National In-Service Teachers' College

NISTCOL

Primary Teacher's Diploma

by Distance Learning

Module 3

Numeracy

Revised Edition 2011

MINISTRY OF EDUCATION ZAMBIA

Acknowledgement

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NATIONAL IN-SERVICE TEACHERS' COLLEGE (CHALIMBANA)

EDUCATION BOARD

PRIVATE BAG E 1

LUSAKA

7th May 2012

South African Institute of Distance Education
P O Box 31822
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South Africa

Dear Tony,

RE: AUTHORIZATION OF PUBLICATION OF NISTCOL PTDDL MODULES INTO OERs

I write in reference to the soft copies of the modules (Module 3, 4 and 5) earlier sent to you. I am glad to confirm that authorization is hereby given to SAIDE to publish the three modules into OERs.

I further suggest that this should be done under the Creative Commons Attribution license to replicate, copy, distribute, transmit, or adapt this work freely provided that attribution is provided as illustrated in the citation. The citation for example for module three – numeracy should be as follows:

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Yours Faithfully,

P. Muzumara
Principal
National In-service Teachers’ College

Preface

The skills presented in the modules of this Primary Teacher's Diploma course will help you as a teacher to refocus and re-orient yourself to the major changes that are taking place in both the colleges of education (the new Zambia Teachers Education Course) and basic schools (the New Curriculum and the Primary Reading Programme). At the end of the course, you should therefore, show competency in pedagogical related knowledge and skills in planning, implementing and evaluating teaching and learning processes.

The approach used in the modules and which you are expected to adopt when working in the classroom is probably radically different from that which you have been exposed to during your college training and school teaching. The approach adopted for the modules is much more practical, requiring you to inquire into and reflect upon what you are doing to a far greater degree than you have possibly been used to.

Throughout the modules and your exposure to different approaches and methodologies, try to apply each approach and methodology to your work in the classroom. We have combined theory and practice; the practice stemming from your own experience in classroom use. So, this is not a "How-to-do-it" module. It is a "How-we-do-it" module. We are not presenting our experiences in order to set ourselves up as models to be emulated, but to ensure that all the ideas are rooted in reality, and therefore, are entirely possible and usable.

Take responsibility for your own learning that the modules offer. You might have probably sat passively for so long in so many learning situations that this change will not be easy. But we know that it is a very exciting development and ultimately you will welcome it. We have no doubt in your resolve to complete this module successfully.

Finally, I want to wish you success in completing this vital course, not only for you, but especially for all primary school children throughout Zambia.

P. M. Muzumara

Principal – National In-service Teachers. College

P/B E1

LUSAKA

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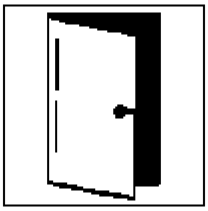

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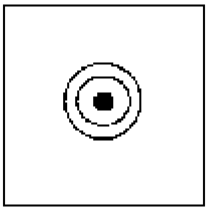

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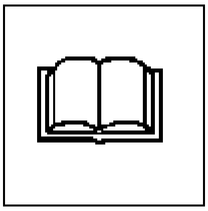

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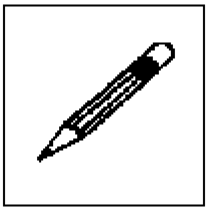

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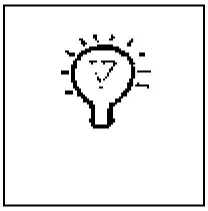
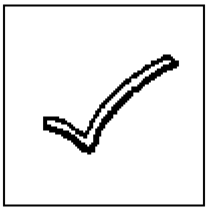


Practice activity in the class



For the assignment file

General introduction

Educating Our Future, the National Education Policy document, states, “A fundamental aim of the curriculum for lower and middle basic classes is to enable pupils to read and write clearly, correctly and confidently in a Zambian language and in English, and to acquire basic numeracy and problem-solving skills.” What do we mean when we say a person is numerate?

This module explores the answer to this question by investigating both WHAT to teach and HOW to teach Mathematics effectively.

Studying through this module

This module is activity based. You learn and develop teaching skills better by doing and trying out ideas yourself. There is a range of activities that have been given in this module. The activities are mainly intended for your own self-assessment, which is to find out how much you have learned, and for your practice in the classroom. You should do as many activities as possible. You will be required to keep some of the activities you do in your teaching file as part of your portfolio.

Support from colleagues

Learning can be a challenging process when you are doing it by distance mode. In the course of studying this module, you will need the support of your colleagues in the school, especially in your Teachers. Group (TG). Some activities will specifically require you to do them either with a friend or with your Teachers. Group members in the usual TG meetings.

Competencies to be attained

After completing this module, you will be able to:

- Communicate effectively with pupils in your class to facilitate learning of mathematics
- Assess and analyse learning difficulties of your pupils and provide the necessary remedies
- Teach lower basic classes mental arithmetic and estimation skills and problem-solving
- Create activities for teaching mathematics practically
- Teach Mathematics effectively to Lower and Middle basic classes: using a variety of learner-centred techniques.

Assessment

Assessment of your performance in this module will largely be based on the practical activities you will be required to do throughout the course of your study. These practical activities involve classroom teaching. Although you may be teaching one class from grades 1 - 7, this course requires you to acquire skills of teaching Mathematics to all Primary School grades. As you study and work through this module, you will find that the topics are progressively developing from grade I to grade 7.

UNIT 1

PHILOSOPHY OF MATHEMATICS EDUCATION

In this unit, we discuss the philosophy of mathematics education. Also, we look at why it is important to teach Mathematics at the Primary School level.

Learning Outcomes

As you study and work through this unit you are expected to:

- explain the philosophy of mathematics education
- state some of the aims and objectives of mathematics education
- discuss the philosophy of teaching and learning mathematics
- explore the nature of mathematics
- discuss some of the approaches of teaching mathematics
- apply critical thinking and problem-solving activities in teaching mathematics

Philosophy of Education

When looking at the philosophy of education, remember that education is about the desire to learn. The process of learning should be pleasurable, interesting, and helpful. Pupils need to construct their own knowledge through understanding via gathering data, exploring the truth, analysing the information and making decision for their own learning.

It is also important to remember that learning is a life-long process. What pupils learn in schools is not enough to prepare for their practical living. They should be able to organise, understand, and apply what they learn in order to deal with the complex and diverse questions of their daily life. They should have the abilities to know how to use what they have learned and to continue to learn and create "new" knowledge to solve problems they meet when they leave school.

Finally, education is life. Education supports life; life (experience) connects to education. They also use their life experience in their learning.

Activity 1.1

As a teacher, use the following three questions to answer why you teach Mathematics:

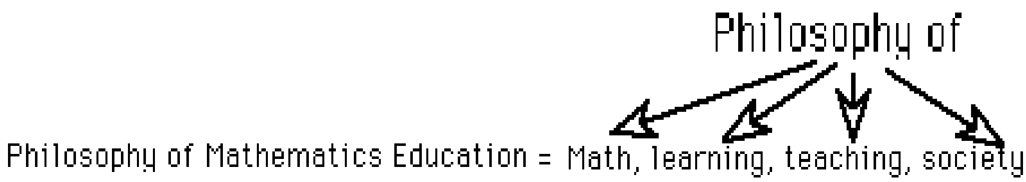
1. *Why do you need to learn Mathematics?*
2. *Why do you need to teach Mathematics?*

What is Mathematics?

Mathematics is the science of patterns and relationships. As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. The abstractions can be anything from groups of numbers to geometric figures to sets of equations. In addressing, say, "Does the interval between prime numbers form a pattern?" as a theoretical question, mathematicians are interested only in finding a pattern or proving that there is none, but not in what use such knowledge might have. In deriving, for instance, an expression for the change in the surface area of any regular solid as its volume approaches zero, mathematicians have no interest in any correspondence between geometric solids and physical objects in the real world.

What is the Philosophy of Mathematics Education?

The Philosophy of Mathematics Education is the practice of teaching and learning mathematics, as well as the field of scholarly research on this practice. Researchers in mathematics education are primarily concerned with the tools, methods and approaches that facilitate practice or the study of this practice.



Philosophy of Teaching and Learning Mathematics

The teacher is important in classroom learning. The teacher organises. The teacher manages. The teacher motivates. The teacher leads. The teacher creates an environment in which learning occurs. Learning is a function of the quality of the teaching.

Your influence as teacher reaches far beyond the subject matter that you discuss from day to day. As an active learner you inspire pupils to attain knowledge. Through your interactions with pupils you influence moral development. Your behaviour affects the pupils' work ethics. Your quest for personal excellence inspires pupils to strive for higher aspirations. Please note that the person, called the teacher, has a far-reaching influence on the lives of pupils, (Muzumara, 2008).

Mathematics plays an integral role in the lives of every member of our society. Numeracy is essential to each functioning member of our society. The need for mathematical knowledge is increasing as our way of life becomes more saturated with technology and information. Everyone in our educational system must have an opportunity to learn mathematics and to achieve a level of knowledge commensurate with his or her personal potential, needs, and aspirations.

