



## Introduction

The Data-Informed Practice Improvement Project (DIPIP) is a Gauteng Department of Education (GDE)-Wits School of Education (WSOE) initiative which was established in 2006. Our goal is to improve teaching and learning in mathematics. In 2006 and 2007 some schools wrote the International Competitions and Assessments for Schools (ICAS) tests. Each test consists of multiple choice questions assessing certain core mathematical knowledge that a learner in a particular grade should have. The performance levels were low across the grades. A group of teachers (from the ICAS schools), Subject Advisors and Wits staff members and post-graduate students has been working towards identifying learners' misconceptions leading to errors in mathematics by analysing the data obtained from the ICAS tests. We hope this will be useful for informing teaching practice. We have two focuses that aim to be of help to the mathematics teacher:

- Develop classroom practices to deal with learner misconceptions
- Produce teacher support materials, which will be directed towards understanding learner misconceptions.

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One way of communicating with teachers in schools will be through this newsletter. We would like your feedback, so that we can begin a conversation about improving mathematics teaching. Our newsletters will include some of our findings and also introduce some ideas for teaching. This newsletter contains some questions that focus on ratio, rate and proportion. We also discuss the errors that the students have made; and try to identify why learners are struggling with particular concepts. We also suggest teaching strategies that may be useful in correcting some of the errors.

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## Misconceptions are useful!

Past research has shown that learners' errors are often very logical to the learners and are usually carried out consistently. According to the research, errors also tend to be very resistant to attempts by the teacher to correct them. Also, errors can have a positive effect for teachers, in that they can reveal incompleteness in learners' knowledge; and thus enable the teacher to contribute additional knowledge, or better still, guide the learner to realise for him or herself where s/he is going wrong. Errors often show us learners' misconceptions. Misconceptions are learners' conceptual ideas that explain why they (learners) might produce a particular error or set of errors. For example, in generating equivalent fractions, learners often say 'what is done to the numerator should be done to the denominator' of the original fraction. Following this rule, learners may say  $\frac{a}{b}$  is equal to  $\frac{a \times 2}{b \times 2}$ , which is correct, and at other times,  $\frac{a}{b}$  is equal to  $\frac{a+2}{b+2}$ , which is incorrect. The language used to speak about finding equivalent fractions has enabled these learners to form this misconception. As teachers we need to keep a close watch on how we speak about and introduce mathematical ideas. Note that misconceptions may also sometimes lead

to *correct* answers, although the mathematical thinking producing these answers is incorrect.

Misconceptions are part of the knowledge that learners develop, and hence form part of their current knowledge. As teachers, we need to build on learners' current knowledge. This means that we need to listen to, work with and build on learners' misconceptions as well as their correct conceptions. As Prof. Karin Brodie at the Wits School of Education notes, learning actually entails transforming current knowledge. Therefore, if misconceptions exist, the existing knowledge will need to be transformed and restructured into new knowledge. How we as teachers listen to and work with learners' thinking is related to the information available to us about:

- the type of conceptions learners have,
- how to build onto these conceptions, and
- how to use misconceptions to transform learner thinking and to inform our own teaching.

