receipt known as an employee's tax certificate (IRP 5 certificate) where it shows that tax was deducted from his/hers income.

11 Who should complete the tax forms?

Many individuals fill out their own personal tax forms. There are also registered tax agents who can assist in this process, although the responsibility for the correctness of the information entered on the form still rests with the individual.

The table below gives the Tax Rates for Individuals for the tax year 2008/2009.

Taxable Income (Rand)	Rates of Tax
0 – 122 000	18% of each R1
122 001 – 195 000	R21 960 + 25% of the amount above R122 000
195 001 – 270 000	R40 210 + 30% of the amount above R195 000
270 001 – 380 000	R62 710 + 35% of the amount above R270 000
380 001 – 490 000	R101 210 + 38% of the amount above R380 000
490 001 and above	R143 010 + 40% of the amount above R490 000

Rebates:	**Tax thresholds:**
Primary: R8 280 Additional (Persons 65 and older): R5 040	Below age 65: R46 000 Age 65 and over: R74 000

Tax Year http://www.oldmutual.co.za/markets/south-african-budget-2008/income-tax-calculator.aspx *(13-08-2008)*

Framework for calculation of tax liability:

Gross income		R…
Less exempt income		(R…)
	Income	**R…**
Less allowable deductions		(R…)
Add taxable capital gains		R…
	Taxable Income	**Rx**
Tax per tables % of Rx (taxable income) :		(R…)
Less rebates		(R…)
Less SITE and PAYE		(R…)
	Tax payable	**R…**

Take a look at the following simplified example:

An employee who is not yet 65 years of age received the following income for the period 1 March 2007 to 29 February 2008 (that is the 2008 year of assessment):

GROSS INCOME: R238 000
Tax was deducted and paid to SARS (by the employer) during the year of assessment as follows:

SITE	R3 060
PAYE	R41 770

Find the amount of tax payable to SARS by the employee.

SOLUTION:

Gross income received	R238 000
Less: exempt income	R0
Income	R238 000
Less: allowable deductions	R0
TAXABLE INCOME	**R238 000**

The income tax payable on the taxable income of R238 000 is calculated by applying the tax rates for the year of assessment ending 28 February 2008 (see table above). The taxable income of R238 000 falls within the interval R195 001 – R270 000.

Therefore the tax on the first R195 001 is		R40 210
The tax on the remaining R43 000 (R238 000 - R195 000) is 30% of R43 000		+R12 900
Normal tax payable		R53 110
Less: Primary rebate		-R8 280
Net normal tax payable		**R44 830**
Less:	SITE	-R3 060
	PAYE	-R41 770
TAX PAYABLE ON ASSESSMENT		**R0,00**

LEARNING ACTIVITY 2.11

Use the information in the table for tax rates for individuals to answer the following questions (assume all incomes were earned during the 2008 tax year).

1 Find the tax payable for a 34-year old person on the following taxable incomes:
- **a** R80 890
- **b** R252 471

2 Find the tax payable for a 71-year old person on the following taxable incomes:

  - **a** R160 780
  - **b** R302 371
3. From an income of R370 457, allowances of R12 569 are deducted. Find the tax payable on the assessment if the person is a senior citizen.
4. A man is 46 years old and earns R500 200 per year. He is allowed a relief of R1 890 on his pension fund payments. Determine the tax payable on his assessment.

GROUP ACTIVITY 2.12

1. Complete the IT12S SARS form below by making use of the given IRP 5.

   Use the following web site to help you to complete the form:
   http://www.capegateway.gov.za/Text/2007/9/how_to_fill_in_your_it12s_form
   (06-03-2009)

NEED MORE HELP?
- Call 0860 12 12 18
- Visit www.sars.gov.za or visit any SARS branch

South African Revenue Service

How to fill in your IT12S

INTRODUCTION

This guide is designed to help you to accurately and properly complete your Personal Income Tax return (IT12S). A comprehensive guide is available on the SARS website (www.sars.gov.za). If you need help to complete your return visit any SARS branch or look out in your local newspapers for details of where and when our staff will be visiting shopping centres, community halls and other public places in your area.

YOU MUST COMPLETE AND SUBMIT THE RETURN BY 31 OCTOBER 2007

COMPLETING THE RETURN

Use a black or a blue pen to complete the return and keep your writing within the spaces provided. Do NOT strike through the squares that do not apply.
Please do not use correcting fluid if you have made a mistake. We also request that you do not fold your return as it will delay the process of assessing your return.

You MUST complete all relevant parts of the return. Any incomplete return will be sent back to you and will be marked as "not submitted" until you send it in fully completed. This could result in penalties for the late submission of your return.

The following fields on the return are mandatory:
- Signature: The tax return is a legally binding declaration which you make to identify all the income, tax and deductions for the year. Without your signature it is worthless;
- ID number or Passport number or Date of Birth (at least one must be completed);
- Personal particulars (Name, address and contact details);
- Banking details;
- Income received (if you received no income during the year you must still complete this section by entering a zero [0]).

GETTING STARTED

To complete the return you will need the following documentation:
- Details of your banking particulars;
- Your IRP5 and/or IT3(a) certificates;
- Certificates you received for local interest income you earned (if applicable);
- Details of medical expenses and claims (if applicable);
- Information relating to retirement annuities (if applicable);
- Information relating to business travel expenses (if you get a travel allowance);
- Any other documentation relating to income you received or deductions you want to claim.

Please note that although you will be using the documentation to complete your return NONE of these documents must be attached to your return when you submit it to SARS. You will however be required to keep ALL relevant documents for a period of five years in case SARS calls for them.