As a teacher you should believe that everyone can learn mathematics. Not all pupils have the same motivation or aptitude for learning mathematics and Science. Not all learn at the same rate. Not all learn in the same way. Not all process information at the same rate. Not all comprehend abstractions with the same level of understanding. But all can learn when you provide the conditions that meet the individual learner's needs.

Learning is a multifaceted process. Learning occurs from visual and audio stimuli. Learning occurs from reading. Learning occurs when writing or communicating orally. Learning occurs from experience. Pupils learn not only from the teacher but from each other. You must acknowledge, and respond to, the fact that learning occurs in many varied, yet equally important, ways.

Activity 1.2

Learning Mathematics like any other subject occurs from ***visual and audio stimuli****. Learning occurs from r*__*eading*__*. Learning occurs when* ***writing or communicating orally****. Learning occurs from* ***experience****. Give at least two examples in each of the stated situations.*

The Nature of Mathematics

The Nature of Mathematics therefore can be summarised as:

- the study of number, form, arrangement and associated relationships using clearly defined literal, numerical and operational symbols;
- science of space and number
- what mathematicians do
- an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality

Use of Mathematics in Every Day Life

Mathematics is universal in a sense that other fields of human thought are not. It finds useful applications in business, industry, music, historical scholarship, politics, sports, medicine, agriculture, engineering, and the social and natural sciences. The relationship between mathematics and the other fields of basic and applied science is especially strong.

Activity 1.3

(a) From your teaching experience give four examples on the relation between Mathematics and other fields of basic and applied sciences.

(b) Explain how Mathematics finds useful applications in business, industry, music, historical scholarship, politics, sports, medicine, agriculture and engineering.

Mathematics in a Classroom

Three things are essential in a good mathematics classroom:

Counter intuitive thinking
Counter intuitive thinking Pupils need experience with counter-intuitive problems to develop logical thinking skills. If you do something that is counter-intuitive, you are doing something that seems like it is the opposite of what common sense would tell you to do.

Manipulative
Manipulatives not only help pupils understand concepts more clearly, they also give concrete examples of how mathematics surrounds us. For instance, baby toys can teach higher level mathematics, sewing patterns are derived from statistics, sharing a bottle of coca cola drink can be used to measure volume, spreading the arms or walking can be used to measure length, Fun examples are everywhere.

Technology
Calculators and computers allow pupils the freedom to explore the beauty of mathematics without the time consuming calculations. Children can understand complicated mathematical ideas even though they cannot yet perform the cumbersome arithmetic that makes those concepts possible.

Mathematical Inquiry
Using mathematics to express ideas or to solve problems involves at least three phases:

- representing some aspects of things abstractly,
- manipulating the abstractions by rules of logic to find new relationships between them,
- seeing whether the new relationships say something useful about the original things.

Abstraction and Symbolic Representation
Mathematical thinking often begins with the process of abstraction—that is, noticing a similarity between two or more objects or events. Aspects that they have in common, whether concrete or hypothetical can be represented by symbols such as numbers, letters, other marks, diagrams, geometrical constructions, or even words. Whole numbers are abstractions that represent the size of sets of objects and events or the order of things within a set. The letter A may be an abstraction for the surface area of objects of any shape. The symbol + represents a process of addition, whether one is adding apples or oranges, hours, or kilometers per hour. Abstractions are made not only from concrete objects or processes; they can also be made from other abstractions, such as kinds of numbers (the even or prime numbers for instance).

Approaches of Teaching Mathematics

The Three Approaches to Teaching Mathematics

Skills	**Conceptual**	**Problem Solving**
Aim: To foster the mastery of basic skills- memorization of arithmetic and geometric facts, rules, formulas, and procedures **Focus:** Procedural *content* (e.g., *how* to divide with fractions **Roles:** Teacher-directed; pupil passive **Organising principle:** Sequential instruction based on hierarchy of skills- build from simplest to most complex skill **Methods:** ·Direct instruction; primarily teacher talk ·Practice, with an emphasis on written worksheets **Instructional Implications:** Teacher needs to imbed process standards and critical thinking, Vertical school design to insure pupils receive instruction for all mathematical areas	**Aim:**To foster meaningful learning- understanding of facts, rules, formulas, and procedures **Focus:** Meaningful *content* (e.g., *why* you invert and multiply when dividing fractions) **Roles:** Teacher-directed; pupil active **Organising Principle:** Sequential instruction usually presented as a spiral curriculum based on the readiness of pupils to construct understanding **Methods:** ·Teacher-directed use of manipulatives, (e.g., linking concrete models to symbolic mathematics ·Discovery-learning activities **Instructional Implications:** Vertical school design to insure pupils receive instruction for all mathematical areas	**Aim:**To foster the development of mathematical thinking **Focus:** *Processes* of mathematical inquiry: problem solving, reasoning, and communicating **Roles:** Teacher-guided; pupil active **Organising Principle:** Posing problems that are neither too simple nor too difficult, whether or not pupils have received formal instruction on the content involved **Methods:** ·Pupil discussion of problems, solution strategies, and answers Content instruction done incidentally as needed **Instructional Implications:** Teacher needs to focus on content standards & supplement curriculum with additional math skills

Adapted from Problem Solving, Reasoning, and Communicating: Helping Children Think Mathematically by Arthur J. Baroody (2002)

Learning through play

Usually teachers tend to dominate discussions, so that a child's ability to grasp certain methods of calculating problems is stifled. Teaching mathematics at foundation and primary level should be fun. Tell stories; give simple puzzles and written or mental games. Relate numeracy to play in your sessions. Use activities for their play to teaching pupils mathematics.

Activity 1.4

Beat the teacher — try this group activity as an alternative to general worksheets. Teachers can ask quite difficult questions covering a range of mathematics topics. If the children get the answer right they get one point. If they get it wrong, the teacher gains a point. The first to score five gets a permanent point on a scoreboard. This is a great way to motivate pupils and to further develop their mathematics skills by recapping on what has been learnt in previous lessons.

Detective trails — this activity adds a physical element to mathematics learning. It involves grade 2 or 3 pupils using mathematics tools like compasses, rulers and string to solve questions set around the school grounds. A great way to take learning out of the classroom, teachers can invent a scenario such as the theft of a school trophy, with children divided into groups according to set questions. Each answer can be used to crack a code to form a word revealing the whereabouts of the 'stolen' item.

Using Information and Communication Technology (ICT)

Information and Communication Technology (ICT) might sound a bit frightening, particularly when the aim is to link it with teaching mathematics at primary level. Using computers to teach, to plan your work, research and refine your work and allow your pupils to use the computer to learn are some of the benefits in ICT. However, when put into the context of DVDs, CD-ROMs, MP3, Videos — all of which come under the ICT umbrella — perhaps some creative thinking can come out of the different types of resources and software that can be used to solve various mathematics problems(Muzumara, 2011).

ICT at primary school level usually covers five main areas:

- learning from feedback
- observing patterns
- exploring data
- teaching the computer (through pupil-designed activities)
- developing visual imagery

Linking ICT with mathematics or any other subject can develop pupils' interest and understanding of ICT in a way that straightforward use of PCs might limit. Special calculators can be used with overhead projectors to teach children the basic key functions as an addition to their methods of using jottings or mental mathematics to find answers to problems.

Reflection

In what way has your study of this unit made you look at the teaching and learning of Mathematics differently?

UNIT 2

COUNTING SYSTEMS

"Young children should not be expected to move too quickly to written recording in mathematics. The forming of figures correctly on paper is a skill that needs to be learned. Until it has been mastered, attempts to carry out written calculation can inhibit the development of mathematical knowledge." Cockcroft (1982).

The above quotation asks to reflect on how you have been teaching Mathematics to children at the primary school level.

Numbers form the basis of all mathematics. The ability of young children to construct for themselves ideas about numbers depends on their acquisition of efficient counting procedures. Therefore, counting should be fully developed in children before they learn more about numbers.

In this unit you will learn about counting (or number) systems. You will also learn about pre-number activities and counting skills in base ten, five and eight.

Learning outcomes

As you study and work through this unit you are expected to:

- explain how numbers started.
- demonstrate the knowledge of the egyptian, babylonian, roman and the hindu-arabic number systems.
- teach pre-number activities.
- use a variety of activities to teach counting skills and number relations in base ten, five, and eight.

How did numbers come about?

In Zambia, before the coming of the missionaries, each society or tribe had its own way of counting. The Lozi people used pegs to 'count' the number of cows or cattle they had. Each animal was tied to a peg. If an existing peg is vacant then one animal is missing. If all the pegs are occupied and there are some animals still without pegs then the extra animals are from another family.

Activity 2.1

(a) Find out from colleagues in your Teachers' Group, other forms of counting in traditional societies in Zambia.

(b) Discuss with members of your Teachers' Group how the traditional counting systems worked, their strengths and weaknesses.

(c) Keep notes of this activity in your activity file.

The Roman system (AD 100)

The Roman numeration system is similar to the Egyptian system except that it used both addition and subtraction e.g. IX means one before ten, which is nine, i.e., 10 -1 = 9. The following are the Roman numerals.

Roman	Present
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

Here are some examples of numbers in Roman numerals.

