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7730

# Preface

**Junior Primary Mathematics** is part of the PREP Education Series developed by Primary Education Project based in the School of Education at the University of Cape Town and published by Juta. The book is a result of a two-year pilot study which tested the material for effectiveness and relevance.

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**Junior Primary Mathematics** was developed for in-service training of junior primary/foundation phase teachers in South Africa in the late 1990s. However, with the exception of Chapter Three, the topics and approaches will be useful for the training of junior primary mathematics teachers in other African countries. In order to adapt the book, Chapter Three could simply be replaced with a chapter covering the scope and expectations of the national curriculum in the particular country for which it is intended.

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# SECTION 1: INTRODUCTION TO GOOD MATHEMATICS TEACHING

By the end of this section you should be able to:

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| SAIDE icons_2.png | * Consider the relevance of mathematics;
* Acknowledge the need for children to design their own methods of calculation;
* Recognise some of the essential learning outcomes of the Junior Primary Phase;
* Organise your class in various ways;
* Design lessons for whole class teaching and group work teaching;
* Design tasks that suits group work;
* Recognise the necessity for talk during group work.
 |

**There are five chapters in this section.**

##### Chapter 1: Why teach mathematics?

##### Chapter 2: Active learning

##### Chapter 3: The junior primary mathematics curriculum for outcomes-based education

##### Chapter 4: Classroom organisation

##### Chapter 5: Planning a mathematics lesson

Allow about eight hours for this section.

# Chapter 1: Why Teach Mathematics?

By the end of this chapter you should:

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| SAIDE icons_2.png | Recognise the relevance of mathematics to people in their everyday lives;Recognise your responsibility in helping children to become numerate (able to work with numbers);Recognise that mathematics is about observing, representing and investigating patterns and relationships. |

Mathematics is a subject in the primary school that has received a great deal of attention. Interest in the subject comes from public feeling that mathematics opens doors to the sciences and technology and from the fact that the primary school is where the foundations of sound mathematical understanding are laid. In fact, we know that you cannot even train for some professions and careers unless you have mathematics.

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| **SAIDE icons_3.png** | **Activity One**List eight tasks skills in the employment field that require calculation. |
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| ***SAIDE-icons_4c.png******Comment on Activity One*** | In order to build a bridge, plan a house, invest money or make furniture certain calculations have to be made. Learning maths can, of course, also provide pleasure and intellectual stimulation. |

In addition, control of family finances presents increasingly complex problems for all adults. People have to manage their taxes, hire purchase payments, bonds, rent, investments, budgets, etc. Failure to manage these financial affairs competently places a burden not only on the family but on the community as a whole.

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| **SAIDE icons_3.png** | **Activity Two**Write down the payments that **you** make **every** month, e.g. Food R 485. |
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Add these amounts and subtract the total from your income.TOTAL: \_\_\_\_\_\_\_\_\_\_ INCOME: \_\_\_\_\_\_\_\_\_\_ BALANCE : \_\_\_\_\_\_\_\_\_\_\_\_Are you left with money to spend?

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Do you owe money?

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | This was a very simple example of how we use arithmetic or mathematics in everyday life. But it serves to show how important it is that children learn to be numerate. |

All teachers who teach mathematics have to take responsibility for teaching mathematics well. This mathematics course is aimed at helping you to improve the ways in which you teach mathematics. We also hope that this course convinces you that it is important for both you and your learners to gain confidence in working with numbers.

#### What is Mathematics?

Many of us think of mathematics as something to do with numbers and shapes. That is so, but mathematics is about representing (in writing and symbols and graphs and diagrams), observing and investigating patterns and relationships.

We can say that mathematics is the science of patterns. People who are good at maths are able to notice patterns in number, in shapes, in science and in their imaginations. Mathematicians use these patterns and the relationships between patterns to 'explain' natural phenomena.

So mathematics is not actual reality, mathematics is all in our minds. The source of mathematical knowledge is not in physical reality but in the patterns and relationships created in our minds. For example, the physical properties of bananas (colour, mass/weight) are in the bananas themselves - we can observe the colour and observe the mass of the bananas. But when we start putting the bananas in order according to size, then we are introducing a mathematical idea because the order (smallest, bigger, biggest, etc.) is not in the bananas themselves but in the relationships we have created between the bananas. The idea of small, smaller, smallest, and so on, is just that - an idea - and in this case a mathematical idea.

In conclusion we can say that the colour of a banana is a physical quality of the banana - it is physical, observable knowledge. The size of the banana, the number of bananas, the similarities and differences between the bananas are examples of logico-mathematical knowledge. And that is mathematics.

#### What kind of thinking is involved in mathematics?

Mathematics always involves abstract thinking of some sort. Why? Because, as was explained in the previous paragraph, size, number, position, similarities and differences are all ideas that we have created over time in our minds. Ideas are abstract, so to think mathematically is to think in the abstract. We observe similarities between certain things in our environment or between numbers or shapes. Then we represent what we have observed in some way by using words, symbols or diagrams. For example, 'five' is abstracted from all collections of objects which can be matched (one to one) with the fingers of one hand; a bar graph represents relationships such as the children in the class and their heights.

Once we learn some of the rules of doing mathematics and once we learn some of the ways of thinking in mathematics, we can begin to investigate and represent relationships between mathematical objects. Mathematics provides precise methods of representing patterns and relationships.

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| **SAIDE icons_3.png** | **Activity Three**1. What is the pattern? 45, 47, 49, 51, 53, 50, 59, 61.

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2. What shape is missing to complete this pattern?1.png3. Continue the pattern one more time. |
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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | There is a link between patterning and repetition, whether it is in number or shape. The link between patterning and number in the first example is the pattern of digits in the unit’s column (1.3.5.7.9) and the repetition of these digits. The link in the second example is that the shapes are repeated but with one of the shapes being excluded from the repeat pattern. |

When we begin to see mathematics as a series of patterns and relationships, we can say that we are beginning to think mathematically. And when we can recognise and also generate intricate patterns ourselves, we really are thinking mathematically. So children must be provided with many opportunities to discover pattern in number and space.

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| **SAIDE icons_3.png** | **Activity Four**Find some natural objects to use in an introductory lesson on patterning with your class, e.g. *mealies, sunflower heads, watermelon slices, pine cones, seed pods*.Create a lesson where these objects can be used to discuss and recognise pattern. Write down the questions that you will ask and some comments that you will make to stimulate discussion and discovery of patterns in the natural environment. |
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| ***SAIDE-icons_4c.png******Comment on Activity Four*** | The point about discovering patterns is that you really have to study and discover the patterns for yourself so that you learn how patterns work. Simply being told what the pattern is doesn't help you to learn the skill of recognising patterns. So lessons have to be planned in such a way that children are given the chance to observe, think about and talk about the patterns among themselves. |

The maths lessons should always include time for studying and recognising and generating patterns in mathematics. As you work through this course, you will notice how we emphasise the recognition of pattern when teaching mathematics.

# Chapter 2: Active Learning

By the end of this chapter you should realise that:

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| SAIDE icons_2.png | All children do not learn in exactly the same way;In order for children to become independent thinkers, they must be active learners;Children must learn to verbalise their thinking. |

* All children do not learn in exactly the same way;
* In order for children to become independent thinkers, they must be active learners;
* Children must learn to verbalise their thinking.

Do we all calculate in exactly the same way? Teachers sometimes teach as if we do in fact all calculate in the same way.

But researchers who asked this question found that

* of the people they questioned, roughly two thirds used their “own” methods.
* these own methods' had a very high success rate, much higher than when the same people used school-taught methods.

In fact, when forced to use school-taught methods, many of the people questioned failed to come up with the right answer. Yet, when using their own methods, they were able to explain very clearly how they had worked out the sum.

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| **SAIDE icons_3.png** | **Activity One**This would be my “own” method for the sum 47+ 39:2.pngI was not taught this method at school.Write down the method you would use to do this sum:346 divided by 29 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
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| Were you taught this at school or did you choose to calculate this way? |
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A popular view of learning held by many teachers is that children learn through repetition and recitation. They believe that knowledge is transferred directly from person to person. However this this view has been discredited (become highly questionable). Cognitive psychologists suggest that most mathematical knowledge has to be constructed by the children themselves with the help of a teacher. If the teacher tells or shows a child how to solve a problem without letting the child participate in the understanding of the activity, the child is unable to use this knowledge in a new or different situation. If the teacher drills a certain method for calculating without checking whether all the children understand and can do the method, it blocks children from learning calculation skills which can be used independently.

Allowing children to develop their own methods for calculating and problem solving, means that teachers have to organise their classrooms to support this. This type of classroom is a place where pupils are involved in sharing, inventing (doing something new or different), evaluating (judging the value of something), negotiating (explaining something to someone else and trying to convince them, but also listening to their opinion), and explaining. This learning environment allows children to experiment with different ways of finding a solution and talking to each other about possible solutions. In this way the teacher supports and encourages independent thinking. Independent thinking is important as it teaches children to think for themselves in such a way that they are eventually able to operate successfully on their own (they become self-regulating).

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| **SAIDE icons_3.png** | **Activity Two**Which skills do you think children need in order to invent or construct a range of methods for problem solving? Write down some ideas.  |
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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | I cannot say what you wrote down, but maybe you came up with things like bonds and tables, numbers, how to add, how to subtract, how to multiply ... You may also have suggested that children need to hypothesise (theorise, have an idea), use their prior knowledge, predict, investigate, or test their mental frames (the ways in which they understand the world). |

Children need to be given the opportunity to develop all sorts of skills to make them competent and confident in number work. So we have to design lessons which develop these skills.

Activity Three is one idea for a lesson in which children can be encouraged to think for themselves. Try it with your class and see what happens.

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| **SAIDE icons_3.png** | **Activity Three**Write a problem on the board and ask the class to try to find the answer without using a method that you have shown them. Don't make the problem too easy; make sure that it challenges the children to think. Get the children to explain to each other how they worked out the problem. Listen carefully to the children's explanations and write two of their explanations here. |
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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Remember that encouraging children to explain in words how they calculated is good teaching. Listening to their explanations helps you, the teacher, to understand how the children are thinking and gives you opportunities to respond and to develop their thinking. Remember, that children need to be given an opportunity to verbalise their thinking so that they can communicate their ideas to themselves and to others. Children must practise the language used to explain thoughts. In this way children become active learners - they actively participate in the construction of knowledge. They learn to understand their own thinking and why they use the methods they choose to use. |

In Chapter 3 of this mathematics course, we will come back to the argument that children must learn to develop their own methods for problem solving because there are often many solutions to a problem and also because people need to be inventive in finding solutions to problems.

# Chapter 3: The Junior Primary Mathematics Curriculum for Outcomes-Based Education

The purpose of this chapter is to give you an overall picture of all the aspects of mathematical teaching in the Foundation Phase/Junior Primary Phase. This overview can help you to plan lessons.

By the end of this chapter you should recognise that:

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| SAIDE icons_2.png | There are specific outcomes to be achieved by learners by the end of Grade 3;There are certain performance criteria that help teachers to ensure that learners become numerate;Different number ranges are used to allow learners to work with numbers that they can manage in order to develop a concept of number;Number knowledge has many aspects. |

The learning area Mathematical Literacy, Mathematics and Mathematical Sciences in Curriculum 2005 had ten specific outcomes. Learners were expected to achieve these outcomes during their school years.

The ten specific outcomes are listed below:

1. Demonstrate understanding about ways of working with numbers.
2. Manipulate number patterns in different ways.
3. Demonstrate understanding of the historical development of math­ematics in various social and cultural contexts.
4. Critically analyse how mathematical relationships are used in social, political and economic relations.
5. Measure with competence and confidence in a variety of contexts.
6. Use data from various contexts to make informed judgements.
7. Describe and represent experiences with shape, space, time and motion, using all available senses.
8. Analyse natural forms, cultural products and processes as representa­tions of shape, space and time.
9. Use mathematical language to communicate mathematical ideas, con­cepts, generalisations and thought processes.
* Use various logical processes to formulate, test and justify conjectures.

*(Department of Education and Training, Curriculum 2005, Pretoria, March 1997)*

To achieve these outcomes, teachers in each of the different phases of schooling are expected to teach learners certain skills and concepts that lead to the ten specific outcomes.

The most important skills and concepts for the Foundation Phase for each of the specific outcomes are listed below.

1. Demonstrate understanding about ways of working with numbers.

Learners are able to:

* Express numbers words and symbols
* Understand the importance of place value
* Estimate length, heights, capacity, mass, time
* Add, subtract, multiply and divide whole numbers
* Add, subtract, multiply and divide money
* Share and divide, halves and double
1. Manipulate number patterns in different ways.
* Identify and/or copy simple number patterns in rows, columns and diagonals
* Show a knowledge of skip counting, starting at any number
* Identify and/or copy linear patterns using two and three dimensional shapes
* Identify artistic patterns in South African cultures
* Arrange numbers in a logical sequence
* Identify missing terms in number and geometric patterns
* Extend or create linear patterns in two or three dimensions
* Use concrete objects to extend create and depict tiling or grid patterns
* Generate steps patterns
* Explores tessellation
1. Demonstrate understanding of the historical development of math­ematics in various social and cultural contexts.

Learners are able to:

* Demonstrate counting and measurement in everyday life
* Illustrate at least two mathematical activities at home
1. Critically analyse how mathematical relationships are used in social, political and economic relations.

Learners are able to:

* Demonstrate understanding of the use of mathematics in shopping
* Show understanding of price increases
* Show understanding of family budgeting
* Show understanding of saving
* Map immediate locality
1. Measure with competence and confidence in a variety of contexts.

Learners are able to:

* Show more knowledge of nonstandard forms of measurement
* Demonstrate understanding of reasons for standardisation
* Demonstrate knowledge of SI units
* Compare masses of objects
* Understand money as a unit of measurement
* Show knowledge of how to read time
* Explain the difference between hot and cold
1. Use data from various contexts to make informed judgements.

Learners are able to:

* Choose methods of data collection
* List and arrange data in a logical order
* Sort relevant data
* Represent data using graphs, charts, tables and text
* Explain meanings of information
* Make predictions
1. Describe and represent experiences with shape, space, time and motion, using all available senses.

Learners are able to:

* Represents objects in various forms of geometry
* Shows links between algebra and geometry
* Transform and tessellate shapes
* Show understanding of the concept of point of reference in two and three dimensions
* Show understanding of perceptions of an observer from different reference points
* Demonstrate an understanding of changes of perceptions of space and shapes through different media
* Visualise and represent object from various spatial orientations
1. Analyse natural forms, cultural products and processes as representa­tions of shape, space and time.

Learners are able to:

* Represents cultural products and processes in various mathematical forms, two and three dimensional
* Represent nature in mathematical form
1. Use mathematical language to communicate mathematical ideas, con­cepts, generalisations and thought processes.

Learners are able to:

* Share observations using all available forms of expression, verbal and nonverbal
* Represent ideas using mathematical symbols
* Use mathematical notation
* Formulate expressions, relationships and sentences
* Use abstractions to simulate word problems
* Represents real life or simulated situations in a mathematical format
1. Use various logical processes to formulate, test and justify conjectures.

Learners are able to:

* Demonstrate reasoning processes of association, comparison, classification and categorisation
* Report mathematical reasoning processes verbally and visually
* Recognise familiar or unfamiliar situations
* Infer from known experiences
* Demonstrate respects for different reason approaches
* Choose relevant data as a basis for prediction
* Test validity of judgement

The following grid is a summary of some important aspects of the Maths curriculum for Grades 1 to 3.

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Notice that the grid includes number ranges. This is because a Grade 1 child would be learning all of these skills in a lower number range than a Grade 3 child. The teacher could have a double-sided page like this for each child in the class in her planning book. At the end of the year, these pages could be passed on to the next teacher. The new teacher then has an overview of the work done and of the number ranges in which the child is working. In this way each child would be working with the number range that he or she can manage.

We will comment briefly on the first seven aspects listed on the grid. The others (word problems, number patterns, measurement) are dealt with in other chapters in this book.