DO YOU HAVE THE RIGHT RETURN?

To make it easier for you, this year SARS has introduced two new income tax returns for individuals.
The IT12S is the standard income tax return for individuals. It is intended for completion by employees who earn a salary and travel allowances and deductions such as medical, and pension. The form provides for taxpayers to:
- Indicate all their income earned from salaries and allowances;
- Indicate all the tax they paid during the year as reflected on their IRP5/IT3(a) (up to three IRP5s and IT3(a)'s);
- Indicate their medical expenses during the year;
- Indicate their pension or retirement annuity fund contributions during the year;
- Provide the information for the calculation of the travel expenditure. Should you choose that the calculation be made using actual expenditure the relevant information will have to be completed. Alternatively the fixed scale of costs will be applied.

The IT12C is the complex income tax return for individuals who have income such as rental income, foreign income, business income farming income and capital gain/losses. In addition to the same fields as the IT12S as discussed above, the form also provides for taxpayers to disclose any other income that the IT12S does not cater for.

IMPORTANT:

If you have received income or want to claim deductions which are not provided for on the IT12S form, you must complete an IT12C form. You can obtain one from www.sarsefiling.co.za or from any SARS branch.

Personal details

- Verify the correctness of the printed details and if they are incorrect use the white blocks to the right of the printed details to fill in the correct information.
- Marital status as at 28 February 2007: Mark the applicable "y" or "n" block with an X and if married indicate whether you are married in community of property or out of community of property with an X

Address information

- Verify the correctness of the printed details and if they are incorrect use the white blocks to the right of the printed details to fill in the correct information.

Work address

- Please complete the work address details of your current employer.
- If you had more than one job for the period 1 March 2006 to 28 February 2007, fill in the address details of your main employer - that could be where you worked the most hours. If you are no longer employed leave this field blank.

SARS
At Your Service
www.sars.gov.za

Bank account information

- Verify the correctness of the printed details and if they are incorrect use the white blocks to the right of the printed details to fill in the correct information.
- If no details are printed in this section you need to complete all the applicable fields with the relevant details.

Electronic transfers of refunds are effected using the branch number and not the name of the bank. You are therefore not required to fill in the name of the branch at which you transact.

Note: SARS only issues cheques in exceptional circumstances so you must provide banking details in order to receive a refund. No refund will be paid into the bank account of a third party or agent.

Preferred means of contact

Indicate your preferred means of contact by marking the applicable block by using the numbers 1 to 5. The number 1 indicates the most preferred option whilst 5 indicates the least preferred option.

Tax practitioner information

If you make use of a tax practitioner to complete your return this information should be completed by the tax practitioner.

INCOME RECEIVED

INCOME SECTION of an IRP5 / IT3(a) certificate

CODE/ KODE	DESCRIPTION / BESKRYWING	RF IND / UFD IND	AMOUNT/ BEDRAG
3601	Salary	Y	100,000
3605	Annual payment	N	20,000
3713	Other allowances - taxable	N	3,000
3705	Subsistence allowance - non taxable	N	1,000

The code in the first column reflects the source of your income and the last column reflects the amount you have received. These are the only two columns that you will use to complete this section of the return. If you are only in receipt of one IRP5 you will complete the amount and then the relevant source code in the section of the return as illustrated below.

INCOME RECEIVED

Complete below the SUM of all your income sources as shown on the IRP5 / IT3(a) certificates (grouped per source codes)

Amount (Rands only)	Source Code (IRP5/IT3(a))
100000	3601
20000	3605
3000	3713
1000	3705

If you received more than one IRP5 you must add the amounts from each IRP5 with the same source code together as illustrated below:

IRP5 Certificate from Employer A:

INCOME SOURCES

CODE/ KODE	DESCRIPTION / BESKRYWING	RF IND / UFD IND	AMOUNT/ BEDRAG
3601	Salary	Y	56,000
3605	Annual payment	N	5,000

3601 - This amount must be added to the R100 000 reflected on the IRP5 from employer B
3605 - This amount must be added to the R20 000 reflected on the IRP5 from employer B

IRP5 Certificate from employer B:

INCOME SOURCES

CODE/ KODE	DESCRIPTION / BESKRYWING	RF IND / UFD IND	AMOUNT/ BEDRAG
3601	Salary	Y	100,000
3605	Annual payment	N	20,000
3701	Travel allowance	N	40,000
3713	Other allowances - taxable	N	3,000

3601 -This amount must be added to the R56 000 reflected on the IRP5 from employer A
3605 - This amount must be added to the R5 000 reflected on the IRP5 from employer A

Your return will therefore be filled in as shown below:

INCOME RECEIVED

Complete below the SUM of all your income sources as shown on the IRP5 / IT3(a) certificates (grouped per source codes)

Amount (Rands only)	Source Code (IRP5/IT3(a))
156000	3601
25000	3605
3000	3713
40000	3701

56 000 + 100 000
5 000 + 20 000
These codes only appear on one IRP5

The same procedure must be followed if you are in receipt of three IRP5/IT3(a) certificates. Please note that if you received more than three certificates you must complete an IT12C return.

Gross retirement funding income

The amounts can be found in the gross remuneration section of the IRP5 certificate.

Use the amount reflected next to the code 3697 to fill in this field - if you have received more than one IRP5/IT3(a) certificate all the amounts reflected next to the code 3697 on the certificates must be added together and the total must be reflected on the return.

If your IRP5 does not reflect an amount next to the code 3697 you need to check whether any of the amounts are indicated as "y" in the RF Ind column. If so, you need to add together all these amounts and enter the total on the return as code 3697.