Present	Roman
3	III
4	IV
9	IX
24	XXIV
537	DXXXVII

Activity 2.2

1.Write the following numerals in Hindu Arabic numeration system
(a) XLIV (b) LXXVI (c) MCMXC (d) MDXXI (e) DCXLVII

2. Write the following numbers in Roman numerals
(a) 28 (b) 84 (c) 867 (d) 1999 (e) 2112

3. What are the principles in the Roman numeration system? Illustrate each principle with an example.

Hindu - Arabic Numeration System

The present day numeration system has its origins from the Hindus in India. The Arabs improved upon it, hence the name Hindu and Arabic.

The present day numeration system has ten basic symbols called digits. These are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Take note of the following points about the present day numeration system.

- All the numbers in the system are constructed from the digits 0 to 9 *(These are also known as cardinals or the basic numbers)*
- The position of a numeral or symbol in a number has a place value with place values based on repeated groupings often.
- There is a representation of zero

Check your understanding of place value by writing the number 2 467 and 9 154 060 using the place values and re-write the number in expanded notation.

Activity 2.3

(a) Write down the advantages and disadvantages of the Babylonian numeration system? You should share your points on this task with your fellow students.

(b) Do you think the Roman system is an improvement to the Egyptians system? Give reasons. Keep your answers in <u>your activity file.</u>

Reflection

It is the beginning of the school year and you have been allocated a grade I or grade V class.

(a) What kind of things will you do to prepare your pupils in readiness to start learning?

(b) What will constitute your first few lessons in Mathematics?

Pre-number Activities

Pre-number (or readiness) activities are given to young children before they begin to learn about number. These activities are intended to prepare them for number work. Pre-number activities include sorting, matching and ordering objects.

All kinds of objects can be used for these activities, objects like stones, empty tins, pencils, cups, and mathematical shapes like circles, triangles, rectangles and squares. Given below are some examples of these activities:

Examples:

Sorting

Different sets of objects are mixed. Children are asked to separate them by putting the same objects together. The 'sameness' can be by colour, shape or size as shown in fig. 2.1

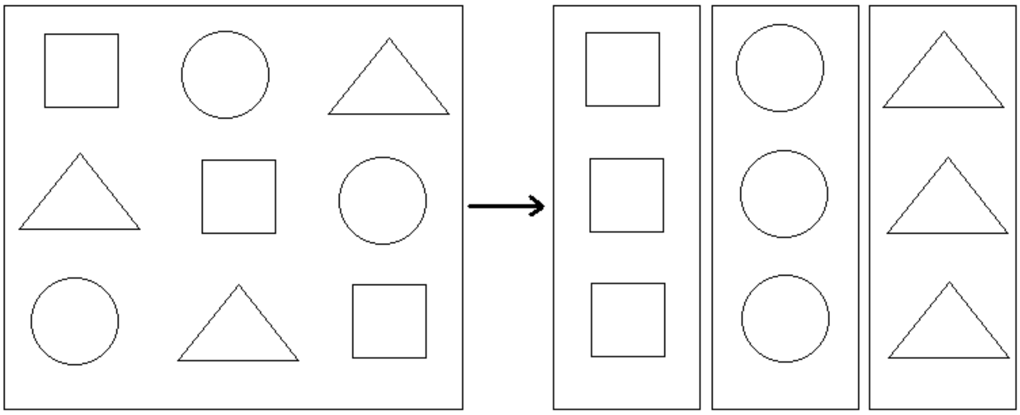


Fig 2.1

Matching

Objects in one set are matched to objects in another set in a one-to-one correspondence.

Put three cups (or tins) on the table in your classroom. Put three pencils on the other side of the table. Ask pupils to take one pencil and put it in one cup until each cup contains one pencil. Ask pupils questions like:
'Are there any pencils left?' 'Are there any cups without pencils?'

In this case the number of cups is equal to the number of pencils.

Later you may use semi-concrete objects like charts, pictures or drawings like the one in fig. 2.2

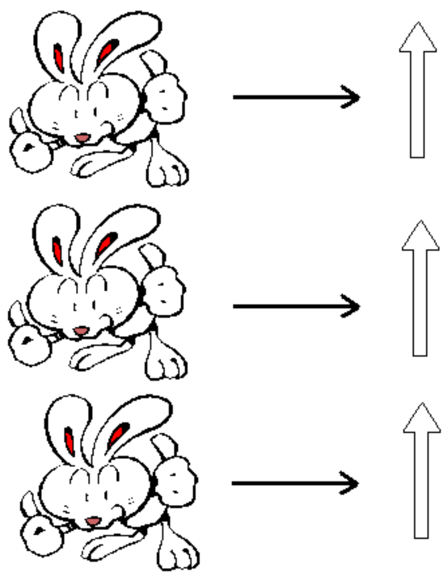
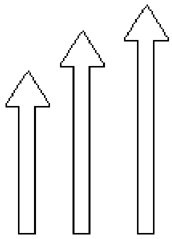


Fig 2.2

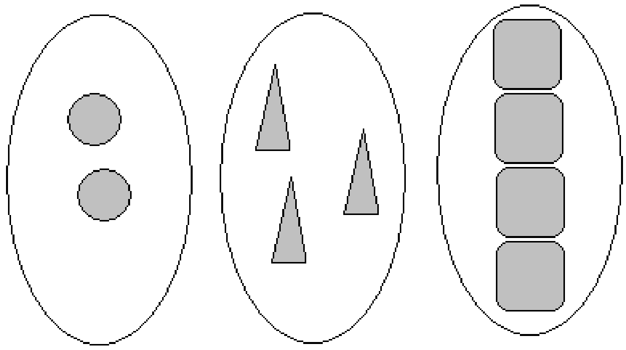
Ordering

In ordering activities, pupils are asked to arrange objects according to size, either from shortest to longest (Fig. 2.3)or smallest to largest. Another form of ordering can be according to the number of elements in given sets (Fig. 2.4)



Objects arranged
according to length

Fig 2.3



Objects arranged according
to number of elements in the sets

Fig 2.4

Do Activity 2.4 for your practice with pupils in class.

Activity 2.4

Prepare a lesson on pre-number activities and teach it to a suitable class. Note down your experiences with pupils in this lesson and keep the notes in your teaching file.

Meaning of Counting Numbers

Another very important aspect of counting is the meaning attached to it. Children should not only be able to count, but they should also know what counting numbers tell.

When you count a set of objects, the number word of the last object counted is the name of the ***'manyness'*** in the set or ***cardinality*** of the set. A class of 40 pupils has cardinality 40. If there are 25 girls in this class, then the cardinality of girls is 25. Therefore, the last number word represents both the count and the manyness of the objects being counted.

Children have acquired the idea of cardinality if, given a set of objects and asked 'how many?', they are able to say the last count.
The set of activities 2.5 will help you to develop the idea of cardinality in your pupils.

Activity 2.5

Prepare a lesson based on the following activities and teach it to a suitable class. Note down all your experiences with pupils in the lesson and keep them together with your lesson plan on your teaching file.

1. With your pupils, count several sets with the same number of objects and discuss how they are alike and different.
2. Ask pupils to make and then count out sets that have a specified number of objects.
3. Ask pupils to count members in a set. Then re-arrange the set and again ask ‘how many they are now’. If they see no need to recount, then the connection is probably made. Discuss with them why they are not recounting If they choose to recount, ask them why they are recounting.
4. Give your pupils a number of different sets and ask them to find pairs that have the same number of members. The sets can be drawn on cards with different arrangements and different objects. A large number of such cards with sets ranging from 4 to 12 objects provides a good activity for four to five pupils at a table or on the floor.

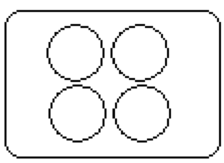
Number Relations

The words ‘more, same, less’ describe basic relationships that contribute significantly to the overall understanding of number by pupils. The following activities will help pupils to develop number relations.

Activity 2.6

1 (a). This is a group activity of about six groups. For each group prepare the following:

(i) 9 cards (on pieces of paper or cardboard) with sets of 4 to 12 objects drawn on them; Take one card and draw 4 circles on it as shown below



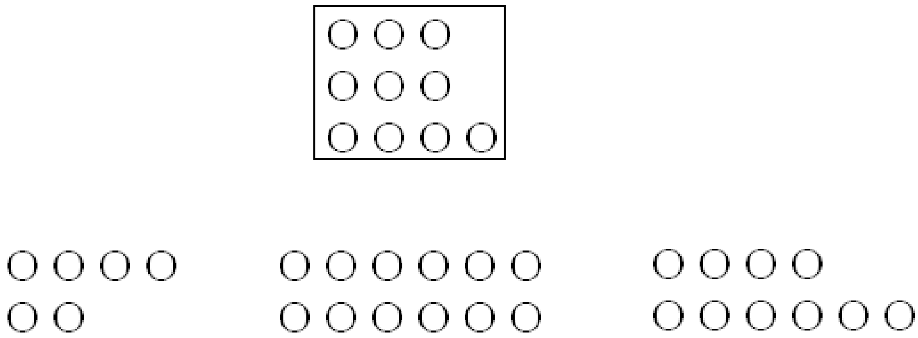
Take a second card and draw 5 circles, 6 on the next and so on until when you have drawn on all the 9 cards.