### 1. Counting

Counting is the starting point of all number concept development. It is an on-going process. Counting should be done daily and, wherever possible, should show learners that mathematics is part of daily life. This means that you can count any time, even if it is not a 'maths lesson'. Some examples of counting activities would be counting learners in a line, counting the windows of the classroom or the school, counting desks, counting fence poles on our way out to play, etc.

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| **SAIDE icons_3.png** | **Activity One**Think of your school day. Find at least three different counting opportunities outside the maths lesson for your learners during the school day. |
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| ***SAIDE-icons_4c.png******Comment on Activity One*** | In order for learners to be able to solve problems later on, they need to see number work as part of daily living. Many experiences of counting must be provided both in the classroom and incidentally. |

### 2. Number names/symbols

The learners have to be given the correct name and symbol for the amount that they have counted. For example, they must know that the name or symbol of the amount represented here is 9.



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| **SAIDE icons_3.png** | **Activity Two**Try this lesson to help to develop the language of mathematics. (Adjust the lesson to suit your class.)Display a set of nine objects (e.g. bottle tops). Let the learners make another set with the same number, but using a different type of object (e.g. crayons). Check that the sets have the same number by one-to-one correspondence. Let the learners make yet another set using different objects (e.g. stones) and check again that there are the same number of objects in the set.5.pngAsk the learners to try to say what is common about these three sets. I am quite sure that they will suggest that what is common is that they all have the same number of objects. Then you introduce them to the numeral 9 and the number nine. Write these on the chalkboard and discuss the difference between 9 and nine. Once you have established this, let each child make sets of nine using whatever objects you have available.You should also make a large card with the numeral, number and cardinal value on it that you can then display on the wall of your classroom. Something like this:6.png |

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### 3. Numerosity

A sense of 'how many' about a number is called numerosity. This develops from the experience of counting and a knowledge of number names and symbols. This knowledge makes it easier for the learner to think about numbers. When a child has worked with many concrete objects, he/she discovers information about that number, such as that forty-one is made up of forty and one, or twenty and twenty and one.

Teachers need to provide a variety of apparatus such as buttons, bottle tops, matchsticks, blocks, etc., so that learners can proceed from the concrete to more abstract thought. When we see that learners have a 'picture' of the number in their head, we move on to apparatus that will extend abstract thinking.

Working with apparatus such as expansion cards helps learners to understand the abstract nature of number. Expansion cards are rectangles of paper which can be pasted onto the backs of cereal boxes as this will make them last longer. Because the cards overlap, a very clear picture of how the number is constructed can be seen. These cards can be used from the third term in Grade 1 right through to Grade 3 and beyond, for example:



 *(The 6 card is placed over the zero of the 40 card to show 46)*



*(The 30 card is placed over the zeros of the 200 card and the 7 card is placed over the zero of the 30 card)*

As you can see, the hundreds cards are three times as long as the unit cards and twice as long as the tens cards so that cards can be placed over the zeros.

In Grade 1 and Grade 2 the tens and ones cards are mainly used. However, as the learners develop a wider understanding of numbers, they will proceed to a higher number range and the hundreds cards can be included.

Here is an example of how expansion cards would be used in a lesson with a group of learners.

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| Petrus is working with the numbers 21-34. He has counted with these numbers and recognises all of the symbols. He has his set of expansion cards packed out. The teacher says: “I am thinking of a number which comes just before 27. Can you build that number?” Petrus takes his card and his card.C:\Users\user\Dropbox\JUNE BITED UCT\FINAL BITED and UCT\FINAL BITED and UCT\UCTJPM DRAFT MT\new uct images\images4.pngHe ‘builds’ 26 C:\Users\user\Dropbox\JUNE BITED UCT\FINAL BITED and UCT\FINAL BITED and UCT\UCTJPM DRAFT MT\new uct images\images3.pngC:\Users\user\Dropbox\JUNE BITED UCT\FINAL BITED and UCT\FINAL BITED and UCT\UCTJPM DRAFT MT\new uct images\images2.pngThe teacher asks: “What is that number made of?” Petrus: “20 and 6”He moves them apart and places one below the other to show this.Tracey looks at what Petrus is doing and says: “26 is 10 and 10 and 6.” Other learners make suggestions too. |

### 4. Addends and minuends

Addends are numbers that are added on and minuends are numbers that are taken away or subtracted.

Going back to the expansion cards, the learners have made 26.

***Teacher***: “Change your number to the one that has three more.”

***Learners*** make 29. (29 is an addend.)

***Teacher***: “Now we have 29, take 5 away.”

***Learners*** make 24. (24 is a minuend.)

The teacher can use many different words or phrases that mean plus and minus, such as 'make it 2 more, make it 1 less, add 5, minus 5, 2 smaller than'. When a child copes well with this task, we say that he/she has a knowledge of addends and minuends.

### 5. Comparison and sequence

To discover comparison and sequence, words such as more and less, between, before, after will be used in questioning (for example, which number comes between ... ?, which number comes after/before ... ?). Learners must learn the sequence of numbers. For example, the learner learns that 26 is between 25 and 27. Questioning learners with apparatus like expansion cards, counting charts, number lines, or even looking at rulers or tape measures shows them the position of the numbers in relation to other numbers.

Learners must also learn to compare numbers. For example, 25 is less than 26, or 300 has 100 more than 200. Remember to keep asking learners about the value of numbers and asking them to compare values.

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| **SAIDE icons_3.png** | **Activity Three**Try working with some learners, each with their own set of expansion cards.Write down eight questions that you would ask the group about numerosity, small addends and minuends, and comparison and sequence. |
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| As you watch the learners, what do you notice about their number concept? Write your comments here. |
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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | When we first start working with any new apparatus, it takes a while for learners to get used to it. We need to allow them some time to play with the numbers. Some strategy for setting the cards out will need to be established. This could be a lesson in itself. |

Notice that by putting questions to learners in different ways, we extend their vocabulary of mathematics at the same time as extending their number concept.

### 6. Halving and doubling

Learners enjoy the repetition of number. In the same way they enjoy the pattern of doubling. We often hear children saying: 'I know that one and one is two, two and two makes four', etc. We can build on this prior knowledge so that children become good at halving and doubling.

Here is an example of how to practise halving.

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| Yolo is in Grade 1, she and Nina are working with 8 bottle tops. The teacher has just asked Yolo to give half of her counters to Nina. Yolo gives 2 to Nina and keeps the rest for herself.\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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| Teacher: | “Nina, do you think that you have half of the counters?” |
| Nina: | “No.” |
| Teacher: | “How do you know?” |
| Nina rearranges the counters so that each has 4. |
| Teacher: | “Why have you arranged them like that?” |
| Nina: | “Because 4 and 4 make 8. We have the same number.” |
| Teacher: | “Yola, do you agree with Nina?” |
| Yola: | “Yes.” |
| Teacher: | “Why?” |
| Teacher: | “What is half of 8?” |
| Yola and Nina | “4.” |
| Teacher: | Yola, if you have 8 bottle tops and give half to Nina, how many will you give her?” |
| Yola: | “4.” |
| Teacher: | “So we can say that half of 8 is 4, 4 and 4 make 8, there are two 4s in 8. |

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The learners match their group of counters against those of their partner in a one-to-one pattern. They discover that they have the same number of counters in their group (or they don't). Many examples need to be done with both halving and doubling.

### 7. Conservation

Conservation means breaking up numbers (decomposition) in many different ways but still recognising (conserving) the value of the number. This leads to the understanding of bonds/sums. Consider the following example.

The teacher asks the learners to pick up seven buttons and put some in their left hand and some in their right hand. Now the teacher goes around and asks some children how many they have in their left hand and then how many they have in their right hand. She records their answers on the chalkboard like this:



Then the children practise by reading and repeating the table.

As the children learn to work in higher number ranges, so the teacher demands more advanced forms of decomposition, for example:



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| **SAIDE icons_3.png** | **Activity Four**Plan a lesson on decomposition of numbers with your class. Notice the learners' growing awareness of the number concept. Write your thoughts here. |
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| How does this activity lead to the learning of sums? |
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| *SAIDE-icons_4c.png**Comment on Activity Four* | The focus is always on discovery together with the use of apparatus. Often learners are given too few opportunities to work with apparatus. This means that sums (bonds) are drilled without learners understanding the cardinal value of a number. |

#### Conclusion

* Number concept develops through a wide experience of number.
* Learners master aspects of number through working with smaller and then greater number ranges.
* Learners learn best when actively involved with number and number apparatus.

# Chapter 4: Classroom Organisation

The purpose of this chapter is to show you that different forms of classroom organisation lead to different kinds of teaching and learning. We hope to show you that teachers need to use a variety of forms of organisation in order to ensure that children get a balanced mathematics programme in which different learning experiences take place.

By the end of this chapter you should understand that:

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| SAIDE icons_2.png | Different kinds of classroom organisation provide for different kinds of learning; Group, paired and individual learning give children a chance to experiment, invent and learn How to be independent thinkers;A classroom must be organised to provide for a variety of experiences and activities;A teacher must use his/her time as efficiently as possible. |

The forms of classroom organisation that we look at in Chapter 4 are:

* Whole class teaching;
* Group teaching;
* Group work.

### Whole Class Teaching

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| ***Read this true story***A teacher (we will call her Ms Mgiba) come to Mandela Primary School (not its real name) in the second quarter of the year. She was given a Grade 4 class. No one told her about the children in the class. when Ms Mgiba looked at the class register, she noticed that next to each child's name were numbers ranging from 120 to 142. She thought that these numbers were the children IQ measurements as tests had recently been done in the school. She was very impressed and quite nervous and she wondered if she would be able to teach such a bright class of children. So she prepared her lessons very carefully. She worked steadily through the year always ensuring that the questions that she asked, the comments and responses that she gave and the tasks that she set stretched the children's thinking and imaginations. At the end of the year her class sat for the examinations and achieved very high marks. The principal came to her and asked how come the children had done so well. She was astonished and replied that of course the children had done well because they were so bright and gifted. The principal could hardly believe his ears and asked Ms Mgiba where on earth she had got the idea that the class was bright because everyone in the school knew that they were not bright. In fact they were the weak class. Ms Mgiba replied that she had seen the pupils' IQ scores alongside their names in the register and that was how she knew that they were gifted children. The principal burst out laughing and said, 'Those are not IQ scores, they are the children's locker numbers!  |

What can we learn from this story?

* We can learn that if we think that children are 'stupid', we will not expect them to perform well.
* We can learn that if we think that children are capable of being challenged, as teachers we will challenge them.
* We can learn that children can only perform as well as we teachers allow them or expect them to perform.
* We can learn that human beings have a great deal of individual potential and that this potential has to be developed through good teaching.
* We can learn that it is dangerous to label children because labels stick.
* We can learn that every child in the class deserves to have the best teaching.
* We can learn that as teachers we cannot judge exactly how and when children learn so we must always give them the benefit of the very best teaching.

Perhaps you can think of other things that we can learn from this story. Whole class teaching (by that we mean teaching all the children the same new thing at the same time) can be very effective when teaching something new. The reason for this is that each child benefits from the interactions that take place during the lesson. So, if the teacher is trying to ask questions which push the children's thinking, all the children listen to the responses and learn from the demonstrations of how different people think. If the teacher is good at taking what children say and developing these responses into more challenging ways of responding and thinking, then she is teaching children how to think critically and how to think in more abstract ways.

The following are the main advantages of whole class teaching:

* Children can benefit from each other's thinking.
* Children can be pushed to think in more abstract ways than they would on their own.
* Teachers can experiment with difficult and unusual questions and use the different responses to build up a satisfactory answer.
* Teachers do not have to teach the same thing a number of times to a number of groups.
* Children are given a fresh chance each time to develop their potential.
* Teachers can use the prior knowledge of a range of children to develop new ways of thinking.
* Teachers can discover which children have not grasped the new knowledge and can then take only those children as a group for further teaching.

Remember that we are not talking about rote teaching and learning. We are suggesting that sometimes teachers can plan whole class lessons which involve children in active learning. The story of Ms Mgiba is a good example of this kind of classroom organisation.

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| **SAIDE icons_3.png** | **Activity One**To be good at maths, children have to learn certain number facts and learn to transform (change) numbers. For example, children must learn bonds and multiplication tables, they must learn to count, to halve and double numbers, to recognise number patterns, to read and work with big numbers, to count in twos, fives, tens, hundreds, to calculate mentally, and to work with whole numbers and fractions of numbers. Whole class teaching is a good form of classroom organisation to develop these understandings systematically and creatively.Here is one way in which to involve the whole class in gaining control of number knowledge. Try it with your class adapting the questions to suit the learners.Use a number chart that you have made with numbers from 1 to 120.1. Point to a number on the chart and ask:
* What number is 10 more than it?
* What number is half of it?
* What number is 3 more and doubled?

Now design three more questions that you would ask your class while pointing to number on the chart.

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| 1. Ask the children to call out all the numbers on the chart that could be in the table of three (remember that 120 is also in the fable of three).

Now design three more challenging questions that you could ask your class using the number chart.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Children need to learn to calculate numbers quickly and accurately. These methods of whole class teaching should be used very regularly. In section 2 we give further examples of activities which suit whole class teaching. |

### Group Teaching (Teaching one group of children at a time)

Group teaching is another form of classroom organisation. There are a number of reasons why it is sometimes necessary for a teacher to teach just a few children, rather than the whole class.

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| **SAIDE icons_3.png** | **Activity Two**Here are some reasons why teachers choose to teach a group of children. Tick the reasons which you think are good reasons .for you to teach a small group of children.

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|   | I have children from more than one standard or grade in my class. |
|   | I do not have enough apparatus or teaching aids for the whole class. |
|   | I need to group those children who did not understand the whole class maths lesson and teach them agin. |
|   | Teaching only a small group of children at a time allows me to observe closely the progress of my pupils. |
|   | I have a large class so I need to give children the opportunity to verbalise their thinking and to have me respond more directly tothem as individuals.  |
|   | When I teach something new, I like to watch closely to see if each child is understanding |

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | As you can see, a teacher needs to have a specific purpose for teaching a group of children. Teaching in groups just because someone says that that is what you should do is not very sensible because you yourself need to decide why you want to use that form of classroom organisation. You should also be realising that when you know why you want to use group teaching you will arrange your groups according to the purpose. For example, if you are teaching something again that you have already taught to the whole class, you will choose only those children who did not understand the first time or cannot practise on their own what you taught. If you want to teach something difficult and you feel that it is better to teach only a few children at a time, you will arrange all the children into groups and take turns with each group. The groups will keep changing, depending on your purpose in taking a few children at a time. |

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| **SAIDE icons_3.png** | **Activity Three**You have decided to arrange children into groups to teach them how to solve problems. Your purpose for doing so is to give each child a chance to show how he/she is thinking {remember that we have said that children should be given a chance to develop their own methods for solving problems}. You also want to watch how each child is working, so you want a few children at a time, rather than the whole class.Think up a challenging problem and write this on the chalkboard. Tell the children to think about the problem and to discuss among themselves how they think it can be solved. Encourage the children to ask each other to check their calculations (not just to rely on you). After a while ask the children to explain their own methods to the group.Write some of these explanations on the chalkboard. Then let the children discuss the various methods and evaluate the effectiveness of each method. After you have tried this activity write down here what you see as your role in this kind of group teaching.

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | You may well have decided only to read out the problem and then to write up the solu­tions or you may decide to comment on the children's explanations and evaluate each response. In the second option, commenting and evaluating, you are demonstrating how to share, negotiate, explain and evaluate. Remember that you are commenting and evaluating, not just saying 'right' or 'wrong' - you yourself must demonstrate how we explain what we are thinking, and so on. |

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| **SAIDE icons_3.png** | **Activity Four**What are the pupils learning and experiencing during the problem-solving lesson described in Activity Three?* Pupils talk to each other.
* Pupils work through the problem in a practical/way.
* Pupils explain their method using language with which they are familiar.
* Pupils watch one another.
* Pupils share their knowledge.
* Pupils invent way of solving problems.
* Pupils evaluate each other's methods.

Add two more things to the list.