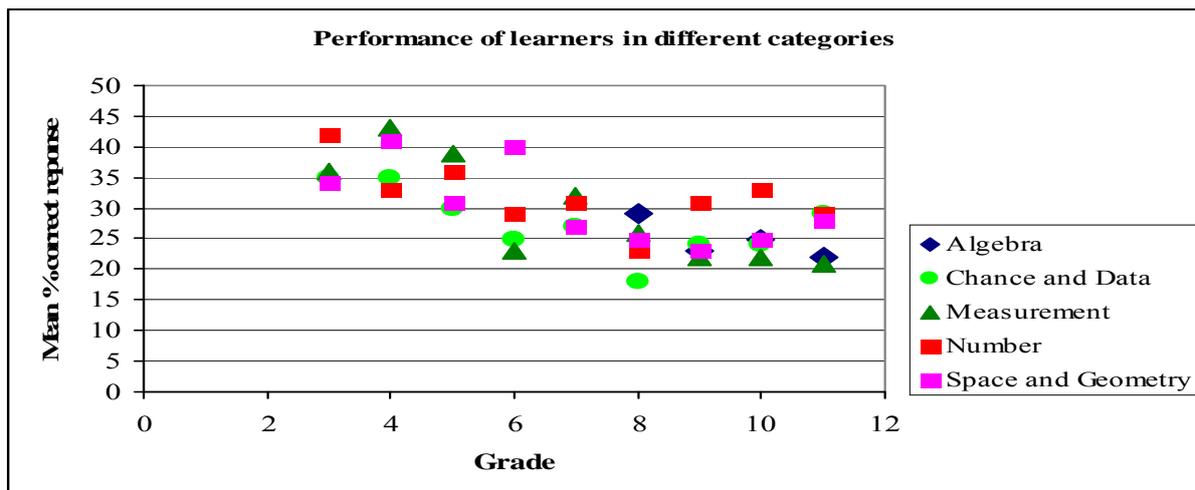
The latter is the main purpose of DIPIP.

## Learner errors in the ICAS tests

As we began to think about how to make your learners' thinking more visible to you, we felt that it is worthwhile to give you a bigger picture of the data with which we work. The ICAS tests consist of 40 items focusing on different categories for each grade level. These categories are similar to our own 'learning outcomes' outlined in the NCS: number, algebra, chance and data, measurement, and space and geometry. Learners from Grades 3 to 11 wrote the tests.

The results showed that although the overall performance across the grades was low, learners in the higher grades did worse than those in the lower grades.

We looked at learner performance for the different categories assessed. The graph below shows a summary of the percentages of correct responses across the different categories and grade levels.



There were two general problems with our learners' performance:

1. Generally low performance across all categories (under 50% correct), and
2. The performance varies widely within the different areas. For example, in the category of *measurement*, the percentage of correct responses ranged from 21% in Grade 11 to 43% in Grade 4. For *Number*, performance ranged from 23% in grade 8 to 42% correct in grade 3.

These poor results led to a call for workable solutions for the teacher - to address learners' misconceptions. Our project hopes to merge identification of learners' reasoning and misconceptions with the designing of tasks, teacher support materials and teaching strategies. We will discuss some of them further in this letter. First we address the errors we found in the area of ratio, rate and proportion.

## Understanding learner errors

In this first issue of the Newsletter, our focus is on Learning Outcome 1 (LO1) from the National Curriculum Statement: *Numbers, operations and relationships*. We saw evidence of misconceptions when learners answered items on ratio, rates and proportion within LO1, and we address this topic in this letter. The specific assessment standards (ASs) addressed are:

Grade 7: '*... when the learner ... solves problems that involve ratio and rate*'

Grade 8: '*... when the learner solves problems that involve ratio and rate*'. (Proportional reasoning was not specified but is necessary to solve some ICAS problems)

Grade 9: '*... when the learner ... solves problems that involve ratio and rate and proportion (direct and indirect)*'. (Ratio as a scale was not specified.)

Below are three ICAS items on ratio, rate and proportion, with analyses of their correct answers and learners' errors.

### **Grade 8 item: Ratio**

*Kent has a recipe for salad dressing that uses 12 spoons of oil and 3 spoons of vinegar.*

*Kent decides to use  $\frac{2}{3}$  of the amount of oil, but the same amount of vinegar. What fraction of Kent's dressing will be vinegar?*

**(A)  $\frac{3}{11}$  (B)  $\frac{1}{3}$  (C)  $\frac{3}{8}$  (D)  $\frac{8}{11}$**

Only 15% of learners arrived at the correct answer (A)

44% (almost half) chose option B

25% chose option C

14% chose option D

*2% of learners did not answer this question*

*Our analysis of the correct answer shows that to get the correct answer, learners need to note that Kent's new recipe would have 8*

*spoons of oil (two-thirds of 12 spoons) and 3 spoons of vinegar. They also need to write this as a ratio; resulting in the oil to vinegar ratio as 8:3 (OR vinegar to oil ratio as 3:8). They then need to recognise that the whole mixture in this case is  $3+8=11$  spoons. Therefore, 3 parts out of 11 are vinegar (and 8 out of 11 will be oil).*

In our error analysis of this item, we identified misconceptions about correctly identifying the 'whole'.