(ii) A set of counters (bottle tops, straws, seeds, etc.)

(iii) 3 (three) word cards labeled **MORE, SAME, LESS**.

(b) Divide your class say into six groups, one group sitting around a table another sitting on a set of desks drawn together the other sitting on the floor and so on.

(c) For each card with circles, pupils make three collections of counters: a set that is **MORE,** one that is **LESS** and one that is **SAME.** Pupils should then place appropriate labels for the sets they have collected as shown in Fig 2.5



less **more** **same**

Fig. 2.5

Introducing Zero (0)

There are many ways in which you can introduce zero to your class. For example, ask your pupils this question ‘How many of you have three hands?’

They will answer ‘no one’ or ‘none’. Now write the symbol ‘0’ to represent ‘no one’ or ‘none’ or nothing.

Activity 2.7

Suggest two other ways of introducing zero. Write down your suggestions and put them in your activity file.

Patterns

Number patterns play an important role in developing number relations such as odd, even and triangular numbers, just to mention a few. Discovering number patterns is one of the vital strategies to problem-solving.

When developing the concept of number patterns in early stages, the patterns for numbers 1, 2, 3, 4, 5. can be designed as in Fig 2.6. For example, in the patterns for the number 5 in figure 2.6 the middle pattern brings out the relation that 5 is made up of 2 and 3, the first one tells you that 5 is made up of I and 4. The last pattern simply brings out a recognisable pattern for 5.

You may introduce number patterns by giving each pupil say about five counters or stones. Show the class one pattern of five dots with number 5 written on it. Ask the following questions “How many dots did you see? How are they arranged?” Tell them to make the pattern they had seen. Show them the other patterns of five and each time giving them time to make the patterns. Do this with other numbers. See fig. 2 .6 below.

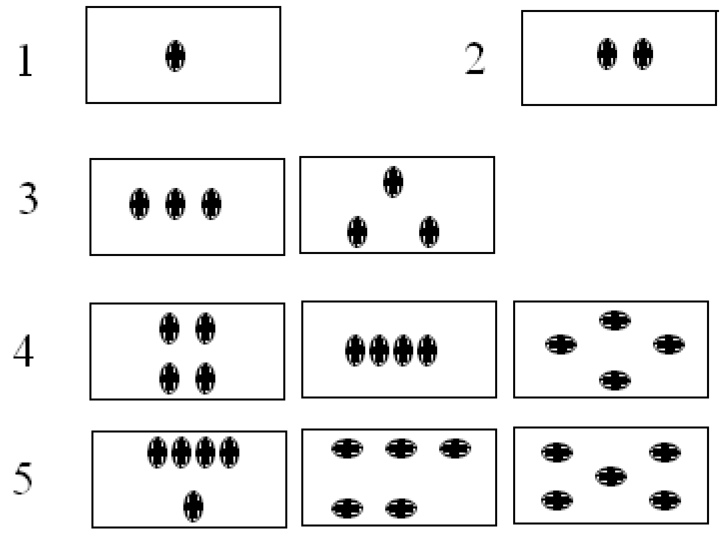


Fig. 2.6

 Activity 2.8

(a) Make different patterns for the numbers 6, 7, 8, 9 and 10 similar to those in fig. 2.9, as follows:
For 6, make 4 patterns; for 7 make 4; for 8 make 5; for 9 make 4; and for 10 make 3 patterns.

(b) Ask your pupils to make patterns for the numbers 6, 7, 8, 9, and 10, as you did yourself in (a).

(c) Show pupils materials from the local environment where patterns have been used.

Making patterns with mathematical shapes also adds fun to children's learning of mathematics. Fig 2.10 shows some patterns that can be made out of a circle.

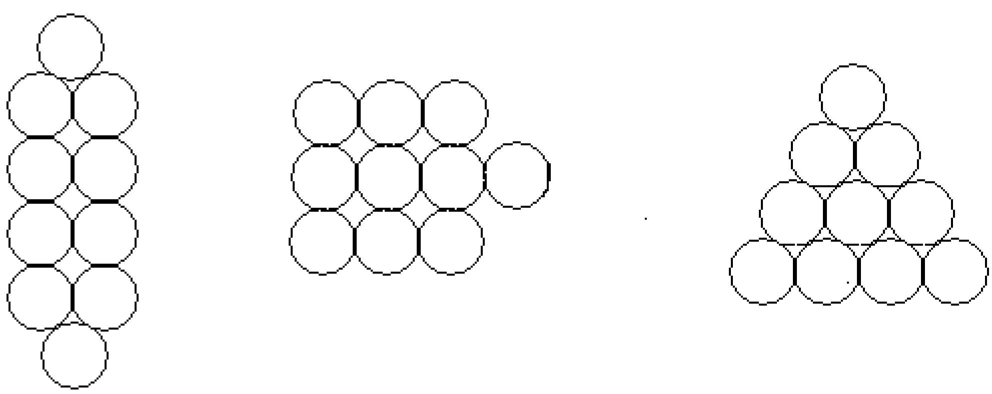


Fig. 2.10

(d) Give pupils at middle basic a homework of finding a situation in their environment that portrays a pattern. Tell them to use illustrations or explanations.
Note: If it is not possible for the pupils to find number patterns in their environment then allow them to identify behaviour patterns or phenomenon patterns or to create number patterns.

Place value

In the previous section we learnt that the Hindu-Arabic system proved superior to the other systems because of the use of the place value, and the use of zero as a placeholder. These made it possible for numbers of any size to be written using only the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. In this section you will learn about place value and the strategies to teach it.
You already know that the place values that we use when writing numbers are ones or units, tens, hundreds, thousands, ten thousands, hundred thousands, millions, etc. What does each digit in the number 2 675 567 stand for?

Place value models
There are a number of place value models that you can use to develop the concept of place value. One type is known as *proportional models (Fig 2.13 to Fig 2.15).*

In these models the Ten model is ten times larger than the model for Ones (units), and the Hundred model is ten times larger than the Ten model.
These are shown below.

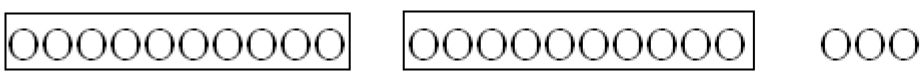


Fig 2.12 Counters or bottle tops (showing 23)

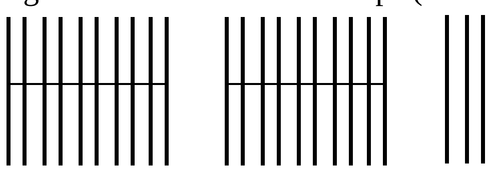


Fig. 2.13 Bundles of straws showing the number 23.

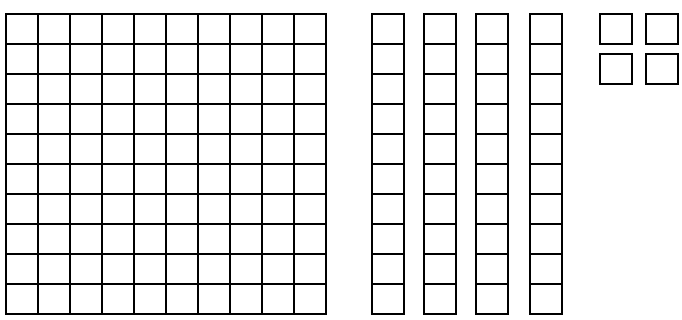
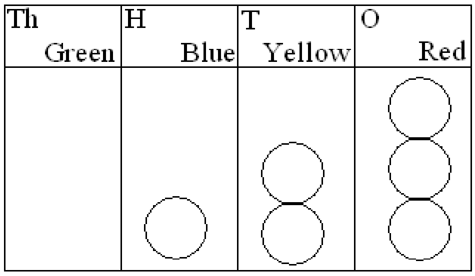


Fig.2.14

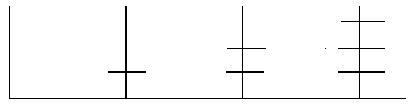
One hundred Forty four

Now, place value cards showing the number 144.

Fig 2.15 below shows another type of model known as *non-proportional models.*



Showing 123



Abacus showing 123

Fig. 2.15

In non-proportional models for base ten, the models for ones, tens, hundreds, etc., are not proportional as on an abacus.

The Abacus

An abacus is a frame of rows with counters used for counting and is also used to demonstrate place value.

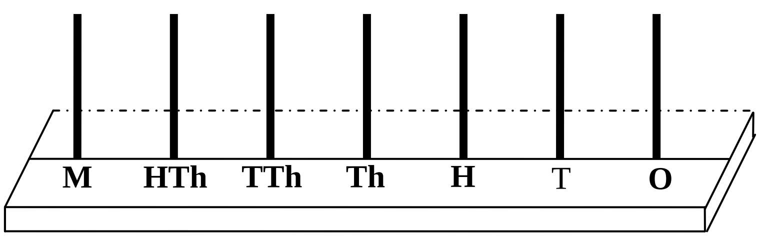
In this section you should make abacuses from the local materials that are available. Your District Resource Centre may have some.