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| Pupils  |  |
| Pupils |  |

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| ***SAIDE-icons_4c.png******Comment on Activity Four*** | As you can see, teaching one group of children at a time allows you to help each child to develop the thinking skills necessary for mathematics. Teachers have to learn the teaching skills necessary to help children to think in more abstract ways. By continually changing your style of teaching, and experimenting with different kinds of questions and different ways in which to respond to what children say, you become more skilled at getting the best out of each child. |

### Group Work (Children working cooperatively in a group or in pairs without the teacher in constant attendance)

Group learning and paired learning are very effective forms of organisation. When learners work together on an activity they are able to learn from each other. They learn by talking, listening and sharing ideas. In this way they are able to create meaning together.

There is a wide range of activities that can be used by children working in pairs or groups in maths lessons. The activities fall more or less into three categories: (I) Revision or practice tasks; (II) Investigation tasks; and (III) Games.

### (i) Revision or practice tasks

While you are busy with a group teaching problem solving or number concepts, the other children can be working in groups or pairs. Some groups (or the rest of the class) are given worksheets or exercises or tasks involving practice work. Revision or practice work reinforces number concepts that have already been taught. The worksheets or exercises could also relate to oral work that was done with the whole class.

For example: The Sub A class has been counting to 50 in twos and fives. The children may be asked to fill in the circles on the worksheet.





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| **SAIDE icons_3.png** | **Activity Five**Think up a practical exercise for Grade 3 children and a group of Grade 0 (or Grade 1) children. Make sure that the activity is not too easy or too difficult, or the children will become bored.Grade 3:

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| Grade 0 (Grade 1)  |
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If children are allowed to learn from each other, the teacher is free to concentrate on one group of children at a time. The teacher has one group with her while the rest of the class work on tasks and activities that enhance number development.

### (ii) Investigation activities

Investigation activities are designed to help children explore and experiment with number concepts. They are practical activities that require children to actually do something, and learn from the doing.

For example, for Grade 0 or Grade I you may have a number of bags of objects that are labelled with the letters of the alphabet. Each bag contains a different number of objects. Each child is required to fetch one of the bags, count out the objects and fill in his or her answer on a printed form which has all the letters of the alphabet to match the bags of objects, like this:



After counting each bag and filling in the form, the children must decide which bag has the smallest number of objects, which bag has the most, which bags have the same amount, which bags have half as many as other bags, which have double the amount, etc. Children (working in pairs or groups) can compare answers and check each other's calculations and show each other how they got to the answers that they did.

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| **SAIDE icons_3.png** | **Activity Six**Design an investigation activity that would help to develop multiplication skills for a Grade 2 class.

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| ***SAIDE-icons_4c.png******Comment on Activity Six*** | Remember that an activity which requires investigation does not have an answer which is obvious to the children. It is an activity which encourages children to explore and experiment, to discuss and to discover. An investigation activity should encourage children to be curious and to enquire. Children should see the process (the working out, the way in which we think to work out) of the activity as more important than simply finding an answer or one solution. For example, an investigation activity for multiplication could be to ask children to discover which numbers appear in the three times-table and the four times-table, and to say why they think these numbers appear in both tables. |

Here is an investigation activity that helps children to hypothesise and predict. Try it with your class. Remember that even the Reception Year children can do some of this activity. What you may expect depends on the standard in which the children are. Remember, too, that children are working in pairs or groups so they can learn from each other. Of course, children can also work on their own at first and then talk to each other when they have finished or when they are stuck.

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| **SAIDE icons_3.png** | **Activity Six**18.png**Instructions to children:*** Compare the triangles. Discuss among yourselves what you notice.
* Complete the table below.

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| TRIANGLE  | NUMBER OF MATCHES  |
| 1 | 3 |
| 2 | 6 |
| 3 |  |
| 4 |  |
| 5 |  |
| 11 |  |
| 16 |  |
| 50 |  |
| 100 |  |
| 500 |  |
| X |  |

Tell each other how you worked out your answers. |

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| ***SAIDE-icons_4c.png******Comment on Activity Seven*** | If children are learning to notice patterns in number, then after making a few triangles and seeing how many matches are used each time they can begin to predict how many matches would be needed to make triangles 5, 6, 7 and so on, and then to predict without even having made the triangles how many matches would be needed for any size triangle. They can, of course, test their predictions by making the actual triangles. When they begin to understand the pattern, they can try to hypothesise (give a theory) as to the pattern that emerges when they use the matches to make the triangles. |

Here is another investigation activity to try with your class.

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| **SAIDE icons_3.png** | **Activity Eight**Prepare this work cord (or write Hon the chalkboard). Write the table of nine on a card, with the answers, like this:19.pngTell the children to study the table and to notice as many patterns as possible. Allow them to talk to each other about the activity and to write down as many patterns as they can discover.When the children have tried the .above activity ill their groups, copy down some of their discoveries here.

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| ***SAIDE-icons_4c.png******Comment on Activity Eight*** | If the learners are in Grade 0, the above will be too difficult. Maybe you could try another number pattern activity. Activity Eight is fine for Grades 1 to 3, but you would expect the Grade 3 children to discover more patterns than the Grade 1 children. |

You will notice that activities which are designed for investigation purposes allow children to talk, to share, to explain, to experiment, to invent, to hypothesise, to predict and to observe. Very often, too, such activities teach children to notice patterns in mathematics. The matchstick triangles activity teaches patterns in sizes of triangles, the table of nine teaches children patterns in number. (Looking at the digits in the ones column of the table of nine - what do you notice? Why do you think this is so? etc.) Once children begin to discover and recognise patterns, they are better able to predict and estimate correctly.

### (iii) Games

Games are good for developing and practising number concepts. Again, the game must suit the needs of your class so that there is something to learn from it. Games are usually made for two or more players (puzzles are more suited for one child). The teacher usually plays no role other than to explain how the game is played because the children themselves 'police' the rules of the game.

Here is an example of a game.

Bingo is a game played by two or more players. Each player has a board in front of him/her with numbers on it. The size of the board is about 15 x 20 cm. Each board has different numbers on it. The idea is that children can practise addition or subtraction or multiplication or division through this game.

For example, you may want your Grade 1 class to practise bonds, so you would make boards like those below with the answers to the bonds written on them:



You will also have to make a series of cut-out rectangles to fit each of the rectangles on the boards. The things you write on the rectangles are the bonds.

Like this:



The rules of the game are:

* Turn the bonds face down on the desk or floor.
* Each child takes a turn at turning over one of the little rectangles and calling out the bond.
* The first child to find the number corresponding to the answer on his or her card calls out the answer and shows it on the card.
* The rectangle with the bond is given to that child who then covers the answer on the board with the little rectangle.
* Then the next little rectangle is turned over and that bond called, and so on
* The first child to cover all the numbers on his or her board calls 'Bingo' and is the winner.

Of course, you could use a bingo game for any standard by making the bonds more difficult, for example division or addition of fractions.

Card games are also useful. Packs of cards can be bought very cheaply and many games can be played which help children to learn to strategize in order to win. Ask educational book shops or other organisations which visit schools to help you with ideas.

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| **SAIDE icons_3.png** | **Activity Nine**Write down how you think games help children to learn· mathematics.

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| ***SAIDE-icons_4c.png******Comment on Activity Nine*** | A lot depends on the game and what the game is trying to teach or what skills are practised while playing the game. Children who need to practise bonds or tables will benefit from games designed for this purpose. Other games can teach children to explore alternative ways of doing things. |

Some teachers have never tried group work and group teaching in their classrooms. If you need help in organising your classroom to make sure that things run smoothly and well, the following diagram of classroom organisation may help you.

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| **12 minutes**  | **Whole class teaching** |
| **12 minutes**  | Group teaching of first group | Practice task | Investigation activity |
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| **12 minutes**  | Investigation activity | Group teaching of second group | Investigation activity continued |
|  |
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| **12 minutes**  | Investigation activity continued | Practice task continued or game | Group teaching of third group |
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| **12 minutes**  | Checking up and tidying up |

(If children finish their activity or task or game before the time is up, they must know where to fetch another activity or game or task, or do some other activity, such as reading.)

As you can see, the class is divided into three groups for the lesson. The groups may be ability groups or mixed ability groups, depending on what you are trying to teach and what you want the children to learn. If you have more than one grade in your class, you may want to divide the class by grade, or you may want to mix children from different grades according to individual need. You will notice that in this one-hour lesson, time has been given to whole class teaching, to group teaching, to group work and to checking up and tidying up.

It is important that teachers train children not to interrupt them while they are focusing on the teaching group. This training also helps children to become independent and to learn to ask for help from the other children in the class. Remember that this kind of classroom organisation only works well if you train the children in how to work without you, if you give them interesting tasks to do, if you have suitable teaching aids and if you have a clear purpose in arranging a lesson like this.

If you have never tried group work or you have tried it and it doesn't seem to work, do the next activity.

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| **SAIDE icons_3.png** | **Activity Ten**Watch the PREP Education Series video called *Classroom Organisation for Group Teaching*. The video programme is about 24 minutes long. On the video, Heather Howell talks about how to plan and organise your classroom so that you can teach children in small groups. You will see different teachers using group work in their classrooms. Read the observation schedule below before you watch the video.**Observation Schedule**While watching the video, notice: * When each teacher chooses to teach a small group of children rather than the whole class;
* When the children are sitting in groups but each child is working on his or her own;
* When the children are working together in groups or pairs on an activity;
* How the teachers organise their classrooms;
* How the teachers organise their time.

As you watch the video, think about:* How small group teaching is different from cooperative learning;
* How you could use small group teaching in your own classroom.

Talk to your colleagues about your ideas after you have watched the video. |

After all of that discussion about classroom organisation, it is time for you to experiment.

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| **SAIDE icons_3.png** | **Activity Eleven**Look at the lessons that you have planned for your class for next week. (If you are a Grade 0 or reception class teacher, you should look at the activities you have planned for the whole class to do together.)Choose one lesson or activity in which you could use small group teaching to help the children in your class to learn better.Why do you think that this lesson will be better if you use small group teaching?

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What will you be teaching each small group and how will you do this?

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What will you be teaching each small group and how will you do this?

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What activities will the other children be doing while you are teaching each small group?

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| Now try this lesson with your class. When you have tried it, write your opinions of the lesson in the space provided here - how you liked the arrangement, its strengths, its disadvantages, etc. Show this to your tutor or facilitator. |

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# Chapter 5: Planning a Mathematics Lesson

In Chapters 3 and 4 we spoke about important aspects of mathematical teaching and classroom organisation. The purpose of chapter 5 is to help you plan lessons thoroughly to ensure that the learning outcomes are achieved.

By the end of this chapter you should recognise that:

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| SAIDE icons_2.png | * Systematic planning ensures balanced lessons;
* Lessons must take account of pupils' prior knowledge;
* Lessons are planned to suit the needs (outcomes) of the learners;
* A variety of activities are necessary to develop number concepts.
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Planning lessons for a week (or for a whole topic) saves time and ensures that there is variety in the activities. Planning ahead also helps you to design lessons which challenge the learners. For example, by looking at every aspect of the number concept, we select an activity for the week's lessons from aspects 1 to 9 on the grid (listed in chapter 3). The number ranges you choose for different activities may vary (for example, children may be counting to 100, but only calculating up to 75).

In chapter 3 you learnt about some of the important skills and concepts that have to be taught and learnt in the Foundation Phase. Obviously you will plan your lessons to include the learning of these concepts and skills. Let us go through some of the things that you need to consider if you are planning a week's lessons.

### Points to consider in planning lessons

1. Awareness of the number range of the learners
* How far can each learner count with understanding?
* Up to what number is each learner able to calculate?
* What number values do the learners have to learn next?

After you have decided what number range suits the needs of each learner, plan the lessons using these number ranges. You may have to select more than one range to suit the needs of different groups of learners in the class.

1. Counting
* Which counting activities must be used in the lessons?
* What apparatus will help counting?
* Is there variety?
* What manner of counting will be used - forwards, backwards, in fours, sevens, odd, even, etc.?

Select a counting activity for each day of the week, paying special attention to variety and development of the number range.

1. Estimating
* What situations will you provide to help the children to estimate?
* What apparatus are you going to use?
* What questions are you going to ask the children?

Use a different situation for estimation every day.

1. Numerosity
* What apparatus will you use (expansion cards, counters, number line)?
* What do you want learners to know (doubling, halving, decomposition, place value)?
* What words will you use to extend their language of mathematics?

Plan the activities in a suitable number range.

1. Problem solving
* What problems will you give the children to solve?
* Can you relate the problems to any other learning programme. For example, suppose the class is dealing with the topic of insects.
* Problems to be solved could be: how many legs do three flies have?
* There are 200 cells for the queen bee to lay eggs in, how many eggs does she have to lay if she has laid 65 already?
* How will learners be organised for problem-solving activities - in pairs, in groups of four or more, on their own or as a class?
* How much time will be allocated for the learners to work on problems on their own?
* How will you encourage them to talk about the process?
* Which maths vocabulary are you going to include?
* What are you going to look for while the learners are solving problems?
* Who will be selected to talk about their discoveries and strategies?
* How will you encourage the rest of the learners to listen to the solution that is offered?
1. Practice and investigation
* Will you prepare work cards and games which encourage children to practise what you have taught them?
* What measurement activities or investigations will extend their mathematical knowledge?
* Will the learners work individually, in pairs or in groups?

While some learners are busy with these activities, you may be teaching another group. Bear in mind that learners have to manage the work on their own. It must be revision of known work (practice), or investigation work which requires self-discovery.

1. Assessment
* How will you assess yourself?
* How will you assess the learners?
* What exactly will you assess?

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| SAIDE icons_3.png | **Activity One**Plan maths lessons for one week. Fill in the scheme below to show what you plan. (You might not want to do all of these things, of course, so leave those blank.) Teach the lessons and then complete this activity.

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|  | GROUP 1 | GROUP 2 | GROUP 3 |
| Counting  |  |  |  |
| Number names and symbols |  |  |  |
| Numerosity |  |  |  |
| Addends and minuends |  |  |  |
| Comparison |  |  |  |
| Sequence |  |  |  |
| Doubling halving |  |  |  |
| Number patterns |  |  |  |
| Bonds (decomposition) |  |  |  |
| Problems |  |  |  |
| Measurement |  |  |  |

Did you complete all that you had planned?

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If not, why do you think this happened? What will you do next week?

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What did learners gain from these lessons?

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How do you know this?

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What difficulty did any of the learners experience?

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Write down anything you would like to improve on next time you teach these aspects of the curriculum.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Initially, planning lessons will take time. After a while, you should find that the learners are very eager to do maths. They enjoy the variety and stimulation of well-planned and prepared lessons. The fact that you are working in the number range which children are learning (not already knowing) means that the learners feel challenged to build on previous knowledge. You may find that you did not manage all the week's work. If so, simply carry that work over to the next week. |

### Advantages of lesson planning

* Careful planning ensures that learners are given the best possible opportunities for discovering and learning number concepts.
* Planning and preparing ahead helps the teacher to feel 'safe', as he/she has an overview of what is happening in his/her classroom all the time.
* It ensures that the teacher works steadily, step by step, from the known to the unknown.
* The teacher models how learning is organised and managed.
* Planning and preparing helps you to develop your teaching style.
* Planning and preparing ensures that learners learn at an appropriate pace with understanding and engagement.
* Planning ahead also means that you can make the best use of apparatus and activities.

### Conclusion of Section 1

This section has shown the need for innovative organisation and planning. It has also shown that children learn both from the teacher and from each other. Teachers must design work that motivates and extends children's thinking. Teachers should look carefully and critically at the syllabus and plan lessons to help children to think creatively and independently.

# SECTION 2: TEACHING NUMBER CONCEPTS

The aim of this section is to enable you to have a clear understanding of basic number concepts and of their relevance to the development of mathematical skills.

By the end of section two you should be able to:

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| SAIDE icons_2.png | * Explain what is meant by ‘concept’;
* Understand the importance of developing a conceptual understanding of mathematics;
* Design sorting, matching and ordering activities to develop basic mathematical concepts;
* Prepare and present lessons designed to help children construct knowledge.
 |

**Section 2 has two chapters**

* Chapter 6: What is a concept'?
* Chapter 7: Good practices for teaching number concepts

Allow about seven hours for this section.