1. For option B, it is possible that the learners understood that if  $\frac{2}{3}$  of the dressing is oil, then the rest ( $\frac{1}{3}$ ) must be vinegar; and thus they would choose option B. This was most likely the case, given that almost half of the learners chose the option. This was a subtle misinterpretation of the question, in that they thought that  $\frac{2}{3}$  **of the mixture** was oil, instead of understanding that  **$\frac{2}{3}$  of the original amount of oil** was used.
2. For option C, the new amount of oil was possibly correctly identified as 8 parts of oil; and learners kept the original amount of Vinegar (3 parts). From here they said that the ratio of vinegar to oil is 3:8, or 3 out of 8 parts ( $\frac{3}{8}$ ), not understanding that the whole mixture contained 11 spoons of liquid. Option C implies that the whole was 8, and 3 parts of it is vinegar, thereby representing vinegar by  $\frac{3}{8}$ .
3. For option D, the learners correctly calculated the new amount of oil to be 8 spoons. They also probably recognised the 'whole' in this case to be 11. Somehow they forgot that the question required the **fraction of vinegar** in the mixture, and chose the distractor showing the fraction of oil  $\frac{8}{11}$  as the answer.

Our discussion of ratios in the next section will present some general teaching issues around ratio.

**Grade 7 item: Rate**

Sam and Kevin are bricklayers. Sam lays 150 bricks in 60 minutes. Kevin lays 20 bricks in 10 minutes. Working together, how many minutes will it take Sam and Kevin to lay 180 bricks?

(A) 25    (B) 40    (c) 70    (D) 100

Only 15% of learners chose the correct answer (B)

11% chose option A

46% (almost half) chose option C

25% chose option D

3% of learners did not answer the question

Our analysis of the correct answer shows that learners have to have a clear understanding of rates and how and when they can be compared and used together. To arrive at the correct answer, they need to be able to write the rates in ways that allow them to be able to combine them. To do this, they need to keep the time quantity the same. They can do this by:

- (a) finding how many bricks each layer would lay per minute as 2.5 for Sam and 2 for Kevin. They would then combine these to get  $2.5+2 = 4.5$  bricks/minute for the two bricklayers. This will then lead to 180 bricks in 40 minutes between the two layers.

**OR**

- (b) finding how long it will take each layer to lay one brick and obtain 0,4 minutes per brick for Sam and 0,5 minutes per brick for Kevin. They would then find how many bricks each of them would lay in any given length of time, and get, e.g. 0,5 bricks in 0,2 minutes for Sam and 0,4 bricks in 0,2 minutes for Kevin. They would then combine these to get  $0,5+0,4=0,9$  bricks in 0,2 minutes for both Sam and Kevin, which would give 180 bricks in 40 minutes.

We recognise that there might be other ways, but they still would have to find how many bricks each bricklayer would lay in a given length of time before combining them. So, the conceptual issue here is when and how to reduce as necessary and combine rates. Our error analysis confirms this.

For option C, it is possible that the learners only added the times and numbers of bricks laid that were given. This could be as a result of misinterpreting the 'together' in the question as addition. Sam's 150 bricks in 60 minutes and Kevin's 20 bricks in 10 minutes would combine to become  $150+20 = 170$  bricks in  $60+10 = 70$  minutes. The fact that the question asks 180 bricks and they got 170 bricks didn't bother them because they see 170 as very close to 180, so they settled for 70 minutes. This was a popular choice among learners.

For options A and D, the reasoning underlying their choices are harder to analyse. It would be interesting to give this item to your learners and ask them what they did if they choose option A or D. You can send your finding by emailing us at [DIPIP@wits.ac.za](mailto:DIPIP@wits.ac.za)

## Grade 9 item: Ratio and proportional reasoning

In the toy car shown the diameters of the back wheels are one-and-a-half times the diameters of the front wheels.