Local Interest

Gather all the certificates received from banks or other financial institutions where you have any money invested. Determine whether these certificates reflect any foreign interest or foreign dividends as income. If so, you must complete an IT12C return.

If you have earned local interest then you must proceed as follows:

If you are married in community of property
Add together the amounts reflected as local interest on the certificates received by both yourself and your spouse. You must fill in the total of the gross amounts of interest received by both you and your spouse - the exempt portion will be calculated by SARS.

If you are not married or married out of community of property
Add together the amounts reflected as local interest on the certificates you received.

Fill in the total of the gross amounts of interest - the exempt portion will be calculated by SARS.

Main Income Source Code

Using the table below, select the sector in the economy from which your main income is derived. Enter the relevant last two digits.

	Description
3534	Agencies and other services
3501	Agriculture, forestry and fishing
3511	Bricks, ceramics, glass cement and similar products
3523	Catering and accommodation
3509	Chemicals and chemical, rubber and plastic products
3505	Clothing and footwear
3510	Coal and petroleum goods
3520	Construction
3527	Educational services
3519	Electricity, gas and water
3525	Financing, insurance, real estate and business services
3503	Food drink and tobacco
3506	Leather, leather goods and fur (excluding footwear and clothing)
3514	Machinery and related items
3529	Medical, dental other health and veterinary services
3535	Members of CC/Director of company
3512	Metal
3513	Metal products (except machinery and equipment)
3502	Mining, stone and quarrying work
3518	Other manufacturing industries
3508	Paper, printing and publishing
3532	Personal and household services
3526	Public administration
3531	Recreational and cultural services
3528	Research and scientific institutes
3522	Retail trade
3517	Scientific, optical and similar equipment
3530	Social and related community services
3533	Specialised repair services
3504	Textiles
3516	Transport equipment (except vehicle, part and accessories)
3524	Transport storage and communication
3515	Vehicle, parts and accessories
3521	Wholesale trade
3507	Wood, wood products and furniture

PAYE & SITE paid

Note: This is the only section in the return where RANDS AND CENTS are used

To complete this section you again need your IRP5/IT3(a) certificates. This form allows for three certificates and is therefore divided into three sections.

The following information is required to fill in this section of the return:

- If you only received one IRP5 or IT3(a) certificate you will only have to complete the first section. You will be required to fill in the IRP5/IT3(a) certificate number which is located in the top left hand corner of the certificate. If you were issued with a duplicate certificate you will find that the certificate carries two certificate numbers. The number that you must fill in is the one listed next to the original certificate number.
- The PAYE reference number is located directly below the certificate number.
- The gross income is located next to the code 3699 in the gross remuneration section.

The tax calculation information section on the certificate reflects the "periods in year" as well as "periods worked". If the information is not available the fields must be left blank on the return.

- The amounts next to the codes 4101 and 4102 must be filled in separately in the boxes provided for SITE – code 4101 and PAYE – code 4102 on the return. If you received an IT3(a) certificate there will be no deduction for SITE or PAYE on the certificate. In this case the applicable fields on the return must be left blank.

MEDICAL DEDUCTIONS

State the number of members and dependants for whom contributions were made.

The information required refers to the contributions that you paid to the medical fund as at 28 February 2007, in respect of yourself and the other persons (dependants) that are covered by the fund. This information is usually reflected on the medical statement that you received from your medical fund.

Did the number change during the year of assessment?

Mark the 'Yes' or 'No' block with an "X".

If yes, state the total number of members and dependants per month.

Provision has been made for all twelve months of the year of assessment starting with "M" for March as the year of assessment starts on 1 March 2006.

State your medical fund contributions

- Contributions paid by your employer on your behalf will be reflected next to the code 4005 on the IRP5.
- Contributions paid by yourself i.e. via your cheque account or debit order will be reflected on your medical statement.

State your qualifying medical expenses not recovered from the medical fund

If you belong to a medical fund you will find the amount for the medical expenses that you have not recovered from the medical fund on the medical statement. This is the amount to be used together with the amount of the claims that you did not submit to the Medical fund due to the fact that you have exceeded the limits in respect of certain procedures. The amounts in respect of special dependants do not qualify as a deduction and must not be filled in on your return.

For more information refer to the comprehensive guide available on the SARS website www.sars.gov.za.

State your employer's contributions

This information will be reflected next to the code 4474 on your IRP5/IT3(a) certificate.

Physical disability expenses (not recovered from the Medical Fund)

If you, your spouse or child have a physical disability the amount of expenditure relating to the physical disability must be inserted here providing it was not covered by the Medical Fund.

Handicapped expenses

A handicapped person refers to a blind person, a deaf person, or a person who, as a result of a permanent physical disability, requires a wheelchair, calliper or crutch, to assist him or her to move from one place to another, or a person who requires an artificial limb. It also includes a person who suffers a mental illness as defined in the Mental Heath Act.

Note: Please refer to the comprehensive guide available on the SARS website www.sars.gov.za to determine what supporting documentation must be retained for a period of five years to substantiate your claim.

RETIREMENT CONTRIBUTIONS

State your Pension Fund Contributions

This information is on your IRP5 /IT3(a) certificate under the "Deduction" section.

Current - Use the amount next to code 4001 on your certificate.
Arrears - Use the amount next to code 4002 on your certificate.
Non-Statutory Forces Pension Arrears - Use the amount next to code 4026 on your certificate.