# Chapter 6: What is a Concept?

Mathematics is part of a child's world. It is not just about addition or multiplication; mathematics is about concepts or ideas. Understanding mathematical concepts helps children to master mathematics. Conceptual understanding is displayed through symbolic (abstract) and verbal language.

By the end of this chapter you should understand that:

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| SAIDE icons_2.png | * Facts are not concepts;
* Concepts develop over time;
* Teaching for conceptual understanding requires special teaching styles and approaches
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### What is a concept?

Though the term concept is widely used, it is not easily understood, nor is it easy to explain what you mean by the term. This is because the term 'concept' is an abstract idea - you cannot point to a concept and say 'there is the concept'.

Let us try to explain what we mean by 'concept'. We will borrow from Hayakawa's book Language in Thought and Action (George, Allen & Unwin 1974) to help us explain.

We will use a cow as an example. We will call the cow Rontawul. Rontawul is a living organism, constantly changing, eating grass and breathing in air and getting rid of digested food. But, because of Rontawul's appearance, we observe a resemblance in her to certain other animals. We apply the word 'cow' to these animals. Although Rontawul is unique (no other cow is exactly like her), we notice that some of her features are like the features of other cows, so we abstract those features of hers which are common to all cows and we classify her as a 'cow'.

When we classify Rontawul as a cow, we are noting her resemblance to other cows and ignoring differences. What we are doing is moving from the real, actual object – Rontawul - to an idea or concept named 'cow'. We have abstracted (used abstract thinking) to move from the real thing to an abstract idea or concept.

And the process of abstraction can go even further. We can move from Rontawul the real cow, to 'cow', and then abstract further to say that cows are part of 'livestock'. The concept or idea 'livestock' selects or abstracts only the features of Rontawul and other cows that they have in common with pigs, goats, chickens and sheep. 'Livestock' refers to all animals which live on farms. A further level of abstraction is to say that the livestock on a farm represents 'assets', similar to with tractors, trucks, fences, windmills, and so on (it has financial value).

As you can see, learning language is not simply a matter of learning words - it is a matter of relating words to the ideas, things and happenings for which they stand. The general term 'house' cannot describe each separate house in a village, but the term 'house' makes it possible for us to have conversations about houses (those presently built and those that may be built in the future or which existed in the past). The concept of 'work' is a general term for describing certain things that people do. You can only point to someone hoeing mealies, or planting sugar cane, or fixing a motorcar, or teaching in the classroom and say that that is 'work'.

The process of abstracting is very important in mathematics. Why is the word 'calculate' used in mathematics? The word 'calculate' comes from the Latin word calculus, meaning 'pebble' (little stone). 'Calculate' derives its meaning from the ancient practice of putting a pebble into a bag or box to represent each sheep as it left the kraal. This was done so that the herdsman could tell whether any sheep had been lost by checking the sheep returning to the kraal at night against the pebbles in the bag. This example shows how mathematics works. Each pebble is an abstraction representing the 'oneness' of each sheep - its numerical value. There is a one-to-one correspondence between the sheep and the pebbles. The pebbles represent the numerical value or number of sheep. So the numerical facts about the pebbles are the same as the numerical facts about the sheep. Numbers, signs, shapes, patterns, rules, algorithms and theorems are all part of mathematics. Operations such as addition and subtraction are abstract ideas which derive from and relate back to the real world.

Many teachers have found that certain kinds of knowledge cannot be memorised unless the learner understands them. For example, some children can't use the concept of a 'week' or a 'month' properly as they do not have an idea (concept) of time itself. Or perhaps some pupils struggle with the abstract concept of multiplication because they don't understand addition.

The point is that conceptual development is a long and continuous process; we are learning to understand more about certain ideas each time that we learn something new or different about the idea or concept. Let us use the story of Zinzi as an example of a preschool child learning about number.

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| **Can Zinzi count?** Zinzi is three-and-a-half years old. One day her mother gives her three pieces of bread. First, Zinzi puts all the pieces of bread in a line. Then she points to the first piece of bread and says 'l', she points to the second piece and says ‘2', and then she points to the third piece and says '3'. When Zinzi has• finished pointing to each piece of bread and giving it a number, Zinzi's mother says: “Yes, three.”After Zinzi's mother says “three”, Zinzi eats the first piece of bread from her line of bread... Then Zinzi's mother asks Zinzi how many pieces of bread are left, Zinzi says: “Three, look here’s ‘2’, (Zinzi points to the piece of bread that she counted as number 2 before) and here is ‘3’, (she points to .the piece of bread that she counted as number 3 before).” |

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| **SAIDE icons_3.png** | **Activity One**Zinzi seems to know something, about counting, but she does not seem to think about numbers in the appropriate way for mathematics. Write down here what you think Zinzi must still learn in order to have a more developed concept of number.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Zinzi does not seem to understand that we use numbers to think about amounts - in other words she doesn't understand that if she has three pieces of bread and she eats one, she will no longer have the same number of pieces to count. It is important to realise that we don't know exactly what Zinzi thinks or understands because we can't see inside her mind. But we have worked out from the things that she says and does that she does not seem to understand the purpose of counting or how to use numbers appropriately for counting. |

Why does Zinzi say that there are still three pieces, even after eating one piece? Remember, Zinzi says: “Three, look here's ‘2’, (Zinzi points to the piece of bread that she counted as number 2 before) and here is ‘3’, (she points to the piece of bread that she counted as number 3 before).”

If this is what Zinzi says and does, how is she thinking about numbers? It seems that Zinzi thinks that numbers are like names. She calls the second piece of bread '2' and the third piece of bread '3'. So, even though she eats '1', she has still got '2' and '3' left. For Zinzi this is just the same as if she sees boys called Steve, Tito and Mike. If Steve were to go away, Tito and Mike would still be left. She would say: 'Look, here is Tito and here is Mike.'

If Zinzi thinks that numbers are like names, she will not understand that numbers work with amounts.

* If we think about numbers as names, when something with a name is taken away, all the other things that are left will still keep their names. The names of the things that are left will stay exactly the same. (Tito is still Tito; '2' is still '2'.)
* If we think about numbers as amounts, when something we have numbered is taken away, there are fewer things left. There won't be the same amount of things left, so the numbers that we give to the things that are left will have to change. (If the first piece in the line is taken away, the piece that was second in the line will become ‘1’.)

So Zinzi still has a great deal to learn about the concept of number, as all junior primary children do. Let us look now at a Grade I child, Thandi, learning to count.

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| **Can Thandi count?**When Thandi was in Grade 2, her teacher gave her a ‘counting on’ task to do. The teacher used a peg board with twelve pegs in it but she covered up five of the pegs with a box so that Thandi could see only seven pegs. The teacher asked Thandi: “How many pegs are there?”Thandi pointed at the pegs, counted “1, 2, 3, 4, 5, 6, 7” and said: “Seven pegs.”Then the teacher lifted the box and put it on the other side of the board so that the first seven pegs were covered and the lost five were uncovered. She asked Thandi: “Now, how many pegs are there?”Thandi pointed at the pegs, counted: “1, 2, 3, 4, 5” and said: “Five pegs.” The teacher left the box on top of the first seven pegs and asked Thandi: “How many, pegs are there altogether?”Thandi looked puzzled and tried to lift the box to see how many pegs were underneath the box. But, the teacher would not let Thandi do this, so she sat looking confused. Then suddenly Thandi's eyes lit up, and she got a happy expression on her face. Thandi pointed to the box and said:“There were 7 under there when I counted them before. So now I can count them all, look.”First Thandi pointed to the box and said: “Seven.” Then she pointed to the pegs outside the box and said: “8, 9, 10, 11, 12. There are 12 altogether.” |

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| **SAIDE icons_3.png** | **Activity Two**What do you think Thandi had to learn in order to complete the counting task? Write your thoughts here.

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | Thandi used the counting skills she had used before; she pointed at pegs and counted them. She also worked with the idea that counting must start at number 1. When Thandi was not allowed to see the pegs under the box she was puzzled. She knew that she had to work out how many pegs altogether but she realised that she could not count all the pegs together in the same way as she did before. She could not work out how many pegs altogether by counting them from 1 to 12. She could not do this because she could not see all the pegs. Thandi had to form the idea that she did not always have to start counting at number 1. She formed this idea when she remembered that there were 7 under the box. She realised that she could point to the box and say 'seven' and then count on from there. This was a new way of counting for Thandi because she had never done this before; she had always started at 1 before. We can explain Thandi's difficulty by saying that Thandi had to develop her conceptual understanding by including the idea that she could start counting from the number of pegs that she remembered were under the box. |

Thandi used two different ways of counting. She started counting every single one - 1,2,3,4,5,6,7. Then she learnt more about counting because she discovered a new and possibly faster way of counting - counting on. This is how children develop their conceptual understanding of number. Number concept develops as children have new experiences with number. In later counting lessons children will learn to group and count in tens and then use this knowledge to calculate more efficiently.

Let us look now at Thandi learning more about counting.

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| **Thandi learns more about counting**The teacher says to Thandi: “There .are 29 red motorcars and•15 blue motorcars. How many motorcars are there altogether?”Thandi writes on her small chalkboard: 20+10 = 30 9 + 5 = 14 30 + 14 = 44Thandi has learnt to do the calculation without counting the objects.Thandi explains to the teacher: “I know that 29 is 20 and 9 and I know that 15 is1 0 and 5. So I put 20 and 10 together and that made 30. And 1 added the 9 and the.5 together and that made 14. Then 30 and 14 are 44.” |

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| **SAIDE icons_3.png** | **Activity Three**What did Thandi have to learn in order to complete this task? Write your thoughts here.

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Thandi had to develop her conceptual understanding to include the idea that she could count the tens separately from the ones. She had to understand the place value of the digits. |

Number concepts keep developing as children meet with further counting experiences. This is why teachers have continually to be devising new and different and more challenging counting tasks.

There are other mathematical concepts that children have to develop besides a concept of number. Mathematics classifies things in certain ways. For example, things to do with weight, heaviness and lightness are classified under 'mass'. Things to do with hours, minutes and days are classified under 'time'. 'Mass' and 'time' are abstract concepts which allow us to calculate and work with things in certain ways.

### How to form concepts

Children can be helped to form concepts in mathematics. Teachers can provide appropriate experiences and the appropriate language to use to describe the mathematical experiences. Children must learn to sort and classify according to common properties (things that are the same). The following sorting and classifying activities are useful in mathematics concept formation.

Matching objects or ideas, e.g.

* This stick is shorter than that one, but these are the same size.
* 2 X 3 = 6 is based on the same idea as 2 X 4 = 8.

Sorting objects or ideas, e.g.

* All of these sticks go together as they are the same size and all of those sticks go together because they are the same size.



* All of these sums go together because they are about multiplication and all of those sums go together because they are about subtraction.



Pairing objects or ideas, e.g.

* There is a chair for each child in the classroom (one child = one chair).
* The sum (answer) of 32 + 6 is the same as 6 + 32.

Ordering objects or ideas, e.g.

* The horse comes before the cart/wagon.
* Four comes before five, but after three.

Why are matching, sorting, pairing and ordering useful for mathematical understanding?

*Matching* activities develop concepts of:

* Length: (as long as, shorter than, the same length as, etc.)
* Mass: (heavier than, lighter than, the same weight as, etc.)
* Distance: (further than, closer than, the same distance as, etc.)
* Time: (slower than, faster than, the same amount of time, etc.)
* Capacity: (more volume, less volume, the cups hold the same amount, etc.)
* Amount: (more than, less than, little, lots, etc.)

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| **SAIDE icons_3.png** | **Activity Four**Design a matching activity for your pupils to teach the concept of length. Try if with your pupils.

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*Sorting* activities develop concepts of:

* Sets (these are all blocks), and
* Subsets (these blocks are blue, those blocks are red).

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| **SAIDE icons_3.png** | **Activity Five**Design a sorting activity for your pupils’ that teaches the concept of sets and subsets. Remember that the activity must challenge the children. Try it with your pupils.

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| ***SAIDE-icons_4c.png******Comment on Activity Four and Five*** | The activities that you are designing should be suitable for the class that you are teaching. So, if you have Grade 3 children in your class, you would design a more challenging task using fractions, for example, or more complicated geometrical shapes. |

*Pairing* activities teach concepts such as:

* As many as
* More than
* Too few
* Just enough
* Remainder

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| **SAIDE icons_3.png** | **Activity Six**Design a pairing activity suitable for your class.

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| ***SAIDE-icons_4c.png******Comment on these activities*** | Have your tutor check Activities Four, Five and Six with you. Tell your tutor what you thought of the lessons. Say what the children learnt. Would you teach the lessons in exactly the same way next time? |

*Ordering* activities teach ideas such as:

* First
* Last
* Twentieth
* Before
* After
* Between
* In the middle
* Next to, etc.

An ordering activity suitable for Grade 0 or Grade 1 pupils could be:

Here is a row of coloured beads.

* What colour is the first bead?
* What colour is the bead that is sixth?
* What colour is between the red bead and the yellow bead? Etc.

An ordering activity suitable for Grade 2 or 3 pupils could be:

Here is a row of numbers.

* Which number comes between 215 and 217?
* Which number comes after 3 456?
* Which numbers come before 159, but after 150?

So matching, sorting, pairing and ordering objects and ideas helps children to see relations between objects and between numbers and between shapes. Recognition of the relations between numbers helps to develop a concept of number.

Children need to learn that objects or ideas or numbers can be classified according to common properties.

We can say that:

* Objects can be grouped together because they share characteristics (e.g. colour, shape, size ...);
* Objects can be grouped together because they are commonly associated with each other (e.g. animals);
* Objects can be grouped together because they serve similar functions (e.g. vehicles).

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| **SAIDE icons_3.png** | **Activity Seven**Create a set of three objects which:* Share some characteristics;
* Are commonly associated;
* Serve similar functions.

Say why you have created the set in the way that you have and what common properties do the objects share?

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What characteristics do they share?

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Why are they commonly associated?

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What similar functions do they serve?

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Now try this with your pupils. See if they can describe the common properties of the set.

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# Chapter 7: Good Practices for Teaching Number Concepts

Certain practices in the classroom can help us to teach number concepts more effectively. Here are five good practices.

### Using the prior number knowledge of children

All children come to school with some understanding of number, for instance: how old they are, how far the shop is, how many goats or chickens they have, how much a loaf of bread costs, how much a taxi ride costs.

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| **SAIDE icons_3.png** | **Activity One**What number knowledge do the children in your area have when they come to school? List some examples here.

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Each teacher must find out what experiences the children have already had with numbers and see what he/she must do to use this preschool number knowledge to help children to work with mathematics. Remember that we have already seen how prior knowledge affected what Zinzi did and what Thandi did. And, as we saw with Zinzi and Thandi, teachers must make an effort to understand how children are thinking. Activity Two is one way of starting this process.

Activity Two could be done with the whole class or during group teaching.

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| **SAIDE icons_3.png** | **Activity Two**Ask your pupils the following questions and listen very carefully to their responses. After you have tried this with your class, write down what the children's answers to your questions teach you about children and their number knowledge.Questions to ask the children: * What is the smallest number you know?
* Do you think that there is a smaller number?
* What makes you think that?
* What is the biggest number you know?
* Are there bigger numbers, do you think?
* What makes you think that?

Write your comments here.

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### Doing/acting

Explore the concept that you are teaching by direct experimentation and observation (with the whole class or with a group). Concepts are explored by hearing, seeing, handling and thinking. Young children learn through physical experimentation, through investigation and through discussion and contemplation (thinking about). All children in the end must learn to think about things and ideas abstractly.

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| **SAIDE icons_3.png** | **Activity Three**Try this with your pupils. You will need disks, bottle tops, beans, shells or other kinds of counters.* Give each child a handful of counters.
* Ask them to estimate the number of counters that they have been given.
* Get them to count the exact number of counters and to record the number.

You could ask questions like:* Whose estimation was exactly correct?
* Who estimated more than the amount?
* Who estimated less than the amount?
* How much more / less?
* How many more does Agatha have than Nkosinathi?
* Ask each child to compare his or her amount with a partner or the group.
* What methods of counting do the learners use?
* Do they count in ones or twos?