When the car travels 1 metre the back wheels go around 6 times. How many times do the front wheels go around when the car travels 1 metre?

(A) 4      (B) 6      (C) 9      (D) 12

21% of learners chose correct answer (C)

22% chose option A

17% chose option B

38% chose option D

1% of learners did not answer this question

Our analysis shows that to get the correct answer, the learners must

- know what  $1\frac{1}{2}$  times smaller means;
- relate the size of the diameter to the circumference of circular objects; and
- apply knowledge of proportional reasoning; specifically inverse proportionality.

Learners who got this correct would know that the distance around the smaller tyre would be less than the bigger one, and so to cover the same distance the smaller one would go round **more** times than the bigger one. The question is **how many times more** the smaller wheel will rotate since ' $1\frac{1}{2}$ ' is the multiplying factor,  $6 \times 1\frac{1}{2} = 9$  times

Learners could also have drawn pictures of the situations and draw the distance the one wheel travels in relation to the other, noting that one round trip of the bigger wheel is two-thirds round trip of the smaller one.

To summarise, the conceptual issue here is relating multiplicative factor of diameter to that of circumference of circular shapes. Our error analyses below confirm this.

1. For option D, the learners could see that the front wheels are smaller, and they could also see that the smaller wheel

should go round more times than the bigger wheels. However, they could not conceptualise what  $1\frac{1}{2}$  times bigger/smaller really means, and how that would affect the circumference and the number of rotations of the wheels. Realising that the front wheels had to go around more times than the back wheels because they were smaller, they were left to choose option C or D. They settled for D because it is a multiple of 6 - the number of rotations of the back wheels. It could also be that they rounded up  $1\frac{1}{2}$  to two, or saw from the picture that the back wheels were about two times bigger than the front wheels. Therefore they doubled 6 to get 12.

2. For option A, the learners saw that the front wheels were smaller and thus thought that there should be fewer rotations than those of the bigger wheel. They understood the concept of ' $1\frac{1}{2}$  smaller', but could not relate that value to the number of rotations, by applying the concept of inverse proportionality to the problem. Therefore they divided 6 by  $1\frac{1}{2}$ .
3. For option B, the learners must have believed that since the front wheels cover exactly the same distance as the back wheels, they would have go around the same number of times as the back wheels i.e. 6 times. In their thinking, this would be regardless of the different diameters of the sets of wheels, and how diameter affects the number of times the front wheels will go around, compared with the back wheels.

Our discussion, in the next section, will cover some general issues around the general teaching of proportional reasoning.

Because so many learners struggle with proportional reasoning, we provided some activity sheets that will help with these issues. On the next pages, we explain what some of the problems in teaching those concepts might be, as well as how doing the problems should help learners improve their conceptual understanding.

## Teaching with errors and misconceptions

We have reported three misconceptions in our analysis of ICAS items in the last section. These misconceptions relate to when and how to reduce and combine rates (grade 7); identification of the 'whole' (grade 8) and relating the multiplicative factor of the diameter to that of circumference (grade 9). Although the errors came up in test items for particular grades, the underlying ideas are linked and go across grades e.g. identification of the whole is an issue for teachers from Grade 3 to Grade 12 as are the others. We now present some mathematical issues underlying ratio rate and proportion that relate to these misconceptions. The activities on page 9 to 11 will help you to address the misconceptions with your learners.

### ***Aspects of Rate and Proportion***

Activity sheet 1 (page 9) provides the basic kind of thinking that a grade 7 learner needs to use for formal work in ratio, rates and proportion. More difficult questions appear in activity sheets 2 and 3 (pages 10 and 11), but if you realise that learners are struggling with the basic concepts, you can build up knowledge using sheet 1 first. Closely linked to the concept of ratio is that of **rate**. Ratios involves quantities in the same units. When quantities that relate to each other are in different units, we have rate. For example, if one litre of a brand of fresh milk is sold for R3,75, we say the brand of fresh milk is sold at the **rate** of R3,75 per litre (written as R3,75/l). We can use these relationships to calculate the cost of 2, 3, 4, ... litres of milk. Rates are relationships between quantities in

different units, whereas ratio are relationships between quantities in the same unit. Similarly, given that we pay R11,25 for 3 litres of milk, we can re-write the relationship as  $R\left(\frac{3}{3}\right)$  per  $\left(\frac{11,25}{3}\right)$  litre, which reduces to R1 per 3,75 litres, and then we can easily compute how many litres one can buy with R2, R3, R4, etc.