State your Retirement Annuity Contributions

Current and arrear retirement annuity contributions

Use the certificate you received from the institution to which you made the contributions to complete the return. Only contributions you paid in respect of a policy that you yourself will benefit from can be claimed.

OTHER QUALIFYING DEDUCTIONS

Note: Please refer to the comprehensive guide available on the SARS website to determine what supporting documentation must be retained for a period of five years to substantiate your claim.

In terms of the Income Tax Act only certain deductions are allowable if you earn a salary. Some of these deductions such as pension and retirement annuity contributions have already been addressed in this brochure. The remaining deductions that qualify, providing expenses were incurred, are the following:

Subsistence allowance

If you were in receipt of a subsistence allowance which is reflected next to source code 3704 on your IRP5/IT3(a) certificate, enter the amount calculated next to the code 4017 on your return.

Donations to approved Section 18A Public Benefit Organisations

Use the amount reflected on the receipt you received. Remember the amount will only qualify as a deduction if the receipt states that it is issued in terms of Section18A of the Income Tax Act.

Income protection insurance contributions

If you made any contributions to protect your income you should receive a certificate from the institution to which the contributions were made. Enter the amount next to the code 4018 on your return.

Depreciation

If you own an asset, such as a computer, and you are obliged to use this asset regularly to perform tasks relating to your job, you will be entitled to claim depreciation on the asset. The amount calculated must be filled in next to the code 4027 on your return.

Home office expenses

If you are employed, working for a salary and a condition of your employment is to bear the cost of maintaining a home office as your central business location, you may qualify for a deduction in terms of your home office.

If you believe you are entitled to claim expenditure for a home office the following formula must be used when calculating the amount to be deducted:

A / B x Total costs, where

- A = the area in square metres of the area used for work;
- B = The total area in square metres (including any outbuildings and the area used for work) of the residence;
- Total costs = the total costs incurred in the acquisition and upkeep of the property (excluding expenses of a capital nature).

Other deductions

- You may claim legal expenses incurred in respect of any claim that is directly related to your salary package, such as a CCMA claim, which are included as income as a result of a court order or out of court settlement in respect of labour disputes.
- Bad debts and provision for bad debts.
- Reduction of the fringe benefit for the "use of a motor vehicle".
- Public office holder expenditure.

Should you require further detail contact your local SARS office or access the comprehensive guide available on the SARS website www.sars.gov.za.

TRAVELLING EXPENSES AGAINST A TRAVEL ALLOWANCE

You may only claim for travel expenses if your IRP5/IT3(a) certificate indicates that you received a travel allowance. A travel allowance on your certificate will be indicated by source code 3701 or 3702. If you have used more than two vehicles for the period 1 March 2006 to 28 February 2007 you need to complete an IT12C return.

Did you use a log book?

You have the choice to claim for travel expenses based on actual kilometres travelled or by using the deeming provisions provided for in the Income Tax Act. You must be in possession of a properly completed logbook in order to use actual kilometres. If you do not make a selection by marking the "yes" or "no" block, it will be assumed that the deeming provisions must be applied. In terms of the deeming provisions the first 18 000 kilometres travelled will be deemed private kilometres travelled. In calculating the travel claim the total kilometres travelled will be limited to 32 000.

Please note that the minimum information required for a log book is the following:

- Date on which the travel took place;
- The destination to and from;
- The kilometres travelled ;
- The reason for the travel.

Calculation of the travel claim

To enable SARS to calculate your travel claim it is imperative that you complete the following information:

- Start date and closing date;
- Starting and closing kilometres (odometer readings);
- The business kilometres travelled;
- The cost price or cash value of the vehicle;
- The vehicle registration number.

Please note that without this information SARS will not be in a position to calculate the travel claim and will therefore not consider any travel claim.

Travel expenses against a travel allowance can be claimed according to one of the following methods:

(i) Where accurate records of expenses have been kept

Complete the applicable line items under the sub heading "Where Records of Actual Expenditure were kept" in the return.

(ii) Where no records of expenses have been kept

The cost of scale table will be used to calculate your claim if you did not complete the line items relating to actual expenditure.

CHOOSE THE WAY IN WHICH YOU WANT TO SUBMIT YOUR RETURN TO SARS

Electronically

By registering as an eFiler on www.sarsefiling.co.za you will be able to receive, complete and submit your return electronically.

Post

Using the envelope provided by SARS mail the completed and signed return to SARS.

Remember NOT to include any supporting documents.

Drop off

All SARS branches have drop boxes where you can drop off your completed return.

page four

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                                     WERKNEMERSBELASTINGSERTIFIKAAT                                    IRP 5
WERKGEWER INLIGTING
------------------------------------------------------------------------------------------------------------
HANDELS- OF ANDER NAAM
                                                                              WERKGEWER BESIGHEIDSADRES
IRP 5 NOMMER                                 *    BLADSY   01   VAN   01
VERWYSINGSNOMMER                             *
BELASTINGJAAR                                *
WERKGEWER GENIET DIPLOMATIEKE VRYWARING      *
                                                                            POSKODE
------------------------------------------------------------------------------------------------------------
WERKNEMER INLIGTING

AARD VAN PERSOON    A *  WERKNEMER SE VAN OF HANDELSNAAM                                                   *
EERSTE TWEE NAME                                         *  VOORL.        *      WERKNEMER SE RESIDENSIELE ADRES
IDENTITEITSNOMMER                            *                                                             *
PASPOORTNOMMER                               *                                                             *
GEBOORTEDATUM                                *                                                             *
MAATSKAPPY/BK/TRUSTNOMMER                    *                                                             *
INKOMSTEBELASTINGNOMMER                      *                                POSKODE                      *
                                                                             WERKNEMERNOMMER               *
------------------------------------------------------------------------------------------------------------
BELASTING BEREKENING INLIGTING