Now write about the lesson. Explain how you implemented your lesson, what questions you asked and how the children responded. You should also include some comment on what you thought was particularly successful and what was not so successful.

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Estimation is another very important skill for children to develop in order to refine their sense of number. Estimation is not the same as taking a wild guess - it is about taking a sensible guess. Often teachers regard estimation as just another topic in the learning programme, but in actual fact the better children can estimate, the better they can judge if answers are correct. |

### Talking

Give pupils an opportunity to talk about their observations, their conclusions, their experiments, their hypotheses. This gives them the opportunity to verbalise their thinking and to learn to use (and think with) the language of mathematics.

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| **SAIDE icons_3.png** | **Activity Four**Develop the children's imaginative powers. Imagination is very important in learning because we need to imagine things {abstract thoughts or situations} in our minds. And we need to talk about what we are imagining.INSTRUCTIONSPut some counters / bottle tops in your hand{s} and hold them behind your back. Do not let the children see how many you have in your hands. Tell the children to close their eyes and imagine the bottle tops which you are holding. {Give them some time before the next instruction.} While their eyes are still closed, ask them to count the bot­tle tops which they are imagining.Tell the children to open their eyes and to record the image that they have in their minds either by drawing it or writing it.Now ask the class to tell you what each person has recorded. Record the different estimations on the chalkboard. Ask some of the children to explain to the class how they tried to imagine the number of counters, what / how they were thinking. Ask the pupils how they can make sure that the number that they have recorded is correct. Hopefully they will comment that the counters behind your back should be carefully counted. Get the class to count out the number of counters with you. Whose estimate was nearest? Whose was furthest away from the right number?Do this exercise a few times with the children. After the lesson, write down what you observed about the children. Include in your account things like how the pupils went about the exercise and what you think they learnt from the exercise. Also say what you learnt from the exercise.

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### Writing mathematics

Mathematics is an abstract subject with its own written system. Children have to learn how to write mathematics. They also have to learn how to use written mathematics to help them to calculate.

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| **SAIDE icons_3.png** | **Activity Five**This sign is very abstract: +If you follow steps " 2 and 3 of good practice above, how could you introduce this sign as a symbol for 'plus'? In other words, how would you show children how algorithms are used in maths? Show the steps that you would take here:

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| ***SAIDE-icons_4c.png******Comment on Activity Five*** | Children need to learn that while we can add up all our sweets, counters, etc., and just say how many, we can also abstract from this doing by representing this action/doing in mathematical language. We can use a sign + which we call 'plus' and we can use a sign = which we call 'is equal to' to write an addition sum. As the teacher you need to talk about these signs and introduce them to the children carefully so that they understand that mathematical signs have a very particular meaning and are used to represent abstract ideas / concepts about number. |

### Formalisation

When pupils have gained some confidence in what you are teaching, they must apply the knowledge to various situations and problems. Unless children can apply knowledge to new and different situations, the knowledge is of little use to them. Only when children can apply knowledge appropriately to a new or different situation can we say that they have learnt.

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| **SAIDE icons_3.png** | **Activity Six**Say how you would now use knowledge of the concept of '= is equal to' in a new or different mathematical problem.

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| ***SAIDE-icons_4c.png******Comment on Activity Six*** | Teachers often rush too quickly into using mathematical signs before the children have really understood what they represent. It is good practice to write in words what signs mean, eg * eighteen divided by five is equal to three and three fifths or
* eighteen divided by five is equal to three, with three remaining

Writing in words gives children a chance to use language to help them to understand mathematical signs. It is of no use if a child can copy from the board '18+ 5 = 3 rem 3' if the child cannot explain what this means. |

In this chapter we are describing some of the practices which we think help teachers help children to develop number concepts. While the children's prior knowledge is important, allowing them to act and discover personally, allowing them to talk about the lesson and verbalise their thinking, and helping them to learn to abstract their actions into mathematical terminology are equally important.

In junior primary classes there are certain concepts which are regarded as fundamental to any child's progress in mathematics. Some of these are the concepts of size, shape, length, height, mass, time, capacity, space and amount. Of course, an understanding of place value and the way in which our number system works on a base ten system is also crucial. We are stressing that these concepts can only be successfully taught and learnt if teachers provide relevant experiences.

Activity Seven is designed to help you begin to plan for good classroom practices. Section 3 will take this further.

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| **SAIDE icons_3.png** | **Activity Seven**Each concept is listed with the relevant vocabulary. On the lines that follow each concept, try to think up the sort of apparatus you could find or make to help children develop conceptual understanding.***Size***

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***Shape***Vocabulary: circle, triangle, square, rectangle, cube, rectangular block, ball, cone, the same shape as

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***Length***Vocabulary: long, short, longest, shortest, longer than, shorter than, the same length as

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***Height***Vocabulary: high, low, higher than, lower than, highest, lowest, tall, short, taller than, shorter than, tallest, shortest, the same height as

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***Mass*** Vocabulary: heavy, light, heavier than, lighter than, heaviest, lightest, the same mass as

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***Time*** Vocabulary: a long time, a short time, day, night, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, yesterday, today, tomorrow, morning, afternoon, evening

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***Capacity*** Vocabulary: full, half-full, empty, level, cupful, as much as

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***Space and position*** Vocabulary: below, above, bottom, top, on, under, other, underneath, between, up, down, over, together, in, inside, out, outside, left, right, first, last, front, middle, back, behind, before, after, between

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***Amount*** Vocabulary: many, few, more, less, most, least, the same number as, equals, is equal to, match, estimate, enough, more than, less than

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| ***SAIDE-icons_4c.png******Comment on Activity Seven*** | From Activity Seven you will realise that apparatus, carefully used, can help children to learn mathematics. Collect as much apparatus as you can and add more each year. Use waste material, if possible, as it costs you nothing. Get the children to help you to collect apparatus.The following is a list which might be helpful to you.* Smaller objects (that can be handled by a child at a desk) such as short sticks, cotton reels, old pens, feathers, washed stones, shells, large seeds, seed pods, buttons, clothes pegs, cold-drink can rings, tops or lids(from toothpaste, cans), large beads, sucker sticks, nails, screws, etc.
* Larger objects (that can be handled by a child on the floor or in a group) such as building blocks (carpenter's off cuts), cans (painted), string, toys, scraps of wool and dress material, plastic bottles or jars, margarine tubs, cartons, cardboard boxes of all sizes (eg from soap powder, matches, shoes, fruit), cardboard rolls of all sizes (eg from toilet paper, foil, knitting wool cones), sticks of various lengths and widths, polystyrene trays from packing cases, wrappers, packets and bags (eg from soap, flour, sugar, fruit)
* Pictures - these may be collected by cutting from canned food labels, magazines, advertising brochures, grocery boxes or packets, by tracing from books, by drawing simple pictures, by using rubber stamps bought from toy shops
* Containers - old plastic milk and juice bottles, broken cups and mugs, empty tins of various sizes.
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### Conclusion

Developing conceptual understanding requires that we learn to think abstractly. For example, we learn to abstract the colour 'red' in order to group an apple and a flower together. We learn to group a bus and a car together as 'vehicles'. Later this concept is developed on a higher level so vehicles become part of 'transport', and so on. The idea or concept is abstracted more and more. The same thing happens in mathematics because mathematics is an abstract subject. We have to learn that two sweets and three sweets put together make five sweets, and that this is abstracted to 2 + 3 = 5.

# SECTION 3: EXTENDING MATHEMATICAL KNOWLEDGE

The purpose of section 3 is to help you to ensure that you provide a balanced and challenging mathematics programme in your classroom. This section builds on sections 1 and 2 and tries to highlight the kinds of understanding and practices that are needed to improve the teaching of mathematics.

By the end of section three you should understand that:

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| SAIDE icons_2.png | * It is your responsibility to provide for the language needs of your pupils;
* Children’s thinking must be continually challenged;
* Conceptual understanding is critical to success in mathematics;
* Teachers must use approaches and apparatus which help children to be active, creative learners.
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**There are four chapters in this section.**

* Chapter 8: The language of mathematics
* Chapter 9: Standard units of measurement
* Chapter 10: Place value
* Chapter 11: Why problem solving?

Allow about thirteen hours for this section.

# Chapter 8: The Language of Mathematics

By the end of this chapter you should:

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| SAIDE icons_2.png | * Know that children need to acquire the language of mathematics;
* Know what is meant by the 'language of mathematics';
* Understand some useful mathematical terminology;
* Change your teaching practice to include more emphasis on the language of maths.
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Schools don't usually recognise the importance of language for learning; schools usually put emphasis on facts and information, on content, and only see language as the medium through which we express what we know. So subject teachers worry about the content of their lessons and give little consideration to the way in which language is used to learn to think, talk and write about the subject.

Yet every teacher is a teacher of language because every subject or discipline has particular concepts which need to be understood in order to produce knowledge. And language is the tool we use to think about and produce knowledge. For example, in mathematics division, multiplication, the properties of a circle, sets and subsets, and place value are all mathematical terms. When you use terms like 'place value' then everybody knows that you are talking about maths and that you are using the language of mathematics. And at school words like 'place value' will only be used in the maths lessons, so the maths teachers must be responsible for teaching the language of mathematics.

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| **SAIDE icons_3.png** | **Activity One**Write down seven more terms/words that are used in mathematics - that belong to a maths vocabulary.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Geographers tend to speak about latitude and longitude and crops and economy. Historians tend to be the ones who talk about frontiers, early settlements and unification, and so on. Mathematicians talk about parallel lines, hexagons, algorithms, measurement, numerator, denominator, and so on. |

When we hear children say that they don't understand the maths, it is because they don't have the conceptual understanding necessary to make sense of what they read or hear or try to do. And one of the most important ways in which to develop understanding is through language, through learning how to think about maths with the tool of language.

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| **SAIDE icons_3.png** | **Activity Two**Look at this sentence: **Convert four and one third to an improper fraction.** There are a number of concepts embedded in this sentence. What is meant by 'convert' in this sentence?

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What is meant by 'improper'?

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What is meant by 'fraction'?

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | We have to come to understand what each of these words/concepts means in mathematics before we can calculate. We have to come to understand the ways in which language is used in mathematics in order to think about maths ourselves. Going to the dictionary doesn't help very much because words are used in a very particular way in maths. How easy did you find the above activity? How difficult will children find mathematics if they are not helped to use this language correctly and meaningfully? |

To make things even more complicated, often the meaning in a mathematical problem is entirely different from the meaning which applies in everyday language. An example will help to explain what I mean.

A maths problem could be:

What is 8 from 10? (Answer = 2)

What is 8 minus 10? (Answer = 2)

There doesn't seem to be any confusion with those problems. But look at this maths problem:

What is the difference between 10 and 6?

There are several possible answers for this problem. Here are some of these possible answers.

The digits are shaped differently

10 has a one and a nought and 6 has a six

10 has two digits and 6 has one



Any of these responses could be right because all of them make sense.

But in a maths lesson where a teacher is trying to teach the concept of 'difference' in mathematics, only the response '4' would be mathematically acceptable. But the point to make is that the children may be thinking (or even saying aloud) some other version of difference because the words 'different' and 'difference' are used in many other contexts besides mathematics.

So confusion often occurs (happens) because certain words that are used in everyday language are also used in very specific ways in a subject.

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| **SAIDE icons_3.png** | **Activity Three**Write down four words that are used in everyday language and in special ways in mathematics:

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | The word 'share' and the word 'halve' are often used by children to mean the same thing because these words are used in everyday language. But they have a special meaning in mathematics; 'halve' has to be used precisely in mathematics, not just to indicate a form of sharing. |

Learning to use and understand the language of mathematics helps children to work creatively and sensibly with mathematics. For children who have to learn mathematics through English when they themselves are not English speaking it is even more important to hear and use the right terms so that they progress in their mathematical ability. It is not helpful to a child if a teacher is careless with language in the maths lesson. This is why we emphasise that teachers must ask and listen to explanations given by each child. If a child is using 'halve' in an everyday sense, a teacher can model the correct use of the term 'halve' and explain how it is used in mathematics.

We have already said that children come to school with some mathematical knowledge. They have already been counting, even though they don't necessarily understand the concept of number. Gita may be able to answer “I am three” to the question “How old are you Gita?”. She may even have learnt how many fingers to hold up to show how old she is, but it is very doubtful if she understands the concept of 'three years'. She is simply naming something that has been taught to her by older people around her. Young children learn to chant numbers in order (1,2,3,4,5,6...) but this does not mean that they can count. We saw an example of this with Zinzi in section 2.

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| **SAIDE icons_3.png** | **Activity Four**Try this experiment. Find three young children (Grade 0 or younger) who can chant numbers in order. Ask the children to count the number of chairs in the room or the number of children playing outside. What do you notice?

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| ***SAIDE-icons_4c.png******Comment on Activity Four*** | Very often when children try to count objects they 'say' one number for two or more objects even if they touch them, particularly if they count more quickly than they move their finger or touch the objects being counted. Learning to count accurately and with meaning comes very gradually because children have to develop the concept of one-to-one correspondence. Children have to learn that each number counted (or said) is an abstract representation of objects being counted in the real world or an abstract representation of measurement such as time (age in years), and so on. This is why oral work with the whole class is so important. |

### Language and concepts

Counting is one of the first experiences that children have with mathematics. But the act of counting with understanding so that new understandings can develop involves certain conceptual developments.

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| **SAIDE icons_3.png** | **Activity Five**Write down what is meant by the following mathematical terms:**Numeral:**

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(34 or 175 or 8 are numerals)**Number:**

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(Thirty-four or one hundred and seventy-five or eight are numbers) **Cardinal number:**

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(34 is the cardinal value of a set containing or representing 34 objects) **Ordinal number:**

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(34 comes before 33 and after 35) **Digit:**

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(Ten digits are used in our number system. They are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.) |

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| ***SAIDE-icons_4c.png******Comment on Activity Five*** | The trouble with the English language is that a word such as 'number' is used to represent cardinal and ordinal value, but it is also used to talk about the whole system. We talk about the 'number system' meaning anything to do with numbers in the number system that we use. |

In the first years of schooling children work with small numbers, often only up to ten to start with. So the numeral 6 has only one digit (6) with a cardinal value of 6 and the number 'six'. It is easy to confuse all of this. I do not mean that children have to use these terms, but I do mean that the teacher needs to understand the different terms.

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| **SAIDE icons_3.png** | **Activity Six**Say why you think a teacher needs to understand the meaning of terms such as 'number', 'numeral', 'digit', 'cardinal and ordinal value'.

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| ***SAIDE-icons_4c.png******Comment on Activity Six*** | When children begin to count, saying a number aloud as they touch or move an object, they are not usually aware of the cardinal value - they take the last number spoken to refer to the last object touched. They do not realise that the last number spoken refers to the cardinal value of the whole set (remember how Zinzi did not see the difference between naming and number value). They have not developed sufficient knowledge to understand this. If teachers have an understanding of cardinal value, they can teach carefully enough to allow the correct type of learning to take place. This would be the case for the other terms referred to in Activity Five as well. |

The following three teaching practices are often followed in junior primary classrooms, resulting in poor maths language development.

* Teachers begin using mathematical symbols without teaching children how to read these symbols; for example, they write 3+4=7 without writing the sum out in words 'three plus four is equal to seven'. Children need to see many sums in words - on the chalkboard and on work cards - so that they can build an understanding of maths language.
* Teachers use abbreviations before the children actually understand the whole word/term - for example, using the abbreviation 'rem' for 'remainder'. Children need to see the full word for some time before it is abbreviated.
* Teachers sometimes use the incorrect word for the mathematical symbol. For example, in Xhosa the correct word for 'is equal to' is 'zilingana', but many teachers use the common word 'zenza' this is not a mathematical term.

A Grade 4 child I worked with did not know that the sign = meant 'is equal to', so she was unable to use that knowledge to help her to solve a problem. She also did not know that 'rem' had a meaning (remainder), although she was writing sums 'off by heart'. She was therefore poor at maths because she could not do anything without the teacher telling her what to do. It would have been so much better for her if her teacher had taken the time to explain the language and to write out in full what everything meant.