We use the word 'proportion' to describe the relationships between rates. There are different kinds of proportion. The context of the question is used to determine whether the relationship describes direct or inverse proportion. When it is direct, the relationship shows an increase (or decrease) in one quantity with a corresponding increase (or decrease) in the other. For example, as the quantity of items purchased increases, so does the price. For inverse relationships an increase in one quantity would correspond with a decrease in the other (or vice versa). For example, for men working at the same rate of time, more men working results in less time required to complete a particular job.

This contextual interpretation needs to be stressed in class, so that learners do not erroneously assume direct proportionality for all rate problems. The tasks in activity sheet 2 require learners to distinguish between direct and inverse proportion and to explain the difference. Learners need to be given time to comprehend the context before deciding whether a direct or an inverse proportion is in place. It is important to allow them to argue, discuss and justify their strategies for solving the tasks.

## ***Aspects of Ratio***

Many learners struggle to do problems relating to ratio. We identified the fact that learners could not correctly identify the 'whole'. A fraction such as  $\frac{2}{3}$  means that the whole was divided into 3 equal parts, and that we are concerned with 2 of those parts. A ratio written as 2:3 does not mean the same thing - the whole here is  $2 + 3 = 5$ , and the values 2 and 3 represent different amounts of mixtures in the same units e.g. 2 units of water and 3 units of oil. There are 5 units of the mixture. The mixture is  $\frac{2}{5}$  water and  $\frac{3}{5}$  oil. Learners need to work with the ways of representing parts in the form of ratios. This will be of great benefit to other areas such as science and biology, where they need to make dilutions of liquids. When writing a ratio we need to emphasise to our learners that the quantities are comparisons. They do not represent actual amounts. For example, the ratio of water to oil as 2:3 is interpreted as 'for every 2 parts of water in the mixture, there are 3 parts of oil'. We are not implying that there are, for example, 2 spoons of water and 3 spoons of oil in the cake mixture.

Giving an explanation such as the following would help learners to conceptualise this:

If a particular juice is made by mixing two litres of its concentrate with 3 litres of water, then the ratio of:

- concentrate to water is 2:3. The 'whole' here is  $2+3 = 5$
- water to concentrate is 3:2. The 'whole' here is  $2+3 = 5$
- mixture to water is 5:3. The 'whole' here is  $3+5 = 8$
- concentrate to mixture is 2:5. The 'whole' here is  $2+5 = 7$

An extension of ratio problems in which we noticed that learners were committing errors is equivalent ratios. From the example above, since the whole mixture is 5 units we can consider equivalent ratios by asking learners what quantity of water and concentrate would be required to make 20 litres of the mixture. This would imply that the 'whole' is 20, and the ratio of concentrate to water would be 8:12, showing that  $2:3 \equiv (2 \times 4):(3 \times 4)$ .

Use the discussion generated by this and perhaps a few others to generate the relations below:

$$a : b : c \equiv x a : x b : x c \equiv \frac{a}{y} : \frac{b}{y} : \frac{c}{y} \quad ; x, y \neq 0$$

We have selected the tasks in activity sheet 3 to addresses issues such as (1) what the parts are that make up a ratio, (2) how ratios may be written, and (3) identification of the 'whole'.

Issues to keep in mind when using the activities provided:

- Listen to and work with learners' thinking as they engage with the various activities. When they make mistakes, see if they are the ones we identified.
- The activities have asked learners to make up their own ratio problems. This is valuable for teachers because it gives us another way to assess what they understand and if misconceptions exist.