Teaching tips

(a) You need the following materials; a strip of wooden board, about 1 cm thick 4cm wide and of length convenient to the grade level. For instance, a four digit number will require four (4) sticks or wire rods or any other local material, of uniform size of about 15cm long, 40 counters or bottle tops or whatever you can use from your local environment. The maximum number to be an represented on an abacus should be 9 999 999

Leave 2 cm. at the end of either side of the wooden strip. Fix the rods in a straight line about 4cm apart. Mark the columns O (one), T (Tens), H (Hundreds) and Th (Thousands); TTh (Ten Thousands), HTh (Hundred Thousand) and M (Million).depending on the place value of the grade level.

For example

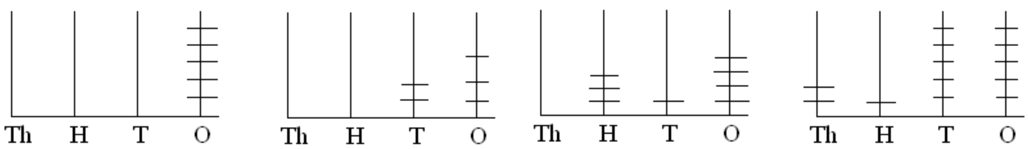


(b) Ask pupils to make abacuses that suit their grade level (5 pupils per abacus). You will need to help pupils with this.

Use of the abacus

Each abacus should have 10 counters per stick or rod . Attach an equal number of pupils to each abacus. Keep one for yourself to use to demonstrate. Start by placing l0 counters one after another on the peg of ones, counting each counter with pupils from 1 up to 10. When you get to 10 counters on the peg of ones you exchange (or trade) them for one counter on the peg of tens. Continue with the process of counting and trading or exchanging until you have 10 counters on the peg for tens that you should then trade (exchange) for one counter on the peg for hundreds. Give pupils enough practice so that they become familiar with the use of the abacus before introducing them to direct presentation of numbers.

Example
Represent the following numbers on an abacus: 5, 23, 314, and 2 155.



Emphasise what each of the numerals in the numbers stand for. Give enough exercise for pupils to practice especially numbers like 102, 406, 203, 460,2011, 3 090 512, etc.

Other Number Bases

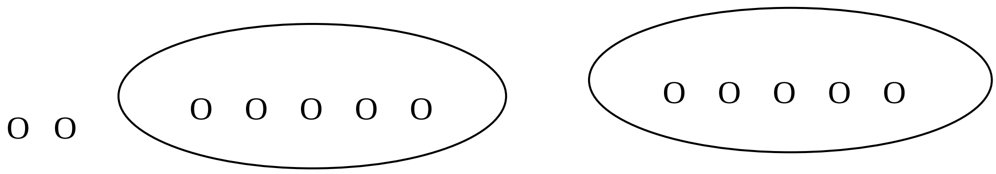
Pupils need to be familiar with numbers in base ten, before they can be introduced to other number bases such as base five and eight. Introduction of such bases would enrich pupils' knowledge of numbers.

There are other bases, which are used, in different situations. For example base sixty, twelve and twenty-four are used in time, base two in computer technology and base five in some Zambian counting systems.

Remember to introduce base five using concrete (real objects) and semi-concrete (pictures) approach.

Base Five

Consider the issue of counting stones in base five. Let pupils count each stone and recognise the sets of five as they count. For example 1_{five}, 2_{five}, 3_{five}, 4_{five}, 10_{five}, 11_{five}, 12_{five}, 13_{five}, 14_{five}, 20_{five}, 21_{five}, 22_{five},



From the above diagrammatic illustration you realise that the number of stones is 22_{five}. Give pupils enough practice until they are able to count with ease in the base.

Summary

In this unit you have learned that:

- the idea of keeping records and of communicating quantities gave rise to numbers and numerals
- numerals are symbols, which represent numbers, e.g. iii, 111, ∇∇∇, 3 are different numerals representing the number three. these can also be referred to as digits.
- early numeration systems had many different deficiencies. the notable ones are lack of place value and a numeral for zero.
- pre-number activities are very important in the teaching of mathematics
- making use of materials from the local environment contributes to pupils' meaningful learning
- number patterns promote creativity
- other number bases enrich and extend the counting systems.

UNIT 3

MULTIPLICATION AND DIVISION

This unit discusses of teaching of multiplication and division to children in lower grades.

Learning Outcomes

As you study and work through this unit you are expected to:
- translate repeated addition into multiplication.
- translate repeated subtraction as division.
- formulate multiplication and division word problems.
- make models of multiplication and division word problems and translate them into symbols for calculation.
- design a variety of activities to teach multiplication and division to lower and middle basic school pupils.

Reflection

Think of how you have been introducing the multiplication and division operations to lower and middle basic school children. How has it been; easy or hard? Keep this in mind as you begin to study through this unit and see whether there will be a difference between what you have been doing and what you are going to learn.

Multiplication

You introduce multiplication through a series of word problems, which give rise to repeated addition that should be expressed as a product. For example, bring to class 4 red, 4 white and 4 green pieces of chalk and line them up as follows:

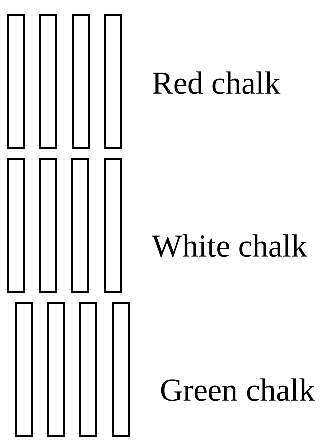


Fig. 3.1

Now ask pupils how many pieces of chalk you have. They will count and say 12. Count with them the 12 pieces of chalk. Then draw the model on the chalkboard as shown in figure 3.1. Explain to pupils that you find the total number of pieces of chalk by adding them up as follows:

$$4 + 4 + 4 = 12$$

Now ask pupils to find the total number of pieces chalk if you have 3 red, 3 white, 3 green, 3 yellow, 3 blue, 3 orange and 3 purple pieces of chalk.

Discuss with them that this gives the total number of pieces of chalk as:

$$3 + 3 + 3 + 3 + 3 + 3 + 3 = 21$$

Ask them what is happening to the sum as the number of things to be added increases. You will notice that the sum increases. We should therefore, find a shorter way of writing and finding this sum.

Tell them that 4 is added 3 times, or in short we say 3 times 4 is 12. Explain to them that this sum is written in short form as a multiplication as: $3 \times 4 = 12$

Therefore, $3 \times 4 = 4 + 4 + 4 = 12$

In the same way, $7 \times 3 = 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21$

Draw the grid of squares on the chalkboard as shown in figure 3.2.

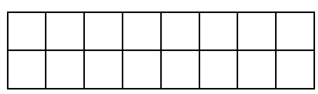


Fig. 3.2

There are 2 rows of 8 squares in the grid. Therefore the total number of squares is $8 + 8 = 2 \times 8 = 16$.

We can also say that there are 8 columns of 2 squares, which gives us

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 8 \times 2 = 16$.

This shows us that $2 \times 8 = 8 \times 2 = 16$.

Activity 3.1

1. Prepare a lesson to introduce multiplication to a lower and middle basic class. Your lesson should include a variety of activities like:

(a) Draw pictures of different objects on the board and ask pupils to write down the products

For example, ask pupils to write down the product for the pictures given in figure 3.3:

Fig. 3.3

The number of pictures in the diagram is $2 \times 4 = 8$ or $4 \times 2 = 8$.

(b) Making multiplication word problems.

2. Teach the lesson and record your experiences, which you should keep in your teaching file.

Multiplication Facts

Multiplication facts should be taught in relation to pupils’ existing knowledge. Therefore, once pupils have grasped the idea of multiplication as repeated addition, multiplication facts can now be consolidated.

For example, $8 + 8 + 8 = 3 \times 8$ is 24; $7 + 7 = 2 \times 7$ is double seven and is 14.

A good number of activities for addition can be modified for multiplication.

Activity 3.2

(a) Design some activities to teach multiplication facts to lower or middle basic pupils. You may refer to the addition facts. Try them in class.

(b) Investigate and write down how you would help pupils to master the facts involving multiplication by 9. Keep your work in your Activity file.

Activity 3.3

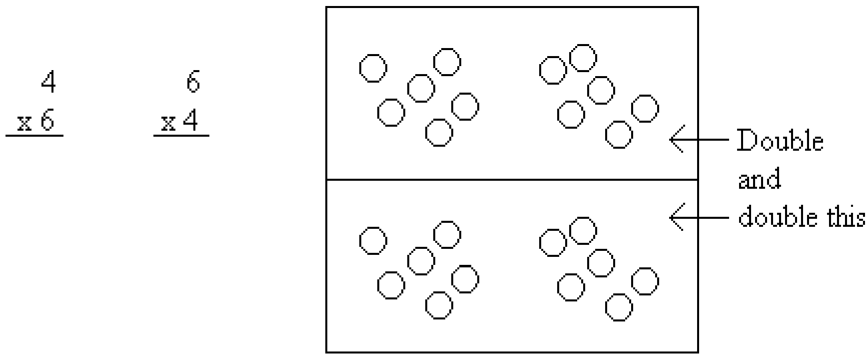
1. Create 10 word problems on multiplication and keep them in your teaching file. For example, Mutumba has 4 bags of oranges and there are 3 oranges in each bag. How many oranges has Mutumba?