TYDPERK IN DIENS VANAF         2006/03/01 *     TYDPERK IN DIENS TOT    2007/02/28 *      VRYWILLIGE OOR-AFTREKKING        N *
PERIODES IN BELASTINGJAAR 12.0000 * AANTAL PERIODES GEWERK      12.0000 * VASTE KOERS INKOMSTE N * AANWYSINGNOMMER         *

INKOMSTE BRONNE
 KODE      BESKRYWING                                       UFD IND               BEDRAG
 3601      INKOMSTE - BELASBAAR                                         *****196 186  *
 3605      JAARLIKSE BETALING - BELASBAAR                               ******38 925  *
 3801      VERKRYGING VAN BATES/VERSEKER. AFSTAND                       *******1 402  *
 3810      MEDIESE FONDS NAMENS 'N WERKNEMER BETAAL                     *******2 201  *

BRUTO BESOLDIGING
 KODE      BESKRYWING                                                             BEDRAG
 3695      BRUTO JAARLIKSE BETALINGS                                    ******38 925  *
 3697      BRUTO UITTREDINGFUNDERINGSDIENS INKOMSTE                     *****211 575  *
 3698      BRUTO NIE-UITTREDINGFUNDERINGSDIENS INK.                     ******27 139  *
 3699      BRUTO BESOLDIGING                                            *****238 714  *

AFTREKKINGS
 KODE      BESKRYWING                                 CLEARANCE NO                BEDRAG
 4001      LOPENDE PENSIOENFONDSBYDRAE                                  ******15 246  *
 4005      MEDIESE FONDSBYDRAE                                          *******3 982  *
 4006      LOPENDE UITTREDINGSANNUITEITFONDSBYDRAE                      *******3 143  *

WERKNEMERSBELASTING AFGETREK
 KODE      BESKRYWING                                                             BEDRAG
 4101      STAND. INKOMSTEBELASTING OP WERKN.- SIBW                   ******3 600.00  *
 4102      LOPENDE BETAALSTELSEL - LBS                                *****40 314.20  *
 4103      TOTALE WERKNEMERSBELASTING                                 *****43 914.20