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| **SAIDE icons_3.png** | **Activity Seven**Explain how you will teach children the **meaning** of 'numerator' and the **meaning** of 'denominator'. Remember that you begin teaching fractions from Grade 7.**Numerator:**

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**Denominator:**

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| ***SAIDE-icons_4c.png******Comment on Activity Seven*** | Although children are not expected to be able to say these words immediately or to explain the meaning in their own words, teachers need to use these words all of the time so that children become familiar with the terminology and see that maths language makes sense. Too many teachers talk about 'the top number' and 'the bottom number' and then wonder why children don't understand that these are not whole numbers but fractions with which they are working. The point is that you - the teacher - must model the correct language. And the language that you are modelling provides samples from which to learn, not examples to learn. |

A final point on learning the language of mathematics is that there is a critical difference between helping a child somehow to get a particular answer, and helping a child to gain some conceptual understanding from which answers to similar questions can be constructed at a future time. Learning the language of mathematics is part of gaining conceptual control because it is that language which you will use to think mathematically at any future time.

# Chapter 9: Standard Units of Measurement

By the end of this chapter you will:

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| SAIDE icons_2.png | * Be able to teach standard units of measure in innovative ways;
* Realise that it takes time for children to understand abstract ideas such as units of measurement.
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Once children have developed a working knowledge of basic number concepts, they can be introduced to standard units of measure. This should be a discovery experience for children. Remember that young children find it difficult to see the purpose of a standard unit of measure because they are still learning to think in abstract ways. Here is a recorded conversation of two five-year-old children talking about how to measure rugs (mats). The conversation is from a book called Classroom Discourse by C. Cazden (Heinemann 1988).

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| *Wally:* | “The big rug is the giant's castle. The small one is Jack's house.” |
| *Eddie:* | “Both rugs are the same.” |
| *Wally:* | “They can't be the same. Watch me. I'll walk around the rug. Now watch - walk, walk, walk, walk, walk, walk, walk, walk, walk - count all these walks. Okay. Now count the other rug. Walk, walk, walk, walk, walk. See? That one has more walks.” |
| *Eddie:* | “No fair. You cheated. You walked faster.” |
| *Wally:* | “I don't have to walk. I can just look.” |
| *Eddie:* | “I can look too. But you have to measure it. You need a ruler, about six hundred inches or feet.” |
| *Wally:* | “We have a ruler.” |
| Eddie: | “Not that one. Not the short kind. You have to use the long kind that gets curled up in a box.” |
| Wally: | “Use people, people's bodies lying down in a row.” |
| Eddie: | “That's a great idea. I never even thought of that.” |

From reading this conversation you can see that Wally and Eddie are trying to discover how to measure rugs (the concept of length). Wally and Eddie have different points of view on what to do. They have not as yet recognised the need for a standard unit of measure. Although Eddie suggests that they should use a ruler (a standard unit of measure), Wally does not agree.

If children are simply told that you measure in metres, they don’t fully understand the purpose of history of why we have standard units of measure. A standard unit of measure (for example, metre, litre, hour) is a unit of measure that everybody agrees to use. It would be no good trying to buy one kilogram of sugar at the shop if you and the shopkeeper couldn’t agree on exactly how much a kilogram was.

Here is an idea for a series of lessons that develop an understanding of the **metre** as a unit of measure.

Adjust the lessons to suit your class, but you can use them with any junior primary class. You may even want to set your class the problem of measuring mats as Wally and Eddie were doing.

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| **SAIDE icons_3.png** | **Activity One**Divide your class into four groups. These groups should not be ability groups because you want the children to help one another. Each group is given a measuring tool. Group 1: A piece of string any length Group 2: A block of wood any length Group 3:A matchbox Group 4: They must use their handspan**Step one**Each group is given something to write on {a sheet of paper or a slate or small chalkboard} and 5 or 6 things that have to be measured, e.g.* height of the door
* length of the carpet width of a window
* length of the desk width of the door
* length of a book, etc. {things that are in your classroom}

Each group moves around recording its findings on the slate or paper. The class then comes together and the teacher records all the findings on a chart that she has prepared, or on the chalkboard. Like this:

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|  | Group 1: string | Group 2: wood | Group 3: matchbox | Group 4: handspan |
| height of door |  |  |  |  |
| length of carpet  |  |  |  |  |
| width of window  |  |  |  |  |
| length of desk |  |  |  |  |
| width of door  |  |  |  |  |
| length of book |  |  |  |  |

Discuss the different measures and show how difficult it is to compare measures or agree on the length. Let the children see the need for one measure that is the same for everyone. Introduce the metre.**Step two**Let the same groups measure the same things with the metre stick or a metre tape. Once again record the findings and it will be noticed that the metre stick does not work for lengths that are shorter than a metre.The discussion must raise the necessity for the metre to be divided into smaller bits to allow for this. This is when centimetres become useful.**Step three**In order for children to grasp the concept of a centimetre square, it is a good idea to let them cut out squares measuring one centimetre by one centimetre that you have prepared and to stick them onto a piece of wood or branch to make their own measuring stick for small lengths.**Step four**Once the idea of metres and centimetres is established, the children can be given many measuring activities.In the space provided, write down any difficulties you experienced with these lessons and any opinions you have concerning these lessons.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Lessons like this can be used to explore the need for litres, kilograms, hours, Rands, etc. Even in Grade 0 we can begin simple activities like this. By Grade 3 children should be able to estimate some measures. |

Developing a concept of time is difficult for children. There is a big difference between being able to tell the time and actually having a good understanding of the concept of time. Time is not something that remains static. It is not observable like a colour. It cannot be felt as mass can. It cannot be measured like length or height. It is not a specific shape that can be named like a square.

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| **SAIDE icons_3.png** | **Activity Two**What do you understand by the concept 'time'? Write your thoughts here. Remember that it is not telling the time.

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To young children time is something that is only relevant when certain things happen, for example:

* The time for going to bed
* The time for getting up in the morning
* The time for going to school
* The time for coming home
* The time for watching TV
* The time for eating
* The time for going to church
* The time for bringing the goats home

Telling the time is the ability to state accurately how many hours and minutes have passed in a particular 24-hour cycle.

For example: It is 20 past 9 in the morning. It is 9:20 a.m.

To young children these figures are meaningless since they are not related to anything in particular in their frame of reference (mental frames). To introduce the concept of time, teachers must work from the things that are most familiar. This will help children to realise that time is the passing of events. Time is a measure of what is past and what is to come. Days of the week, months of the year, and years are some measures of time.

The difficulty with the concept of time is that children have no sense of how the moon and the sun affect time (day and night, seasons of the year, short daylight and longer daylight).

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| **SAIDE icons_3.png** | **Activity Three**Design a set of questions that would establish the concept of 'morning'.

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | 'Morning' is not easy to establish. The length of a morning (how many hours) is often dependent upon when it gets light. In Cape Town, for example, it gets light at 5 am in summer but only at 8 am in the winter. 'Morning' is also often dependent upon when we get up for work or school. |

Very gradually children become aware of the need to establish some unit of measure that lets them be more specific during the morning period. This leads to the idea of the hour as a unit of measure.

There are many different ways of introducing the concept of 'hour'. One of these is to look at how early people told the time and to look at things like shadow clocks, hour glasses and sundials. This helps children to see that in fact we tell the time using the sun and that there is a relationship between the sun, the earth, and the moon and outer space.

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| **SAIDE icons_3.png** | **Activity Four**Design some questions along the following lines to suit your own class:* How long does it take you to dress?
* How long does it take you to get to school?
* How long do you spend at school?
* Guess how long it takes me to get to school?
* Guess how long it takes someone to drive to town?

Write down five more questions like this for your class.

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| ***SAIDE-icons_4c.png******Comment on Activity Four*** | Developing a concept of time takes time! But by continually getting children to estimate (guess) how long something takes (or did take) you help them to develop a sense of time and a sense of the difference between minutes and hours. Running races during the PE lessons and timing these is also a good way to develop a concept of time. Telling the time is easy to learn, but understanding time is much more difficult. |

We hope that these suggestions on how to teach measurement of length and measurement of time will help you teach the other standard units of measure such as litre, rand, kilogram. The point is that these units of measure are abstract ideas and therefore take time to understand. In the same way, place value is an abstract way of developing a number system. Place value is the subject of the next chapter.

# Chapter 10: Place Value

By the end of this chapter you should understand that:

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| SAIDE icons_2.png | * All number systems are abstract ways in which to represent objects;
* The base ten number system that we use is an abstract idea;
* Gaining conceptual control of place value takes time;
* Teachers need to take great care not to confuse children.
 |

In Chapter 9 we spoke about units of measure (hour, metre, etc.) and how these units of measure help us to calculate and communicate measure to each other. Just as we need an agreed way in which to measure, so we also need an agreed way in which to count and calculate. In this chapter we look at our agreed way to work with number. We look at how our number system works according to place value.

The power of a place-value system lies in the way in which digits are used. A finite set of digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) can be combined to represent an infinite number of numerals. The value of the digit depends on the place in which the digit is put. So a 7 in the numeral 723 represents seven hundred, while a 7 in the numeral 78 246 represents seventy thousand. We use a base ten number system, so the digits represent multiples of ten, depending on where the digit is put in relation to other digits. So, in the numeral 77 the 7 on the left is ten times greater than the 7 on the right. In the numeral 777 the first 7 is one hundred times greater than the 7 on the right. A base ten number system means that we record our counting in multiples of ten. Each time that we have a multiple of ten we place that digit in another place (or column). If we were to use a base five number system, we would record our counting in multiples of five!

As we know, children come to school with some number experience. But they have not had to think about place value because they usually have not had to write down what it is that they are counting.

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| **SAIDE icons_3.png** | **Activity One**Explain here when it does become necessary for children to gain an understanding of place value.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | While children are working with and counting real objects, they do not need to understand place value. They have learnt that you simply keep adding to the number you have already counted. But when you want to teach children how to write numerals or how to work with algorithms (a + b = cor c - b = a or a X b = cor c -7- b = a), they are going to need some understanding of place value. |

Children may have learnt that 8 + 7 = 15 and 3 + 2 = 5. They may also have counted 38 sticks and 27 sticks and said 65. But what they also have to learn is that when we use algorithms like addition, there are certain written conventions (rules) to follow - they have to learn that the position (or place) of the digit gives information about the quantity or value it represents. They have to learn that the cardinal value of 38 and 27 added together can be represented in many ways through drawing and continuous addition and bundles of ten, and so on, but finally they have to learn that the conventional way of writing sixty-five is 65.

The fact that our place-value system uses a base ten number system is a cultural and historical fact. In other words, our place-value system was invented and developed by human beings. And it is this cultural knowledge that teachers must explain and demonstrate to learners.

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| **SAIDE icons_3.png** | **Activity Two**Write down how you would teach children to understand and write the numeral 25 conventionally.

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | The children would need an explanation of base ten and how each place is a multiple often. This is not easy to explain and we have to be careful and patient so that we do not confuse children. A teacher might use expansion cards to teach the value of twenty-five: a card showing 20 and a card showing 5 to build 25. Another teacher might use twenty-five sticks and ask the children to make bundles of ten and then count the ones so that they have two bundles often (20) and five (5) which together make 25. Yet another teacher might ask the children to draw twenty-five sticks and then circle groups of ten. Some teachers would use all of these methods to give children a chance to understand in one way or another. The teacher would also need to tell the children to say what they are doing and what the result is of what they are doing for example: 'I have two bundles of ten and five ones so I have twenty-five. ' |

Understanding our place-value system requires logico-mathematical thought (or abstract thought) and young children have to be taught how to think and write using our base ten system. Another difficulty with our system is that we read numerals from left to right, but we often calculate from right to left!

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| **SAIDE icons_3.png** | **Activity Three**Do you think that everybody calculates addition and subtraction from right to left?

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Give one reason why teachers must not teach a method of calculating as though it is the only (true) method.

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Each of us thinks differently about things and each of us calculates according to the way in which we understand the number system. Some people find it easier to calculate in columns, while others find it easier to calculate in rows. Some calculate from left to right and some from right to left. How we calculate depends on how we understand the base-ten system. What is important is that each learner should be able to calculate accurately and quickly, whatever the method used. |

In order to teach place value well, teachers need to use a range of examples and a range of numbers. You cannot expect children to grasp place value if they only work with tens and ones. Children need to see the pattern in the system; they need to recognise how multiples of ten create a pattern in the system - for example that ten ones are recorded as la, while ten tens are recorded as 100 - and that the pattern of zeros will repeat through the system.

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| **SAIDE icons_3.png** | **Activity Four**Try this with your pupils.**Step one**Give each child a large number of objects to count (more than 50). Let them 01/ write down (in any way they like) how many they have. Let each child say **Step two**Pair the children and ask them to calculate how many objects they have altogether. Let them write down the total. Let them say how many they have. How many he/she has.**Step three**Choose two or three pairs to come to the chalkboard and explain and show how they added their numbers and how they wrote their sums. After each explanation, demonstrate to the children how the calculation would be written in the conventional way. Tell them about place value and how we write cardinal values in a special way. |

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | The point to remember is that it is the teacher's task to keep challenging children and to demonstrate how calculations are recorded conventionally using mathematical symbols and notation. You may remember that in section 3 of Education Studies (Juta 1997) we explained that the teacher's task is to teach children new knowledge, to introduce the unfamiliar. We said that children are capable of doing difficult things when they are working with a more capable person like a teacher. So, while you wouldn't expect children to understand place value on their own, you can expect them to work with larger numbers when you are controlling the situation. You would use the opportunity to explain something like place value. So, even if you were working with Grade 1 children, you would still introduce them to numbers greater than one hundred - you just wouldn't test them individually on these numbers. |

We have said that imagination and creative thinking are important in gaining competence in mathematics. This means that teachers must also be imaginative and creative in their teaching of mathematics. If you are able to have calculators in your classroom, you can be even more ambitious in your teaching of place value because children can calculate and work with numbers which are far greater and can see quickly how value depends on the place in which the digit is.

Teachers need to be patient and to take great care when teaching children to write number. Teachers must ensure that children are able to explain what they are doing and writing. If children gain an understanding of the place-value system when working with whole numbers, they have a better chance of also understanding decimal fractions.

# Chapter 11: Why Problem Solving?

By the end of this chapter you should be able to:

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| SAIDE icons_2.png | * Acknowledge that children are able to solve problems if left to do so using their own methods;
* Design problems that will help children to think mathematically in creative ways.
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A problem is a situation that requires a solution. In fact, a problem has two features: an obstacle (something in the way) and an objective (something to be gained). Without these features it is just a task to be done.

The objective is the end result. The obstacle is what has to be solved before the end is reached. In other words, a solution has to be worked out before the end can be reached. For example:

Problem: I have no transport.

Objective: To buy a car.

Obstacle: Not enough money.

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| **SAIDE icons_3.png** | **Activity One**List some solutions for the problem of having no transport and having no money.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | The above problem would simply have been a task if you were able to go and take a car without paying for it. |

We have tried to show so far in this course that children cannot learn if the teacher simply tells them what to say and what to write. Such an approach does not develop problem-solving and thinking skills.

Teaching is most often concerned with content, and less attention is paid to how to develop thinking skills. Yet, to be good at mathematics, a person must be an independent thinker who is capable of looking at a problem creatively and imaginatively. A popular method for teaching children problem solving has followed a set pattern. This is what children usually are told:

* Look at the question and decide what operation it is - addition, subtraction, multiplication or division.
* Now do that operation.
* Now look at the story and decide what it is about.

The result of this method is that children find problems difficult and often cannot see any purpose to the task.

Let me explain by way of an example why this is not a good method.

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| Simon had 13 bananas. He gave five to Nomsa. How many bananas does he have left? Traditional teachers will insist that this is a subtraction because the word 'left' is there. However, a child who did not know that could well say that it is a counting sum because you say five and then count on 6, 7, 8, 9, 10, II, 12, 13 ... so he has 8.Another might say that it is an addition because 5 + ? = 13. |

Solving problems is one of the main ways in which to develop number sense, as long as teachers encourage pupils to choose their own methods of working.