2. Prepare a lesson based on these word problems and teach the lesson.

Children need to understand the basic multiplication facts before they carry out complicated multiplication algorithms. You should systematically build these through the multiplication of the basic numbers from 0 to 9.

Facts with 4

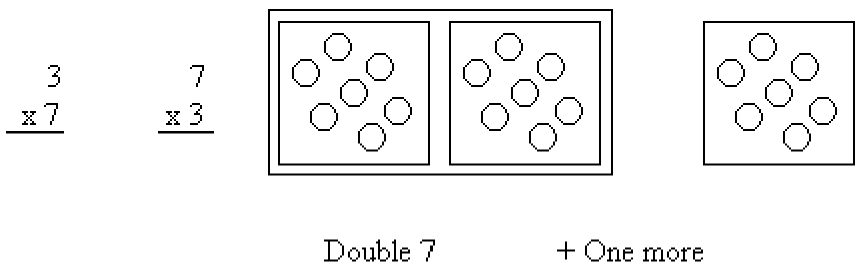
This is a strategy to double and double again:



Double 6 is 12
Double again is 24

Facts with a 3

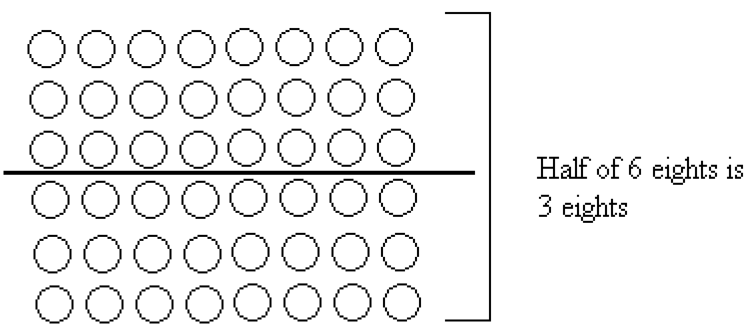
This is a double and one more set:



Facts with an even factor

$$\begin{array}{r} 6 \\ \times \underline{8} \end{array} \qquad \begin{array}{r} 8 \\ \times \underline{6} \end{array}$$

The strategy is to halve, then double



3 times 8 is 24

Double 24 is 48

Activity 3.4

It is expected that at Grade 4 level pupils have developed the concept of multiplication as repeated addition and have mastered the multiplication facts to an extent that they are now able to carry out the basic multiplications mentally.
However, the learning of the mental skills should be an ongoing process and should therefore be revisited whenever necessary for further consolidation.

Prepare separate lessons to teach your class mental skills for learning each of the following multiplication facts:

- Facts with 4, (ii) Facts with 3, (iii) Facts with an even factor, (iv) Any fact

Division

You introduce division using the idea of sharing or grouping. Give pupils a number of stones, bottle tops, and some way to sort them out into groups by specifying the total number of the objects and the number of groups to be formed. Let the pupils report orally like "we separated 10 mangoes into ***2*** groups". We have 5 mangoes in each group". We shared 12 books among 3 pupils and each has 4 books.

Developing division as repeated subtraction
From the sharing activities you can develop the division operation as repeated subtraction as follows:

If 12 books are shared among 3 pupils and each one gets 4 books, then this means that we take away 4 books and give them to the first pupil and 8 books are remaining; then take away 4 more books and give them to the second pupil and 4 books are remaining to be given to the third pupil. That is:

$12 - 4 = 8$
$8 - 4 = 4$
$4 - 4 = 0$

Repeated subtraction of 4 from 12 (this 3 times) means $12 \div 3 = 4$
Vary your activities For instance, if you gave pupils 18 stones, you may initially ask them to sort them out equally into 2 sets, then 3 sets, 6 sets and 9 sets, of equal number of members in each case.

These will lead to the following divisions:

$18 \div 2 = 9$ $\quad$ $18 \div 3 = 6$ $\quad$ and so on.

After the pupils have understood equal partitioning of objects then introduce the word division and its symbol. Emphasise the meaning that $12 \div 3$ means partition 12 objects equally in three groups.

You can also introduce division as the opposite of multiplication by using division as ***'Think multiplication'.*** Most of the activities that were used under multiplication could be used in division.

In your lessons start with division of one-digit number by one-digit number. When you have consolidated this concept move on to division of two-digit number by a one-digit number. Then do division of a two-digit number by a two-digit number.

Activity 3.5

(a) Translate the symbols given to models then words.
   (i) $15 \div 5 = 3$
   (ii) $4 \times 3 = 12$

(b) Prepare translation questions for your class involving models, words and symbols.

Long Division

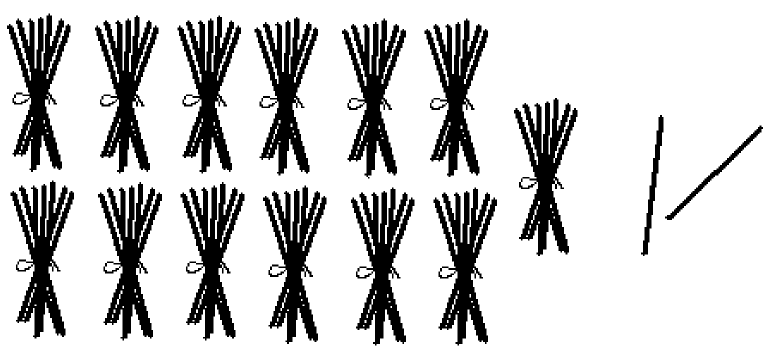
Firstly, you are going to divide 132 by 12, then by 13 using concrete objects before looking at long division.

Materials you need for these activities

- ✓ 132 short sticks
- ✓ strings

What to do

First make 13 bundles of 10 sticks each, with 2 loose sticks



Then make 1 bundle of 10 bundles of 10 sticks each, 1 bundle of 3 bundles of 10 sticks each, and 2 loose sticks.

Hundreds	**Tens**	**Ones**
1	3	2

You are going to divide 132, first by 12, i.e. 132 ÷ 12,
then by 13, i.e. 132 ÷ 13.

1. 132 ÷ 12

This is the same as putting the 132 sticks into 12 bundles of sticks. How many sticks are there in each group?

To answer this question, do the groupings, as follows:

*There is only one bundle of 100 sticks. So we cannot form 12 bundles of 100 sticks. What do you do with this bundle to enable you to form 12 bundles?

*Unbundle the 100 sticks into 10 bundles of 10 sticks. Add the three bundles of ten sticks in the tens place value. How many bundles of 10 sticks do you get?

Ans: 13 bundles of 10 sticks.

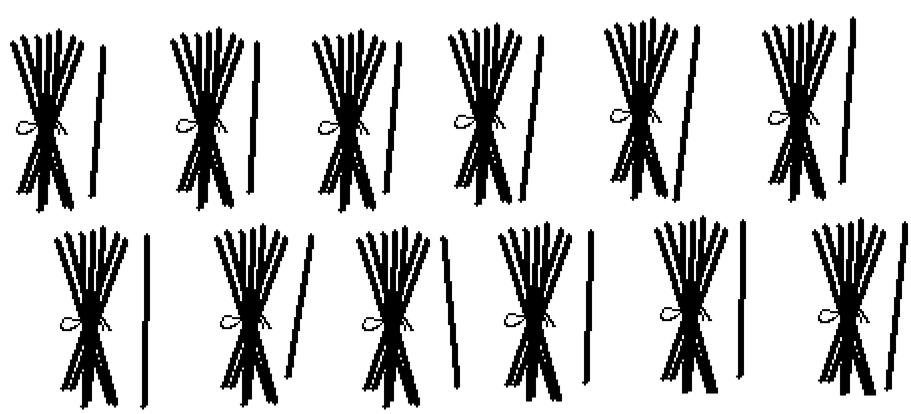
*You can now form 12 bundles of 10 sticks and you shall remain with one

*Unbundle the remaining 1 bundle of 10 sticks and add the 2 sticks in the ones place value. How many sticks do you get?

Ans: 12 sticks.

*You can add one stick to each of the 12 bundles. How many sticks do you finally get in each of the 12 bundles?

Ans: 11 sticks



12 bundles of 11 sticks each
132 ÷ 12 =11

- This activity has given you the answer to 132 ÷ 12 as 11.
- You can now carry out this calculation (without the sticks), using long division, as follows:

In long division, you do the calculation of 132 ÷ 12 as follows:

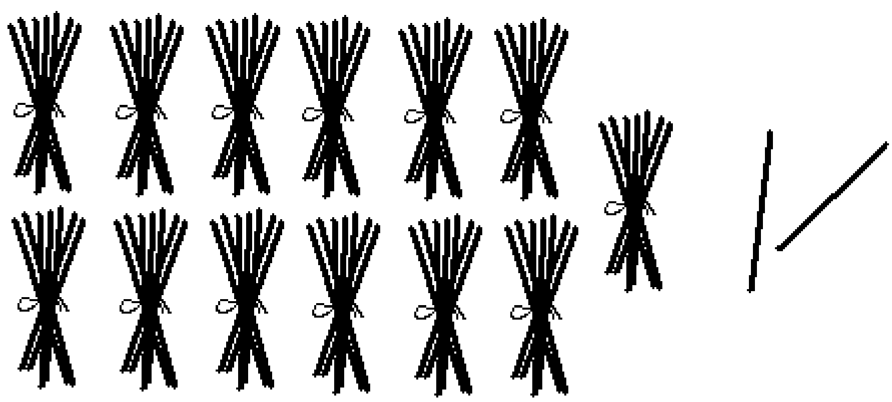
```
         11
    12 | 132      1 divided by 12. Can't
       - 12       13 divided by 12 is 1 (once)
          12      1 x 12 is 12
         -12      plus 2 is 12
           0      (once)
```

13 - 12 is 1. 1ten
12 ÷ 12 is 1
1 x 12 is 12
12 - 12 is 0.