WERKGEWER AFTREKKINGS
 KODE      BESKRYWING                                                             BEDRAG
 4472      WERKGEWER PENSIOENFONDSBYDRAES                               ******24 801  *
 4474      WERKGEWER MEDIESE FONDSBYDRAE                                *******8 201  *
 4486      BEPERKTE BEDRAG DEUR WERKGEWER BEPAAL                        *******6 000  *
```

IT12S 2007

Year of Assessment

PERSONAL INCOME TAX RETURN
(Income Tax Act, No. 58 of 1962, as amended)

I declare that:

- The information furnished in this return is true and correct in every respect; and
- I have disclosed in full the gross amounts of all income accrued to or received by me during the period covered by this return; and
- I have the necessary receipts and records to support all my declarations on this form which I will retain for inspection purposes.

SIGNATURE

Date C C Y Y M M D D

Please consult the SARS Brochure prior to completing this tax return.

TAX REFERENCE NO. / / /

If you have additional income and/or deductions to declare for which space is not provided on this return please contact your nearest SARS branch or call 0860 12 12 18 to obtain the IT12C return.

ID No.

Passport No.

Date of Birth C C Y Y M M D D

PERSONAL DETAILS

Surname | Initials | Home Tel No.

Married Y N | If yes: In community of property | Out of community of property | Cell No. | Fax No.

Email | Bus Tel No.

ADDRESS DETAILS

Postal Address | Postal Code

Residential Address | Postal Code

Work Address | Postal Code

BANK ACCOUNT DETAILS

Account No.

Branch No.

Cheque | Savings / Transmission

Name of Account Holder

PREFERRED MEANS OF CONTACT

Rate your preference as to how you would like to be contacted (Scale of 1 to 5)

Email 0 | SMS 0 | Post 0 | Fax 0 | Through Tax Practitioner 0

TAX PRACTITIONER DETAILS

Did a tax practitioner complete this form? Y N

Tax Practitioner Registration No. PR -

Tax Practitioner's Tel No.

INCOME RECEIVED

Complete below the SUM of all your income sources as shown on the IRP5 / IT3(a) certificates (grouped per source codes)

Amount (Rands only)	Source Code (IRP5/IT3(a))
Gross Retirement Funding Income	3697
Local Interest Income	4201
Main Income Source Code	35

SITE & PAYE PAID

IRP5 / IT3(a) Certificate No. Gross Income – Source Code 3699

PAYE Reference No. SITE - Source Code 4101 , C C

Pay Periods in Tax Year Pay Periods Worked PAYE – Source Code 4102 , C C

IRP5 / IT3(a) Certificate No. Gross Income – Source Code 3699

PAYE Reference No. SITE - Source Code 4101 , C C

Pay Periods in Tax Year Pay Periods Worked PAYE – Source Code 4102 , C C

SITE & PAYE PAID continued...

IRP5 / IT3(a) Certificate No. Gross Income – Source Code 3699

PAYE Reference No. SITE - Source Code 4101 , C C

Pay Periods in Tax Year Pay Periods Worked PAYE – Source Code 4102 , C C

MEDICAL DEDUCTIONS – Rands only

State the number of members and dependants as at 28 February 2007 for whom contributions were made

Did it change during the year of assessment? Y N

If YES, state the total number of members and dependants per month

M A M J J A S O N D J F

State your Medical Fund Contribution

4005

State your Qualifying Medical Expenses Not Recovered From a Medical Fund (other than Handicap / Disability Expenses)

4020

State your Employer's Contribution

4474

State your Handicap / Disability Expenses

Physical Disability Expenses (Not recovered from medical fund)	4022
Handicap Expenses (Not recovered from medical fund)	4023

RETIREMENT CONTRIBUTIONS – Rands only

State your Pension Fund Contribution

Current	4001
Arrears	4002
Non – Statutory Forces Pension Arrears	4026

State your Retirement Annuity Fund Contribution

Current	4006
Arrears	4007

Tax No: Surname: Page No: 02/02

Tax Year: DOB: Form ID: IT12S

ID/Passport: Initials: Time Stamp:

OTHER QUALIFYING DEDUCTIONS – Rands only

Expenses claimed against taxable subsistence allowance	4017
Donations to approved S 18A Public Benefit Organisations	4011
Income protection insurance contributions	4018
Depreciation	4027
Home office expenses	4028
Other	4016

TRAVEL CLAIM AGAINST ALLOWANCE – Rands only

Determination of Business Use

VEHICLE 1		VEHICLE 2
Y N	Did you use a log book to determine your business km travelled?	Y N
CCYYMMDD	Start Date	CCYYMMDD
CCYYMMDD	Closing Date	CCYYMMDD
	Closing Km	
	Starting Km	
	Total Km	
	Business Km	

Where Records of Actual Expenditure were kept

VEHICLE 1		VEHICLE 2
	Fuel and Oil	
	Maintenance and Repairs	
	Insurance and Licence	
	Wear and Tear or Lease Payment	
	Other	
	Vehicle Registration No.	
	Cost Price or Cash Value	

End of section comments

Today's financial world offers consumers a vast variety of products. Due to this degree of choice consumers need to be equipped with the knowledge and skills to evaluate the options and identify those that best suit their needs and circumstances.
We need to plan ahead on the following topics for example:

- budget
- use credit if you can afford it
- provide for unexpected events
- long term (e.g. retirement)
- aware of products that are available
- shop around for best products
- select best option.

This will help consumers understand how to prevent becoming involved in transactions that are financially destructive.