## Activity Sheet 1

### Rates and Proportion

#### Comparing Quantities

(Based on: McMullen A. & Williams JL (1968). On course mathematics for South African schools: Standard 6. Johannesburg: Macmillan)

**Work in groups to solve the even numbers of the following problems. Finish the rest at home. Explain why these problems deal with direct proportion.**

- Complete the following sentences:
  - 15 apples bought for R20 is at the rate of .... per apple.
  - If a tap empties a tank containing 900l in 1 hour 45 minutes, the water flows out at a rate of .... l/min.
  - A car travels 35 km on 4 l of fuel. Its petrol consumption is .... l/ km.
  - A space rocket reaches the moon, which is 384 403 km, in  $13\frac{1}{2}$  hours. The rate at which it travels is .... What is another word you can use to describe the rate of the rocket's flight? How far will the rocket travel in 1 week?
- A workman is paid R1520 for a 40-hour week. Calculate his hourly rate of pay.
- A boy cycles 16 km per hour and a girl runs 4,4 m per second. Who is faster?
- A town has an area of 24 km<sup>2</sup> and a population of 31 000. Calculate the population density of the town per km<sup>2</sup>.
- In pairs, make up three questions of your own, using what you understand about rates and proportion. Provide solutions for your questions.
- Express the following ratios in their simplest forms
  - 40:55
  - 10:4
  - 150:100
- In each case find  $x$ , which makes the proportions equal:
  - $\frac{x}{5} = \frac{4}{10}$
  - $\frac{x}{7} = \frac{3}{21}$
  - $\frac{x}{6} = \frac{3}{4}$
- Express the following ratios in the form 1:n, and then again in the form n:1
  - 3:12
  - 20:15
  - 8:3
  - 1,5:48
  - 0,5:6,6
  - 5:1
- For question 8c above write in words what the ratio in the form n:1 means

#### Using this activity Sheet

This set of tasks has been set up for learners working with ratio and rate for the first time. Although the tasks vary mildly in difficulty, the questions deal with basic conceptual understanding of how quantities relate to each other. The concept of rate as '... per ...' has been broached, and so has the concept of speed as a rate. Simple ratio tasks have been given, but these are very procedural. We strongly suggest you use these in conjunction with activity sheet 3 so that your learners realise how ratios are applied to more meaningful calculations. Note that we have asked learners to explain their understanding in words. You can do this more than once over a work session. This verbalisation is important because learners often believe they understand something, until they have to explain it to somebody else. This is when they realise that they have not understood properly, and now need further intervention.

## Activity Sheet 2

### Direct and Inverse Proportion

#### Comparing Quantities

(Based on: McMullen A. & Williams JL (1968). On course mathematics for South African schools: Standard 6. Johannesburg: Macmillan)

1. In pairs explain to each other, and write down your explanations, how direct proportion differs from indirect proportion. You will be required to explain your understanding of the two kinds of proportion to another group.
2. State which of the following problems contain direct proportion, and which contain inverse proportion? Solve the problems.
  - a) A nursery sells 7kg of compost for R11,50. You will get a discount of 10% if you buy 40kg or more of compost. What will 50kg cost you?
  - b) Six pumps can empty a reservoir in nine hours. How long will it take five similar pumps?
  - c) A shelf will hold 24 books, each 3cm thick. How many books  $1\frac{1}{2}$ cm thick will the shelf hold?
  - d) A wheel of radius 15cm makes 1000 revolutions when travelling a certain distance. How many revolutions would a wheel of radius 24cm make if travelling the same distance?
  - e) A car travelling at 80km/h makes a trip in 7 hours. How long will it take this car if it travels at 60km/h?
3. In your pairs make up three of your own your own direct and inverse proportion questions. Also provide the solutions to your questions, with reasons why they show direct or inverse proportions. Evaluate your questions - If they are good questions, explain why, and if they have weaknesses, try to identify these. Write down your evaluation in table form, with the strengths in one column and weaknesses in the other.