In Section 1 we spoke about active learning. Problem solving or doing a word problem is a form of active learning when children actively think about the problem and decide how to go about finding a solution to the problem.

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| **SAIDE icons_3.png** | **Activity Two**Watch the PREP Education Series video Teaching Abstract Thinking in the Mathematics Lesson. The video shows three teachers teaching mathematics - a Grade I, a Grade 2 and a Grade 3 teacher. Each teacher demonstrates a different way of teaching problem solving and each teacher allows children to think actively about and to talk about the problem. Before you watch the video, read the Observation Schedule below. When you have finished watching the video, discuss the video using this Observation Schedule as a guide.Observation schedule1. Exactly how does each teacher encourage children to use the language of mathematics - what does the teacher say and do?
2. Could the lessons on the video take place without the teacher teaching? What makes the teacher central to each of these lessons?
3. Challenging children to learn and think abstractly is an important part of maths teaching. Say how these teachers were challenging the children.
4. Each of the lessons on the video has a different kind of classroom organisation. The Grade 1 teacher teaches a small group on the mat. The Grade 2 teacher teaches the whole class at the same time. The Grade 3 teacher puts her class into groups of three and works with all of the groups at the same time. Do you think that the children were learning in all these kinds of organisation? Explain your opinion using examples from the video.
5. The Grade 1 and 3 teachers are teaching new knowledge. Notice what kinds of questions the Grade 1 and the Grade 3 teacher ask. Explain exactly how their questioning techniques help children to learn. Would you use this kind of questioning?
6. How does the Grade 2 teacher practise number knowledge with the class?
7. What teaching aids are used in the lessons?
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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | The above problem would simply have been a task if you were able to go and take a car without paying for it. |

It cannot be stated often enough that children need to be given a chance to come to an understanding of number through personal investigation and explanation, through developing a method for calculation with which they are comfortable but also by being allowed to hear how other children have solved a problem. Seeing and hearing how other people think helps us to think more imaginatively, and working together with other people to solve a problem helps us to learn about mathematics.

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| **SAIDE icons_3.png** | **Activity Three**Try solving this problem without using addition, subtraction, multiplication or division. Write down exactly what you have done.Mother collected eggs every day. She collected 5 eggs on Monday, Wednesday and Saturday. On the other days she collected 3. How many eggs did she collect that week?

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| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Perhaps you can compare your solution with those of some of your colleagues. |

### Types of problems

There are two distinct types of maths problems. First of all there are those that are familiar and typical and are straightforward to solve - routine sorts of problems. For example, 'If I have twenty-eight teeth and two fall out, how many are left?' is a routine kind of problem.

The other type would be problems that are not straightforward and that are unfamiliar or unusual or non-routine. For example: eighteen is in the table of two and the table of three. Is eighteen in any other table that you have learnt? This is a non-routine problem and so children to need to think about how they will work it out. There will be a variety of ways in which children could solve this problem.

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| **SAIDE icons_3.png** | **Activity Four**Look at these two problems and decide which is straightforward and which may require more thought. Say why.John had fifteen birds. Eight of them flew away. How many did he have left?

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What is the difference between 8 and 15?

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| ***SAIDE-icons_4c.png******Comment on Activity Four*** | The first example can be described as a typical example. You find examples like this in every maths textbook and teacher's guide. The second example is unusual because it is expressed differently. It is more abstract and uses mathematical language. While both problems can be solved by subtraction, the second example is unfamiliar in the way in which the problem is presented (although it is an important and necessary way in which to present problems so that children learn to see that subtraction is also about difference). Remember the Grade 1 teacher on the video who asked a non-routine question when she gave the problem 'I have twenty-four apples, half of them are rotten, how many are good?'. The children have to think very carefully about that problem, using all the prior knowledge that they have. |

Most teachers are familiar with routine problems - they are typical examples and have a specific method that is followed in order to solve them. The teacher teaches addition and then gives the class a series of addition problems, or teaches subtraction followed by subtraction problems, and so on. The trouble with only doing routine problems is that we don't develop other ways of thinking about maths problems. In order to encourage creative thinking, teachers must present problems which are unfamiliar (or non-routine) so that they may be solved in a variety of ways. By looking at various solutions, children are able to bring personal meaning to the operations and see how different people think and how we can think differently.

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| SAIDE icons_3.png | **Activity Five**Here is a problem. Try to solve this problem in at least three different ways.Mama had 23 biscuits. She gave three to me, ten to my Toto, some to my Dad (and I'm not telling you how many) and there were eight left. How many did my Dad get?Write down three ways in which this problem could be solved.

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| ***SAIDE-icons_4c.png******Comment on Activity Five*** | Here are some possible ways:a) 23 -3 = 20 -10 = 10 - 8 = 2 for Dadb) 3 + 10 + …… + 8 = 23 13 + 8 + ……. = 23 21 + ……. = 23Therefore Dad got 2 biscuits1. M M M

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| SAIDE icons_3.png | **Activity Six**Change the following problem into one that allows for a different way of thinking:Niko had 66 marbles and then he won 15 more. How many marbles does he now have?

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| ***SAIDE-icons_4c.png******Comment on Activity Six*** | You may have thought of things like: Niko had 66 marbles and then won some more which meant that he had 81 marbles. How many marbles did he win? What is the sum of 66 and 15? |

While this may seem very difficult, remember that we can only learn new things if and when we are challenged to think about them. Try to encourage different ways of thinking in your maths lessons. And try to find ways in which to introduce children to the language of mathematics by using terms such as 'sum' or 'difference' or 'total' or 'product', etc.

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| SAIDE icons_3.png | **Activity Seven**Reword the following problem in two different and unfamiliar (non-routine) ways:Thoko was given R 14 by her father. She spent R7 on a necklace and R4 on some soap. How much did she have left?

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Now try these problems with your class. Tell your class to try to solve them in any way they wish. Let them draw or write. Write down some of your thoughts about using problems with your class.

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| ***SAIDE-icons_4c.png******Comment on Activity Six*** | Remember that the purpose of setting problems is to get children to think differently and to see how different children think. So you are interested in the process of solving the problem (how they think). When teaching (not testing), a single method is less important than learning the process of calculation through the exploration of different methods. |

The following are some important points to keep in mind to develop creative thinking.

* Be a good listener.
* Vary the type of problem by asking it in several different ways.
* Evaluate the methods that each child uses.
* Read the problem to the group / class.
* Allow time for the group to solve it.
* Let the children talk to each other.
* Use the errors (mistakes) to teach more about thinking processes.

# SECTION 4: ASSESSMENT

Assessment is an important and essential part of teaching. If teachers are to ask themselves whether what they are teaching and how they are teaching has the desired outcomes, they will need to assess what children are able to do according to a set of criteria. In chapter 3 we listed some of the criteria which indicate what children should be able to do and think in mathematics by the end of the Foundation Phase. Go back to chapter 3 and look at this list again.

By the end of this chapter you should:

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| SAIDE icons_2.png | * Recognise that assessment is an essential part of teaching;
* Recognise that continuous assessment is a means of improving the quality of teaching and learning;
* Know how to record the performance of each learner.
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**There is one chapter in this section.**

* Chapter 12: Assessment in Mathematics

Allow about four hours for this section.

#

# Chapter 12: Assessment in Mathematics

The following are two very important questions that teachers have to ask when teaching.

* Is what I am doing helping children to develop a desire to learn mathematics?
* Is what I am doing teaching children to become numerate?

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| SAIDE icons_3.png | **Activity One**In the space provided, write down why you think it is necessary to assess each learner.

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| ***SAIDE-icons_4c.png******Comment on Activity One*** | Some reasons for assessing learners are so that we can measure whether a lesson has been effective for each learner (whether we have taught the lesson effectively enough); we assess learners to see what each one can do for example which learners are able to calculate change, or which learners are able to solve relevant word problems; we assess learners to see which of the children are ready for a new challenge and which must still practise what has already been taught; we assess learners so that we can plan further lessons that suit the needs of the children. |

There are two main ways in which to assess children - formative assessment and summative assessment. Formative assessment is assessing a learner while the learner is forming the new knowledge; summative assessment is assessing a learner at the end of a lesson, section, topic, quarter or year as a summing up of what the learner knows. So tests and exams are summative versions of assessment. An example of formative assessment would be sitting with a learner while he or she is doing a task (say using a number line to count in groups), watching how the child goes about the task and asking the child to explain how and what he or she is doing. In this way you find out what strategies the child is using and developing and what strategies you should be helping the child with; you are getting direct and instant feedback on how the child is coping and you are able to respond to the situation immediately through re-teaching and explaining again, asking another learner to help, or planning another lesson on that need for the next day.

When both formative and summative assessment are used, that is continuous assessment. In an outcomes-based education system continuous assessment is used. The teacher studies the learning outcomes required of the learners (as listed in chapter 3) and then plans lessons to teach to achieve these outcomes. During the lessons the teacher observes what children are doing and saying and how children are doing a task. The teacher asks for explanations from the children as to what and how they are doing a task. (Remember the Grade 1 and the Grade 3 teacher on the video?) The teacher helps those learners who are confused and continually monitors which learners are gaining control of the skills and concepts (for example: is able to count to 100 independently, is able to calculate change independently). Once a child can do something independently within the number range for that learner, a teacher can say that that child has learnt what was intended by the lesson and so can record that performance as a desired learning outcome for that child.

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| SAIDE icons_3.png | **Activity Two**Make three copies of the grid given in chapter 3. Select three learners from your class. During the next five moths lessons observe each of these learners closely. Tick those areas and ranges on the grid which the learners are able to do independently (without your help). At the end of the five lessons, write what you think of this method of assessing and recording.

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| ***SAIDE-icons_4c.png******Comment on Activity Two*** | Written tests are not always necessary if continuous assessment is being done thoroughly. Remember that continuous assessment means that you are assessing children during the task and asking questions such as: 'What will happen if ...?'; 'How did you work that out?'; 'What do you notice about these numbers?'; What do you mean by ...?'; 'Explain how you thought about'; 'Show me how you do that on your own'. |

Good teachers are continuously assessing children and themselves, ensuring that what they do is actually helping children to become learners rather than just memorisers of facts. Good teachers will keep the learning outcomes in mind and try to ensure that each and every learner can work mathematically at an appropriate level in the Foundation Phase, that each and every learner is becoming numerate. Only through continually assessing the learning in the classroom and planning further learning opportunities can a teacher be sure that children are actually learning those aspects of mathematics which will help each of them to become numerate.

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| SAIDE icons_3.png | **Activity Three**Here is a transcript of a piece of one of the lessons you watched on the video ***Teaching Abstract Thinking in Mathematics***. The teacher is asking the boy to explain how he got to his answer. The teacher is assessing the learner in the context of the task (formative assessment). Read the transcript and write down your assessment of the learner. Say what the child is able to do independently and say what the child still needs to learn about calculation.The children are working on the mat with small chalkboards and chalk. The teacher sets the following problem:

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| T: | Ron had thirteen marbles in one pocket and the same number of marbles in the other pocket. How many marbles did Ron have altogether?  |
| The teacher goes to the boy who has this on his chalkboard:  00000000000000000000000000  13+13 -> 20+6  26  |
| T: | Tell me what you did.  |
| P: | I wrote 13 and I wrote 13.  |
| T: | How did you know to write 13 and 13?  |
| P: | I drew the marbles and counted altogether. Then I wrote 13 and I wrote 13 (pointing to the numbers on his chalkboard),  |
| T: | (pointing to the 20 + 6.) How did you get 20 + 6?  |
| P: | This 1 stands for 10 and this 1 stands for 10 (pointing to the 1 in the tens column of 131) so that is 20. This ‘3’ and this ‘3’ (pointing to the 3s in the ones column of 131) make ‘6’.  |
| T: | Then you said? Say what you did to get this 26 here (pointing to the 26 on the pupil's chalkboard).  |
| P: | I wrote 20 and 6.  |
| T: | Yes, but how did you get 26? |
| P: | I added 20 and 6 to get 26. |

Write down here what you think the child is able to do independently and what he still has to learn about addition.

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| --- | --- |
| ***SAIDE-icons_4c.png******Comment on Activity Three*** | Unless a teacher actually asks a child to explain what he/she is doing and thinking, the teacher will not have a clear idea of what it is the child can do independently and what the child still has to learn. Also, by asking a child to explain, the teacher is able to check for any confusion in the child's thinking. In the above example the child is not confused; the child is able to calculate independently and to record in his own way. The child still has to learn how to write a conventional number sentence. |

### Conclusion

Careful and focused assessment that is continuously and regularly carried on in the maths lessons will ensure that the children are always being challenged to improve their learning outcomes. Regular and continuous assessment also ensures that a teacher plans and teaches lessons which suit the needs of the learners.

# SECTION 5: MATHEMATICS ACTIVITIES FOR JUNIOR PRIMARY CLASSES

Certain kinds of apparatus (such as the abacus, the 120 chart, expanded notation cards, tape measures, and so on) are useful and often essential aids to teaching mathematics. Section 5 consists of a collection of mathematical activities using apparatus. The activities are designed to help learners to become fluent and efficient in mathematical calculations.

After studying section 5, you should be able to:

|  |  |
| --- | --- |
| SAIDE icons_2.png | * Organise similar activities for your learners as part of the daily maths lessons.
 |

**There is one chapter in this section.**

* Chapter 13: Mathematical Activities Using Apparatus

# Chapter 13: Mathematical Activities Using Apparatus

### The Abacus

The abacus can be used individually, in pairs, in groups or for the whole class. The abacus can help children to solve maths problems and is most useful when teaching children to count and to calculate.

#### Counting forwards

* The teacher uses a large abacus so that the whole group can see. The teacher counts forwards.
* The learners count out aloud while the teacher looks and listens to see who is counting correctly and who is still learning to count.

#### Counting backwards

* The teacher uses a large abacus and the learners count backwards. It is useful to count forwards to a particular number first and then to count backwards as this gives the children a starting point; for example count back from 73.

#### Skip counting

* Counting forwards or backwards in multiples, for example counting on in fours; counting backwards from 70 in sevens.

#### Counting on

* The teacher moves a certain number of beads across and asks the children to count on from that number.

#### Odd and even numbers

* Counting in even numbers only.
* Counting in odd numbers only.
* Identifying odd and even numbers. The teacher moves some beads across and asks the children to count them and to say whether the number is odd or even and why.

#### Estimation

* The teacher moves a certain number of beads across and asks the children to guess how many have been moved. Then the beads are counted. If the children have written down their estimations, they can work out the difference between their estimation and the actual count.
* The teacher moves a certain number of beads across and asks the children to estimate how many beads have not been moved.

#### Counting in tens

* Teacher moves 50 across and asks how many beads are left. Teacher moves 120 across and asks how many he or she must move back to leave 100.
* Teacher asks how many tens he or she must move across to have 60.
* Teacher asks how many tens in 20, 30, 80, 120, etc.
* Half of 10, 20, 30, etc.
* Double 10, 20, 30, etc.

#### Patterns in number with addition

* Teacher and children move 10 across. Teacher asks how many more must move across to make 16 (or any other number). Teacher then does the same with 20, 30, 40, etc. Ask the learners what pattern they notice with the tens and units ( 10+6=16,20+6=26,30+6=36, etc.).

#### Patterns in number with subtraction

* Teacher and children move 10 across. Teacher asks how many must move back to leave 8 (or any other number). Teacher does the same with 20, 30, 40, etc. Ask the learners what pattern they notice (10-8=2,20-8=12, etc.).

#### Show me

* Teacher asks children to show him or her 10, 38, 52, 107, etc. on the abacus.

### The 120 Chart

These activities can be done with a large chart and/or with small charts for individual or pair use. The teacher watches how individuals do these tasks.

#### Addends (adding on)

The teacher tells the children to place a counter on a given number, for example 89. The teacher tells the children to add on 5 or 3 or 6, etc. The children count on and place their counter on the new number.

Learners call out the number. Teacher checks.