Feedback

Answers to check how well you are managing your money.

1 b
2 c
3 b
4 a
5 c
6 c

Answers to start-up activity 5.1

1.
- **1.1.** Weekender or Weekender plus
- **1.2.** They are the same
- **1.3.** They are the same

2.
- **2.1.** Companion
- **2.2.** They are the same

3. Depends on when you want to make calls

Answers to learning activity 5.2

1. R120
2.
- **2.1.** R1 520
- **2.2.** R3 520

3. R63
4. 8,75%
5. R2 000
6. 3 years
7. R2 631,58
8. 20%
9. 8%
10. 15,63%

Answers to learning activity 5.3

1. No, you will only have R13 506,57
2. R1 228,79
3. R23,97
4. R3 155,97
5. 6 years

6. 63%
7. Investment gives R3 726,89; Lotto gives R4 540,00, if you are very lucky; It is safer to invest your money
8. R1 863,35
9. 6,8%
10. R874,12
11. 2 years and 4 months

Answers to assessment activity 5.4

1. R498,90 effective rate 7,7%
2. R4 464,44
3. R2 670,01 effective rate 11,8%
4. 6,72%
5. 2,8%
6. 12 years and 8 months
7. 8 years and 8 months

Answers to assessment activity 5.5

1. R25 617,07
2. R621 073,63
3. R132 673,85
4. R5 774,23
5. R248,59
6. 24 years and 3 months

Answers to assessment activity 5.6

1. R7 779,96
2. R68 793,19
3. R19 778,57
4. R624,41
5. R403,49
6. 6 years and 10 months

Answers to assessment activity 5.7

1. R218 787,23
2.
   - 2.1. R5 946,44
   - 2.2. R4 984,86
3.
   - 3.1. R18 428,9

3.2, 3.3 and 3.4.

	Loan amount	100000			
	Interest rate	13% = 0.13			
	Loan term	10			
	Loan payment	18428.96			
Year	**Beginning**	**Total**	**Interest**	**Principle**	**Ending**
	Balance	**Payment**	**Paid**	**Paid**	**Balance**
1	100000.00	18428.96	13000.00	5428.96	94571.04
2	94571.04	18428.96	12294.24	6134.72	88436.32
3	88436.32	18428.96	11496.72	6932.24	81504.08
4	81504.08	18428.96	10595.53	7833.43	73670.65
5	73670.65	18428.96	9577.18	8851.78	64818.87
6	64818.87	18428.96	8426.45	10002.51	54816.36
7	54816.36	18428.96	7126.13	11302.83	43513.53
8	43513.53	18428.96	5656.76	12772.20	30741.33
9	30741.33	18428.96	3996.37	14432.59	16308.74
10	16308.74	18428.96	2120.14	16308.82	-0.08
Totals		**184289.6**	**84289.52**	**100000.08**	

3.5.

	Loan amount	100000			
	Interest rate	13% = 0.13			
	Loan term	10			
	Loan payment	18428.96			
Year	**Beginning**	**Total**	**Interest**	**Principle**	**Ending**
	Balance	**Payment**	**Paid**	**Paid**	**Balance**
1	=SUM(C1)	=+SUM(C4)	=B8*0.13	=SUM(C8-D8)	=SUM(B8-E8)
2	=F8	=+SUM(C4)	=B9*0.13	=SUM(C9-D9)	=SUM(B9-E9)
3	=F9	=+SUM(C4)	=B10*0.13	=SUM(C10-D10)	=SUM(B10-E10)
4	=F10	=+SUM(C4)	=B11*0.13	=SUM(C11-D11)	=SUM(B11-E11)
5	=F11	=+SUM(C4)	=B12*0.13	=SUM(C12-D12)	=SUM(B12-E12)
6	=F12	=+SUM(C4)	=B13*0.13	=SUM(C13-D13)	=SUM(B13-E13)
7	=F13	=+SUM(C4)	=B14*0.13	=SUM(C14-D14)	=SUM(B14-E14)
8	=F14	=+SUM(C4)	=B15*0.13	=SUM(C15-D15)	=SUM(B15-E15)
9	=F15	=+SUM(C4)	=B16*0.13	=SUM(C16-D16)	=SUM(B16-E16)
10	=F16	=+SUM(C4)	=B17*0.13	=SUM(C17-D17)	=SUM(B17-E17)
Totals		**=SUM(C8:C17)**	**=SUM(D8:D17)**	**=SUM(E8:E17)**	

ANSWERS TO GROUP ACTIVITY 5.8

Answers may vary from year to year.

1.

1.1. Transfer cost R17 113.

1.2. Transfer cost is the money you have to pay to register the house in your name.

1.3. I must apply for R667 113

1.4. R8 784,47
1.5. R1 441 159,80
1.6. R2 108 272,80

2.
- **2.1.**
  - **2.1.1.** R204 505,30
  - **2.1.2.** R17 042,11
- **2.2.**
  - **2.2.1.** R7 900,74
  - **2.2.2.** R77 825,00
  - **2.2.3.** R41 541,15
  - **2.2.4.** In year 16
- **2.3.**
  - **2.3.1.** R6 036,87
  - **2.3.2.** R948 849,62
  - **2.3.3.** R1 448 849,62
  - **2.3.4.** In year 15
  - **2.3.5.** Interest R5 420,75; Principle R616,13; Balance R481 227,90

Answers to assessment activity 5.9

1.
- **1.1.** R1 200
- **1.2.** R1 040

2.
- **2.1.** R800
- **2.2.** 10 years
- **2.3.**

Year	Book value at the beginning of the year	Annual depreciation	Accumulated depreciation	Book value at the end of the year
1	8000	800	800	7200
2	7200	800	1600	6400
3	6400	800	2400	5600
4	5600	800	3200	4800
5	4800	800	4000	4000
6	4000	800	4800	3200
7	3200	800	5600	2400
8	2400	800	6400	1600
9	1600	800	7200	800
10	800	800	8000	0

- **2.4.** R1 600

3.
- **3.1.** R126 000
- **3.2.** R11 672,19

3.3.

3.3.1. R10 400

3.3.2. R39 000

3.3.3. 7 years

Answers to assessment activity 5.10

1. R5 324,46

2.

Year	Book value at the beginning of the year	Annual depreciation	Accumulated depreciation	Book value at the end of the year
1	12500	4166.67	4166.67	8333.33
2	8333.33	2777.78	6944.44	5555.56
3	5555.56	1851.85	8796.30	3703.70
4	3703.70	1234.57	10030.86	2469.14
5	2469.14	823.05	10853.91	1646.09
6	1646.09	548.70	11402.61	1097.39
7	1097.39	365.80	11768.40	731.60
8	731.60	243.87	12012.27	487.73
9	487.73	162.58	12174.85	325.15
10	325.15	108.38	12283.23	216.77
11	216.77	72.26	12355.49	144.51
12	144.51	48.17	12403.66	96.34
13	96.34	32.11	12435.77	64.23
14	64.23	21.41	12457.18	42.82
15	42.82	14.27	12471.45	28.55
16	28.55	9.52	12480.97	19.03
17	19.03	6.34	12487.31	12.69
18	12.69	4.23	12491.54	8.46
19	8.46	2.82	12494.36	5.64
20	5.64	1.88	12496.24	3.76
21	3.76	1.25	12497.49	2.51
22	2.51	0.84	12498.33	1.67
23	1.67	0.56	12498.89	1.11
24	1.11	0.37	12499.26	0.74
25	0.74	0.25	12499.50	0.50
26	0.50	0.17	12499.67	0.33
27	0.33	0.11	12499.78	0.22
28	0.22	0.07	12499.85	0.15
29	0.15	0.05	12499.90	0.10
30	0.10	0.03	12499.93	0.07