#### Using this activity sheet

This activity sheet deals with direct and inverse proportion, as the title suggests. We have already mentioned that learners need a mixture of these types of questions, in different contexts, so that they can identify the two types and explain their way through their identification. Again, ideally, if learners can produce their own questions, along with explanations of how their questions are identified as direct or inverse proportion and solutions, we will have a much better idea of the extent of their understanding. Beware of making up contexts that are too 'tidy'. For example, question 2c is what we call 'tidy': all the books have a constant thickness, which is not realistic (think of your own bookshelf). Question 2a is not tidy: we have a much more realistic situation, where there is a discount for buying in bulk. It does add complexity to the question, but this has added value, in that strategic planning of the solution needs to take place, and all the information has to be considered before a solution may be found.

## Activity Sheet 3

### Ratio

#### Comparing Quantities

(Based on: McMullen A. & Williams JL (1968). On course mathematics for South African schools: Standard 6. Johannesburg: Macmillan)

1. If you are told that the ratio of flour to milk in a recipe is 3:1, explain in words what this means.
2. A man buys goods for R142 and sells them for R150. Find the ratio of
  - a. Selling price to cost price
  - b. Cost price to selling price
  - c. Profit to cost price
  - d. Selling price to profit
3. A man mixes sand and cement in the ratio of 5:2 by volume. If he uses  $\frac{1}{2}$  m<sup>3</sup> of sand, how much cement should he put into the mixture?
4. A recipe requires the mixture of A and B in the ratio 3:2. If the final mixture is denoted by M, (i) find the ratio of
  - a. A:M
  - b. B:M
  - c. M:B(ii) Use fractions with an explanation in words to explain the composition of the mixture  
(iii) If Themba decides to use half the quantity of A and the full quantity of B, find Themba's ratio of
  - a. A:M
  - b. B:M
  - c. M:A
5. The ratio of Themba's height to Nozipho's is 13:11. How tall is Themba if Nozipho is 1m 20cm?
6. A hotel charges at a rate of R500 per night in winter and R850 per night in summer. Compare these different rates by finding the ratio of the winter to the summer night.
7. Given that at a particular school the ratio of students to computers is 4:1, determine the following:
  - a. If there are 30 computers, how many students are in the school?
  - b. If there are 100 students, how many computers are there?
  - c. What is the number of computers per student, written as a fraction
  - d. What does the fraction you obtained in question c) mean?
8. Make up three questions about ratio. They need to use the concepts found in questions 2 and 4. You need to provide worked solutions for your questions.

#### Using this activity Sheet

This sheet assesses learners' understanding of a ratio as a comparison of quantities. If a ratio is describing a mixture, then the units of the quantities have to be the same. The learners need to identify the parts that make up the mixture, and what the 'whole' is. If the situation is to be written as a fraction, then learners again need to identify the whole, and write down in fraction form and words. This should demonstrate their understanding of the distinction between descriptions of mixtures as ratios or fractions. On the other hand, if the ratio is not describing a mixture (question 7), then there is no 'whole' and the ratio is interpreted differently. Getting learners to make up their own questions using the two ways ratios are understood exposes them to this subtle difference and encourages them to think more deeply about this very intricate topic. It is probably a good idea to keep asking the question, 'what does this mean, in words?'

We hope this newsletter has been helpful to you for understanding why learners make some of the mistakes they do; and also to give you an idea of some useful tasks to assess where the misconceptions and lack of understanding might be, and try to address problems areas. It does not answer all the questions you might have. If you want to raise some points for discussion, make some suggestions to us or you have some ideas or teaching experiences you would like to share with us, please contact us at [DIPIP@wits.ac.za](mailto:DIPIP@wits.ac.za)

PUZZLE

We hope that you will find some time to try the puzzles out. Our wish is that we can play a part in the enrichment of our teaching community and enjoy such puzzles together. We plan to publish one puzzle at the end of each issue, and provide a solution in the following issue. Try this one:

In the following addition puzzle, different letters represent different digits.

$$\begin{array}{r}
 C \quad A \quad L \quad M \\
 \quad B \quad A \quad A \\
 \hline
 2 \quad 3 \quad 9 \quad 5
 \end{array}$$

If  $C$  is a non zero integer, find the digits represented by each integer and show that this can add up in **only one** way. Send your solution to [DIPIP@wits.ac.za](mailto:DIPIP@wits.ac.za)

### Editorial team



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