#### Minuends (subtracting)

From the given number, the children are asked to make the number less, or to subtract by 4 or 2 or 1, etc. The children then move their counter onto this number. Learners call the number. Teacher checks.

#### Doubling

The teacher asks the children to put their counter on a given number. The teacher tells the children to double the number. The children move their counters to the new number. Learners callout the number. Teacher checks.

#### Halving

The teacher tells the children to put their counters on a given number. The teacher tells the children to halve the number. The children move their counters back to this number. Learners call out the number. Teacher checks.

#### Patterns in number

* Count all the even numbers. What pattern do you notice?
* Count all the odd numbers. What pattern do you notice?
* Count in fives, tens, twenties, etc. What pattern do you notice?

#### Working holistically

All of the above can be used not only as separate activities but also all together, for example:

* Put your counters on the number 14.
* Add on 5. (Children move counters to 19.)
* Subtract 9. (Children move counters to 10.)
* Double your answer. (Children move counters to 20.)
* Make your answer 4 less. (Children move counters to 16.)
* Halve your number. (Children move counters to 8.) Remember to practise within a number range that the children must learn and know, but also to include numbers that will challenge the children.

### Multiplication chart

#### Number patterns

Number patterns help children to recognise multiples and factors. This helps with multiplication and division. The teacher coaches the learners as they notice the multiplication facts on the charts. For example:



The teacher asks the children to count in threes (for example) and asks them to see what pattern is developing. The teacher asks the children: 'Who can say what the pattern is?' The children discuss the pattern.

The teacher then asks questions such as the following:

* How many groups of 3 in 15, 27, 60?
* Are there 12 groups of 3 in 36?
* How do you know that 42 is a multiple of 3?
* If you add together the digits of each multiple of 3, what do you notice?
* (e g 36 ---7 3 + 6 = 9---7 9 is a multiple of 3, so 36 is also a multiple of 3, etc)
* Can you make equal groups of 3 from 45? So is 3 a factor of 45?

#### Multiples

Children count in threes (or colour every third number).

Children count in sixes.

Children count in nines, etc.

* What patterns do you notice?
* Is 12 a multiple of 3? Why?
* Which number is a multiple of 3 and 6 (common multiple)?
* Is 24 a common multiple of 2, 4 and 6? How do you know?
* Which numbers are common to the table of 3, 6 and 9 (common multiples)?

#### Factors

* Look at 24. Can you make equal groups of 2 from 24? How many groups can you make? So 2 is a factor of 24.
* What other equal groups can you make from 24? So 3, 4, 6, 8, 12 are factors of 24.2,3,4,6,8,12 are common factors of 24.
* Look at 25. What are the factors of 25?
* Which are the common factors of 12 and 18?

Learners should work regularly with the multiplication chart so that they become fluent in working with number.

### Dice

#### Addition

Two dice are thrown; the numbers facing upwards must be added together quickly. The first child with the answer wins. Each time that a child wins, he or she gets a point. The child with the most points wins.

The children can take turns to throw the dice. The teacher (or an expert pupil) needs to keep a check on the answers called.

#### Subtraction, multiplication and division

This follows the same process as with addition, except that the children subtract the numbers, or multiply or divide them. The teacher (or an expert pupil) needs to say/check whether the answers called are correct.

#### Challenges

* The children can be challenged further, for example:
* Throw the dice.
* Add the numbers.
* Double the total.
* Halve the total.
* Multiply half the total by 4, etc.

*or*

* Throw the dice.
* What is the difference between the numbers?
* Multiply the difference by 5.
* Subtract 10 from the total.
* Add any odd number to it, etc.

Each child in the group gets a turn to give an answer. The answers can be written on the chalkboard as you go along.

#### Variation

The same as in number 3, but this time the children tell each other what to do next with the numbers. The teacher writes the answers and checks that they are correct.

### Using a tape measure

#### Using a tape measure for counting

Children work in pairs, threes or individually. The tape measure is laid out
in front of the children.

#### Counting on

The teacher chooses a number and the children count on from there while looking at or pointing to the number on the tape measure.

#### Skip counting

Starting at a number and counting on in sets, for example begin on 9 and count forwards in threes.

#### Counting backwards

Count backwards in ones or in sets, for example count backwards in twos from 25. Children should point at each number as they say it.

#### Small addends

The child puts a counter on the first given number, for example 9, and is told to add on 5. He/she then moves the counter to the answer, (that is, 14). The teacher then coaches the child, 9 + 5 is equal to 7 Nine plus how many is equal to 147, etc.

#### Small minuends

The same as in number 4, except that this time it is subtraction, for example 15 - 6 = 7 After each answer the teacher should ask the children to explain how they got their answer. It would only be necessary to ask one or two children after each sum.

The teacher also coaches these, for example:

15 – 6 = 7, 15 – ... = 97, ... – 6 = 97

#### Measuring with a tape measure

#### Let the children measure the same thing.

For example a book, with various tools such as hands, feet, rulers, pencils, etc. Width and length can be measured. A discussion on why everybody measured the same thing but had different answers should follow. Introduce the tape measure as a standard means of measuring (see chapter 9).

#### Repeat the above activity, this time using a tape measure.

The children can work in pairs measuring the list of things to measure. The children record the findings and the group compares the measurement results now that the children have used a standard unit of measurement.

#### Working in pairs

Children measure each other's bodies. The measurements are written down, for example:

|  |  |  |
| --- | --- | --- |
| •  | I am  | cm tall.  |
| •  | My head is  | cm around.  |
| •  | My hand is  | cm wide.  |
| •  | My waist is  | cm around.  |

Afterwards, the class or group can discuss who had the biggest feet, smallest wrist, etc. The teacher can ask questions based on the measurements.

#### Use newspaper or old magazines or waste paper.

Ask the children to measure and cut squares of 9 cm. or a rectangle 8 cm by 12 cm. These can be used for art lessons or paper folding.

#### Measuring can become part of a physical training lesson.

Get the children to jump as far as they can from the same starting place. Each child measures his or her own jump while the others watch and check. Calculations of distance, difference in distances jumped, etc can also be done.

#### Learning metric tables

Teach the children 10 mm = 1 cm, 100 cm = 1 m (1000 m = 1 km, if possible). Then do sums converting mm to cm and vice versa, using the tape measures as a guide. For example:

15 mm = 1,5 cm and / or 2,5 cm = 25 mm

125 cm = 1,25 m and / or 3,56 m = 356 cm

#### Groups and tables

Pie graphs, bar graphs and tables of measure can be designed using the information gained by measuring things (or people) with a tape measure. For example:

* A graph of all hand sizes or foot lengths can be drawn up.
* A graph of the height of each child can be drawn up.

#### Problem solving

Children can be given problems which require them to use a tape measure. For example:

* Who is the tallest in the class?
* How many centimetres taller are you than your partner? (The children can count on the tape measure from their own height.)
* How many centimetres shorter are you than the tallest person in the class?

### Measuring capacity

Apparatus needed:

* Measuring jug
* Old plastic milk and juice bottles
* Empty tins of various sizes
* Buckets
* Anything that can hold water

#### Using capacity

#### Exploratory activities

Children can experiment with capacity by pouring water into different containers and watching which containers hold the same amount, which hold more or which hold less. For example, how many cups of water are needed to fill one bottle?

*Hint: Let the children pour over a basin or a bucket so that the water isn't wasted.*

After plenty of experimentation, discuss with each group the need for a fixed unit of measure for liquid. For example, how do we make sure that we all get the same amount of milk or cooldrink for the same money at the shop? (Litres) How could we all make the same bread or cake without the same measurements for liquids needed in the recipe?
(1/2 cup oil, 11/2 cups milk, etc)

Introduce *l* and *ml*. Tell the children that 1*l* is a specific measurement of capacity: there are 1 000*ml* in 1*l*.

#### Using units of measure

Ask the children to find out the capacity of a variety of containers, for example:

* How many *ml* of water can a cup hold?
* How many *l* can a bucket or jug hold?

#### Using the measurement jug.

Ask the children to fill it with half a litre of water, or 250 *ml* or 3/4 of a litre or 300 *ml*, etc.

#### Estimation

Before pouring water into a jug or cup, the children must estimate how much it will hold. Then they pour and see if their estimation was correct or not. They use the measurement jug to check their estimation.

### Expanded notation cards

Expanded notation cards are useful for teaching number concepts such as place value and cardinal value. For quick and easy access to the cards, keep each group of cards in a separate plastic bag or container. Colour code the back of the cards and the container for easy identification; for example, the tens could be red, the hundreds blue, etc. This also makes it easier to use the cards quickly and correctly.

In the lesson each child should have a pack of notation cards ready to use.

#### Building numbers

The teacher tells the children what number she wants them to make, for example 24.

Each child then takes a 20 from the tens pack and a 4 from the ones pack and places the 4 on top of the 0 so that they form 24. (It is 20 + 4 and not 2 + 4.)



Like this (the 4 covers up the 0)

The teacher and children then talk about the number 24, for example:

* How many tens in 24?
* What does the 2 stand for?
* How many loose ones in 24?
* What does the 4 stand for?
* Which is greater in value, the 2 or the 4 in 24?
* Change the number to 34.
* How many more tens does 34 have than 24?

Teachers would work with numbers to suit the level of instruction of each child/group/class. Grade 3 (Standard 1) would also work with thousands.

#### Addends

The teacher asks the children to build a number, for example 124. Once all the children have that number correct, the teacher continues by asking the children to make it more by 5. The children put back their 4 and take the 9 and put it on top of the 0, making the number 129.

The teacher continues by asking the children to add on 2. The children put back both the 20 and the 9 and take a 30 and 1, making the number 131. So the teacher continues building on, or coaching the children in place value (what does the 1 stand for, what value does the 9 have, etc.).

#### Minuends

As in (2) above, but using subtraction. Place value must be coached again.

#### Doubling and halving

The teacher asks the children to build a number. Once they have built the number, the teacher tells them to double or to halve the number.

The children pack away the original number and pack out the new number.

The teacher asks one or two children to explain to the group how they got to their answer.

#### Multiplication and division

Again the teacher begins by asking the children to build a specific number. The teacher tells the children, for example, to multiply it by 5. The children do-the sum and then pack out the answer. The teacher asks one or two children to explain how they got their answers.

The same thing happens with division. A number is given and the children pack it out. Then the children are asked, for example, to divide it by 7, and the children pack away the old number and pack out the new number. Some children explain how they got their answer.

#### These cards can also be used for making sums:

 e.g. 28 + 15 = ...

The children build the sum and the answer.

Remember that addition, subtraction, multiplication, division, doubling, halving, etc. can all be done at the same time on the same numbers. Remember, too, that children can learn a great deal about place value from using expanded notation cards. Be as flexible and as creative as possible.

### Place value

Note: A fuller understanding of place value can also be obtained by using the other activities outlined in this section.

#### Using sticks to teach place value

#### Bundling sticks

Let the children make bundles of ten sticks (use elastic bands for this). Ask the children to show, for example:

* 12 in sticks (1 bundle of 10 and 2)
* 35 in sticks (3 bundles of 10 and 5), etc

Talk about the number, for example:

* How many tens in 12, 35, ... ?
* How many loose ones in 12, 35, ... ?

This work with sticks must be extended to include bundles of 1 00 sticks.

#### Writing numbers

The children make numbers with their sticks, for example:

43 has four bundles of 1 0 and 3 loose ones.

* How do we write this number?
* How do we read this number?
* What does the 4 stand for?
* What does the 3 stand for?

#### Addition and subtraction

Give the children an addition or subtraction sum and have them work out the answer with the bundles of sticks. Ask the children (with your help) to write the working out as a sum. For example:

* Add 14 and 27
* Write 14 + 27 =
* Show 14 + 27 with the sticks
* Write 14 + 27 = 41

Do the same with subtraction.

#### Working with large numbers

Do the same as in 1, 2 and 3, but show how place value works for hundreds, tens and ones/units. If possible, work with thousands too. For example:

* Show 351 with the sticks. Write this number. How many hundreds, tens, units?
* What is the biggest number that you can make with your sticks? Write the number.
* How many bundles of 100 sticks would I need to make I OOO?
* How many bundles of 10 sticks would I need to make 100? And 1000?
* How many times greater is 100 than 10? Use the sticks to help you.
* How many times less is 10 than 100?

### Counters

#### Estimation

*Group activity*

* The teacher shows the group a number of counters.
* The teacher asks the children to estimate the number of counters.
* The children estimate and write down their answer, either on their chalkboards or on paper.
* The group then counts the counters. (This counting can be done in various ways, for example ones, threes, sixes, etc.)
* The children must each compare their estimation with the actual amount and record the difference; for example, if a child estimates 12, but counts 16, then he/she underestimated by 4.

*Paired activity*

This is done in the same way, except that the children in the pair would take turns to put out a number of counters and ask the other to estimate.

#### Counting forwards and backwards

The teacher asks the children to set out a certain number of counters. The children may count the counters in a variety of ways, for example ones, fives, sevens, etc. The children can also count them backwards.

#### Problem-solving sums

For example: I have 24 counters. How many does each child get if there are 3 children and I share the counters equally?

The children would share out the counters to see how many each child would get.

#### Doubling

The children physically count out how many counters they need; for example, if they are to double 16, the teacher will say, 'Count out 16 counters. Now double your counters.'

The children must then take another 16 counters. They can either count on from the first 16 or they can count 16 and then count both groups of 16 together. They must find the answer 32. The teacher then coaches this. What is double 16? What is 16 plus 16? How many sixteens in 32?

#### Halving

The teacher gives the children a number, for example 38. The children count out 38 counters each. The teacher asks the children to halve 38. The children then work it out in their own way or by sharing. They find the answer 19. The teacher coaches the calculation, for example 38 take away 19 =? What is 19 + 19? What is half of 38? 38 - ... = 19, etc.

#### Multiplication

The children use the counters to make concrete 'groups of', especially in the beginning stages of multiplication, for example 5 x 3 =

 

(3 groups of 5) (5 groups of 3)

The children can build the groups using the counters and then count all the counters. The teacher then coaches 5 X 3 = 15, 3 groups of 5 make 15, 5 multiplied by … equals 15, etc.

#### Division

A story sum is a good introduction to division. Let the children use counters to help them to solve the problem.

e.g. Mother baked 35 biscuits. She had 5 guests for tea. How many biscuits did each guest get?

35;- 5 = ?

The children take 35 counters and share them between 5 guests.

### Extension activities for number knowledge

These activities can be used individually, in pairs or in groups. They can be used as activities for children to do while the teacher works with a group.

#### Double or half



The teacher fills in one number on each square (see example). The children must then complete the square by either doubling or halving the number given.

#### Whole, half or quarter



The teacher fills in one suitable number on each rectangle (see example). The children must complete the rectangle by building up the quarters into halves and the halves into a whole. It would look like this when the children are finished:



Of course, the teacher may put her chosen numbers anywhere in the rectangle. He or she may put the whole number in, or the half, or the quarter.

#### Mind blasters



The teacher fills in the blocks, including the operations that the child must do. The teacher also fills in three numbers, either for the child to start from or the answer and the child must work backwards (see example).

Another alternative is to leave the blocks blank and the child must fill these in too:



#### Estimation and counting

The teacher gives a child, pair, or group a set number of counters, together with the following worksheet (or written on the chalkboard) to complete:

* I estimated: \_\_\_\_\_
* I counted: \_\_\_\_\_
* I over/underestimated by:
* The number I counted is odd / even.
* There are \_\_\_\_\_\_ groups of \_\_\_\_ and \_\_\_\_\_ left over.
* \_\_\_\_\_ more would be \_\_\_\_\_
* \_\_\_\_\_ less would be \_\_\_\_
* Double is \_\_\_\_\_
* Half is \_\_\_\_\_

#### My number is ...

Give the child, pair or group the following worksheet and some counters. The children choose their own number.

* My number is: \_\_\_\_\_
* What number comes before: \_\_\_\_\_
* What number comes after: \_\_\_\_\_
* What number is half of: \_\_\_\_\_
* What number is double: \_\_\_\_\_
* What number is ? more than \_\_\_\_\_
* What number is ? less than \_\_\_\_\_

#### Brain ticklers

Set the children problems which challenge them.