31	0.07	0.02	12499.96	0.04
32	0.04	0.01	12499.97	0.03
33	0.03	0.01	12499.98	0.02
34	0.02	0.01	12499.99	0.01

35	0.01	0.00	12499.99	0.01
36	0.01	0.00	12499.99	0.01
37	0.01	0.00	12500.00	0.00

3.1 R7 500
3.2 R805,31
4. R8 141,66
5. 13,7%

Answers to learning activity 5.11

1.
1.1. R6 280,20
1.2. R49 171,30
2.
2.1. R26 615
2.2. R68 999,85

3. R88 430,80
4. R138 054

Answers to group activity 5.12

The first page will have the student's personal information on.

INKOMSTE ONTVANG

Voltooi hieronder die SOM van al u inkomstebronne soos aangetoon op u IRP5 / IT3(a) sertifikate (groepeer volgens bronkodes)

Bedrag (slegs Rande)	Bronkode (IRP5 / IT3(a))
196186	3601
38925	3605
Bruto Uittredingfunderingsdiensinkomste: 211575	3697
Plaaslike Rente-inkomste:	4201
Bronkode van Hoofbron van Inkomste	3534

SIBW EN LBS BETAAL

IRP5 / IT3(a) Sertifikaatno.	Bruto Inkomste – Bronkode 3699
00001273	238714

LBS Verwysingsno.	SIBW – Bronkode 4101
7120710266	3600,00

Tydperke in jaar	Tydperke gewerk	LBS – Bronkode 4102
12	12	40314,20

IRP5 / IT3(a) Sertifikaatno.	Bruto Inkomste – Bronkode 3699

LBS Verwysingsno.	SIBW – Bronkode 4101
	, c c

Tydperke in jaar	Tydperke gewerk	LBS – Bronkode 4102
		, c c

SIBW EN LBS BETAAL vervolg...

IRP5 / IT3(a) Sertifikaatno.	Bruto Inkomste – Bronkode 3699

LBS Verwysingsno.	SIBW – Bronkode 4101
	, c c

Tydperke in jaar	Tydperke gewerk	LBS – Bronkode 4102
		, c c

MEDIESE AFTREKKINGS – slegs Rande

Meld die aantal lede en afhanklikes soos op 28 Februarie 2007 vir wie bydraes gemaak is: 01

Het die aantal verander gedurende die jaar van aanslag? J X

Indien JA, toon die aantal lede en afhanklikes per maand

M	A	M	J	J	A	S	O	N	D	J	F

	Bedrag	Bronkode
Meld u Mediesefondsbydraes	3982	4005
Meld u kwalifiserende mediese uitgawes nie van u Mediesefonds verhaal nie (uitgesluit uitgawes t.o.v. Liggaamsgebrek / Gestremdheid)		4020
Meld u werkgewer se Bydrae	8201	4474

Meld u uitgawes t.o.v. Liggaamsgebrek / Gestremdheid

Uitgawes t.o.v. liggaamsgebrek (nie van u mediesefonds verhaal nie)		4022
Uitgawes t.o.v gestremdheid (nie van u mediesefonds verhaal nie)		4023

AFTREEBYDRAES – slegs Rande

Meld u Pensioenfondsbydraes

Lopend	15246	4001
Agterstallig		4002
Agterstallige bydraes – "Non-statutory Forces"		4026

Meld u Uittredingannuïteitsfondsbydraes

Lopend	3143	4006
Agterstallig		4007

Belasting No: 0330279142 Van: KLEYNHANS Bladsy No: 02/02
Belastingjaar: 2007 Geboortedatum: 19700607 Form ID: IT12S
ID/Paspoort: 7006070021084 Voorletters: AF Time Stamp:

ANDER KWALIFISERENDE AFTREKKINGS – slegs Rande

Uitgawes geëis teen belasbare verblyftoelaag		4017
Skenkings aan goedgekeurde Art. 18A Openbare Weldaadsorganisasies		4011
Inkomstebeskermingsbydraes		4018
Waardevermindering		4027
Uitgawes t.o.v Tuiskantoor		4028
Ander		4016

REISKOSTES TEEN TOELAAG – slegs Rande

Bepaling van Besigheidsgebruik

VOERTUIG 1		VOERTUIG 2
J N	Het u 'n ritboek gebruik om besigheidskilometers gereis te bepaal?	J N
E E J J M M D D	Begindatum	E E J J M M D D
E E J J M M D D	Einddatum	E E J J M M D D
	Eindkilometers	
	Beginkilometers	
	Totale kilometers	
	Besigheids-kilometers	

Waar Rekord van Werklike Uitgawes gehou is

	Brandstof en olie	
	Reparasies en onderhoud	
	Assuransies en lisensies	
	Slytasie of Bruikhuur	
	Ander	
	Voertuigregistrasienommer	
	Koopprys of Kontantwaarde	

IT12S_Af_B005

Tracking my progress

You have reached the end of this section. Check whether you have achieved the learning outcomes for this section.

LEARNING OUTCOMES	✓ I FEEL CONFIDENT	✓ I DON'T FEEL CONFIDENT
Calculate simple interest using the equation $SI = P \times i \times n$		
Manipulate the simple interest formula to solve for any of the unknowns.		
Calculate compound interest using the equation $A = P(1+i)^n$		
Manipulate the compound interest formula to solve for any of the unknowns.		
Evaluate the effective rate of interest (if the nominal rate is given) using the equation $r = \left(1 + \frac{i}{n}\right)^n - 1$		
Distinguish between the future and present value of annuities		
Calculate the future value of an annuity using the formula $F_v = R \times \frac{(1+i)^n - 1}{i}$		
Manipulate the future value of an annuity formula to solve for any of the unknowns		
Calculate the present value of an annuity using the formula $P_v = R \times \frac{1-(1+i)^{-n}}{i}$		
Manipulate the present value of an annuity formula to solve for any of the unknowns		
Prepare an amortisation schedule by making use of an Excel file		
Calculate annual depreciation using the straight line method		
Calculate annual depreciation using the		

reducing balance method		
Prepare a depreciation schedule for both the straight line method and the reducing balance method		
Calculate depreciation value using the following equation The book value at the end of k years = Initial value × $(1 - \text{depreciation rate})^k$		
Manipulate the depreciation value formula to solve for any of the unknowns		
Solve simple personal tax problems		
Use a computer to determine the monthly instalments of a mortgage		

Now answer the following questions honestly:

1 What did you like best about this section?

2 What did you find most difficult in this section?

3 What do you need to improve on?

4 How will you do this?