Activity 3.6

1. Devise an activity for your pupils to carry out division of a 3-digit number by a 2-digit number, using sticks, or any other suitable material.
2. Prepare some examples on long division that are suitable for each of the Grade levels 1 to 4. Use these examples in your teaching

You can now do 132 ÷ 13, using your sticks, following the same steps as in the first example.



This activity gives you the answer to 132 ÷ 13 as 10 remainder 2

Here is the layout of the long division algorithm:

```
          10 r 2
    13 | 132
       - 13
          02
         - 0
           2
```

1 ÷ 13 can't
13 ÷ 13 is 1 (once)

1 x 13 is 13

13 – 13 is 0

2 ÷ 13 is 0

0 x 13 is 0

2 – 0 is 2

Remainder 2

Mental Skills in Division

Division facts are closely tied to multiplication. So you may approach division from multiplication like 72 ÷ 8. “What number is multiplied by 8 to get 72”?

In division mental and paper computations go well side by side because in both we move from left to right.

Example: 1

684 ÷ 6 is done by dividing 600 by 6 is 100 and then 84 **÷ 6** is 14 then the quotient is 100 + 14 = 114.

Example: 2

24000 ÷ 6: what number do we multiply by 6 to get 24? 6 × 4 is 24. Then quotient is 4000 or 24000 ÷ 6 is 4000.

63000 ÷ 7: think 63 ÷ 7 is 9. So quotient is 9000

When both the divisor and dividend have trailing zeros rely on the base 10-product language of multiplication. e.g.

(i) 36000 ÷ 40: think 4 × 9 is 36, then 9000 ÷ 10 = 900**.** 0r 36000 ÷ 4 = 900

(ii) 189 ÷ 3: think 180 ÷ 3 is 60 and 9 ÷ 3 is 3. Answer is 60 + 3 = 63

(iii) 4836 ÷ 6: think 4800 ÷ 6 is 800 and 36 ÷ 6 is 6. Thus quotient is 806

You may find the following exercise useful for both mental division and multiplication. Find appropriate factors for given products.

Find two factors with product 36

1 12 6 9 3

18 4 2

Find two factors for each product

$\boxed{150}$ $\boxed{84\ 000}$ $\boxed{249}$ $\boxed{15\ 000}$

60 50 140

3 83 30 600

300 5

Activity 3.7

Make a lesson plan or lesson plans to teach mental division to an appropriate grade or grades in the lower basic. Try out these lessons with pupils in class. Take note of your experiences with pupils and keep the record in your teaching file together with the lesson plans.

Reflection

Now cast your mind to what you have learned in this unit. Note down in your diary those aspects in this unit which you feel have contributed to your competency in teaching multiplication and division. Write down what you think should have been included in the unit in order to strengthen it.

UNIT 4

FRACTIONS – PARTS OF A WHOLE

In the previous units we dealt with whole numbers and their properties. But whole numbers alone cannot help us solve mathematical problems which involve parts of a whole. As such, we need to extend number systems to fractions. The content of this unit will be confined mainly to the basic ideas on four types of fractions. These fractions are common fractions, decimal fractions, percentages and ratios that are required for the foundation years of lower and middle basic education.

Learning outcomes

As you study and work through this unit, you are expected to:

- identify different types of fractions,
- demonstrate the meaning of fractions using various models,
- compare fractions using different methods,
- work out problems of addition, subtraction, multiplication and division of fractions,
- work out problems with decimals,
- use ratio and proportion in solving simple problem,
- apply percentage to commercial arithmetic,

Reflection

Think of the ways you have been teaching fractions. Is it one of the topics that you have been finding difficult to teach? Jot down in your diary the topics under fractions in which you have been experiencing difficulties to teach. Keep them in mind and check at the end of your study of this unit if your difficulties will have been resolved.

A part of a whole

The following activities will help you to introduce the idea of fractions to your class. Make a lesson plan based on these activities and teach the lesson. Record your experiences with pupils in this lesson and keep them in your teaching file together with the lesson plan.

Activities 4.1

1. Show the class an orange, a bar of soap or a loaf of bread. Ask the pupils the number of items they see. Explain to the class that when there is one of something, and it is complete, it is called a one whole. Take a bar of soap that is whole; break a piece off and ask the class if it is still a whole bar of soap. Ask why not?

2. Ask pupils to bring mangoes, beans or bottle tops to share. Let pupils choose whom to share with. Tell them to share equally what they have brought. For a pair of pupils, each child will get one part out of two equal parts. We denote 1 part out of 2 equal parts by $\frac{1}{2}$ which is read as one half. Similarly for four pupils, each pupil will get 1 part out of 4 equal parts, which is read as one fourth or one quarter and written as $\frac{1}{4}$

Continue introducing the fractions as per sizes of the groups. For these sharing activities ensure that pupils use a variety of objects.

The first goal in the development of fractions is to help children construct the idea of fractional parts like halves, thirds, fourths, fifths and so on.

For any fractional part there are two requirements;

- there must be the correct number of parts making up the whole.
- each of the parts must be the same size (not necessarily the same shape).

Remember that fractions refer to a part of a group of things like two mangoes out of five mangoes, as well as to a part of a single whole like an orange into parts.

Activity 4.2

Look at the creative halves below.

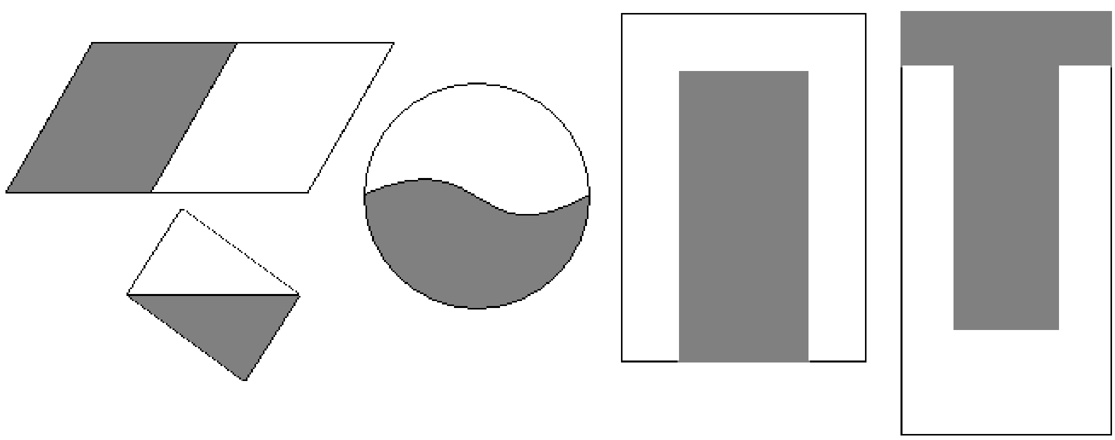


Figure 4.1

Which shapes represent halves that are not necessarily the same shape? How do you know?

Activity 4.3

1. Design practical activities for the pupils to cut whole things into 2, 3, 4, 5, 6, 7, 8, 9 and 10 equal parts. Let pupils use a variety of shapes to cut the same number of equal parts for example, cut a triangle, circle and rectangle. Let them colour or shade one part of each to show the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and so on up to $\frac{1}{10}$.
2. Make displays of fractions using both concrete and semi-concrete materials. Display them in the classroom.
3. Ask pupils to draw diagrams of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and pin them up the classroom.
4. Design your own creative halves and give them to your pupils to work out which ones are halves and which are not.
5. Plan and teach a lesson based on these activities. Record your experiences with pupils in the lesson and keep them in your teaching file.

Common Fractions

Common fractions refer to proper fractions, improper fractions and mixed numbers.

Proper Fractions

From the previous activities, we can say that a fraction is the number of parts selected out of the total number of equal parts. The fraction $\frac{2}{5}$ stands for two selected parts out of five equal parts. We can also say that a fraction is a group of objects selected from the total number of objects.

The number in the numerator is smaller than the number in the denominator. Such fractions are called ***proper fractions.***

Improper Fractions and Mixed Numbers.

When you are introducing improper fractions and mixed numbers, you may proceed as follows:

- Ask the class how many halves make a whole and how many thirds make a whole. Ask them what they would have if they had 3 halves.

- Get two oranges of the same size and cut them into halves. Give three halves to a pupil and ask what fraction of the oranges s/he has. You may get answers like $\frac{3}{2}$, $\frac{3}{4}$ etc.

- Let the pupils try to put the pieces back together and ask them their observation. Explain that there are 3 pieces out of two oranges and if we put the pieces together we will have one whole orange and one half of an orange,

  thus $\frac{3}{2} = 1\frac{1}{2}$

- Ask what $\frac{4}{3}$ is. This is of course one whole and one third or $1\frac{1}{3}$

When the numerator is bigger than the denominator, the fraction is called an improper fraction. When a fraction is made up of a whole number and a common fraction, the fraction is called a mixed number.

You notice that improper fractions and mixed numbers are just two ways of writing the same thing.

Examples

(a) $\frac{5}{2} = 2\frac{1}{2}$ (b) $\frac{13}{4} = 3\frac{1}{4}$

Before you continue, do activity 4.5. Discuss with a friend how to convert (change) an improper fraction to a mixed number and vice-versa.

Activity 4.4

1. Convert the following improper fractions to mixed numbers

(a) $15/2$ (b) $17/5$ (c) $23/8$ (d $52/7$ (e) $33/12$

2. Convert the following mixed numbers to improper fractions

(a) $3\,{}^1/_3$ (b) $4\,{}^3/_5$ (c) $5\,{}^1/_7$

Comparing Fractions

It is not until pupils begin to compare what they think about the relative size of fractional parts. There are various approaches you can use to compare two fractions. Before pupils are introduced to rules such as finding the common denominator or cross multiplication they should be given opportunities to develop understanding of relative sizes of various fractions.

Do you know how to tell which fraction is bigger or smaller than the other?

Do activity 4.6 before you continue.

Activity 4.5

Which fraction in each pair below is greater? Give one or more reasons.

[4/7, 3/7] [3/8, 3/5] [1/4, *2/3*] [3/4, 4/5].

Compare the reasons you have given with the ideas given below.

Ways of Comparing Fractions

- More of the same size parts: since each part in $\frac{4}{7}$ and $\frac{3}{7}$ is of the same size, then 4 parts are greater than 3 parts therefore $\frac{4}{7} > \frac{3}{7}$.
- Same number of parts, but parts are of different sizes: $\frac{3}{8}$ and $\frac{3}{5}$ have the same number of parts but of different sizes. The greater the denominator the smaller are the equal parts. Since $\frac{3}{8}$ has a greater denominator therefore $\frac{3}{8}$ > $\frac{3}{5}$.

Errors Children Make in Comparing Fractions

Children have strong ideas about which numbers are bigger. These ideas may cause them difficulties with the relative size of fractions. Basically larger numbers mean more. The tendency is to transfer this whole number concept to fractions incorrectly, for instance since 6 is more than 2, 1/6 is taken to be greater than 1/2

Equivalent Fractions

You can use the fraction chart in figure 5.2 below to compare fractions and to identify equivalent fractions.

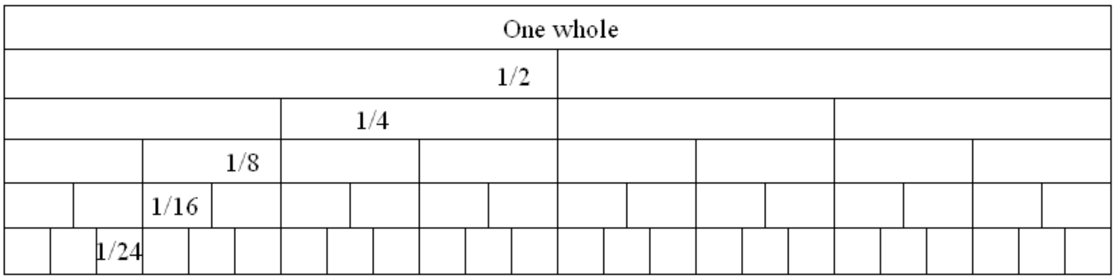


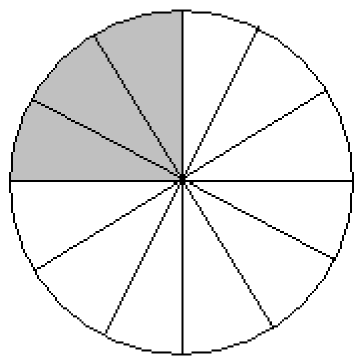
Fig 4.2

Use the chart to determine which fractions are equivalent to:

a) 1/2 b) 1/4 c) 9/24

Two or more fractions are equivalent if they represent the same amount.

Draw a circle on the chalkboard and divide it into, say 12 equal parts. Shade 3 of them, as shown in the diagram on the right. It is possible to give more than one answer to the fraction of the un-shaded area. You could say



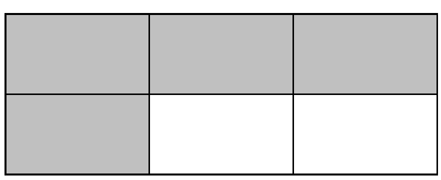
3/4 or 6/8 or 9/12 or …..

Before you continue, do Activity 4.7 given below:

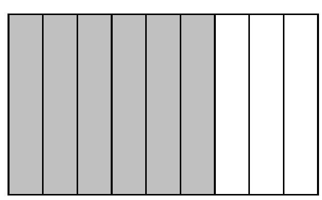
Activity 4.6

1. What fraction in the diagram below is shaded?

2. The diagrams below represent equivalent fractions to 2/3. Write them down.

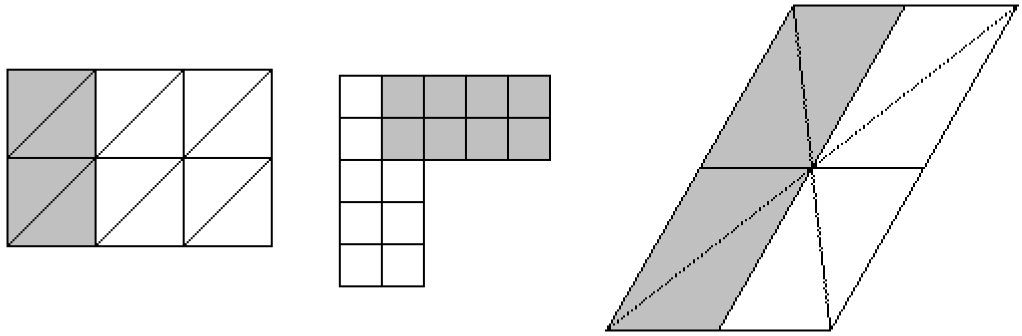


(i)



(ii)

3. Find three fractions that describe the shaded part of each of the following diagrams.



(i) (ii) (iii)

You should now make a lesson plan based on the set of activities 4.8 given below to teach to an appropriate class. Take notes of your experiences with the pupils and keep them on your teaching file together with the lesson plan.

Activity 4.7

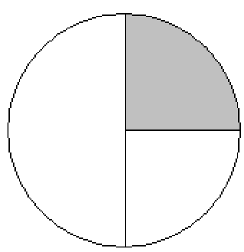
1. Ask pupils to fold a sheet of paper into half and colour one half of the paper. Write down the fraction. Refold and fold one more time. Before opening, ask the class how many sections will be in the whole sheet and how many of these will be coloured. Then open the sheet of paper and discuss what fraction can now be given to the shaded region. Is it still the same? Why? Repeat until the paper can no longer be folded.

2. Repeat activity 4.7 with your class.

Before you continue, do activity 4.9 given below;

Activity 4.8

A grade 3 class was given an exercise on fractions. Mutinta's answers to the exercise are given below. Explain her reasoning for the answers she chose.



Question '1: Write a fraction representing the shaded part in the diagram above.
Mutinta's answer: (1/3)

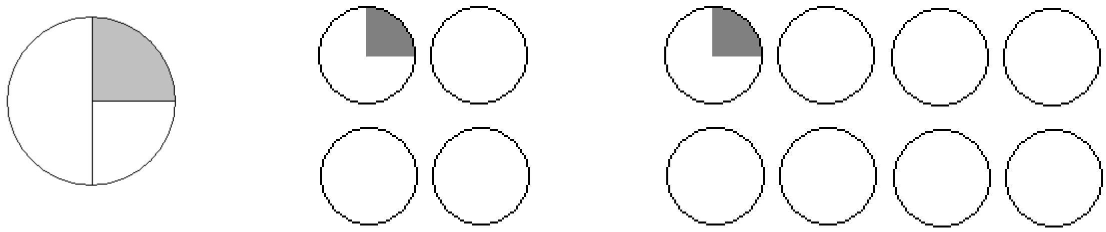
Question 2: Choose the fraction that shows the larger amount

1) 1/3 or 1/4
2) 2/5 or 2/3

Mutinta’s answers: 1) 1/4 2) 2/5

Question 3: For each diagram below colour or shade 1/4.

Mutinta’s answers:



1 2 3

Compare your responses to the following explanations:

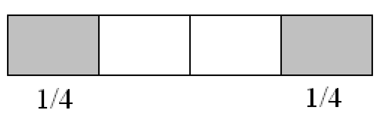
Question 1: She took one part out of three not realising the fact that the three parts are not equal.
Question 2. She chooses a fraction with the larger denominator.
Question 3. She defines all fractions as a part of one whole.

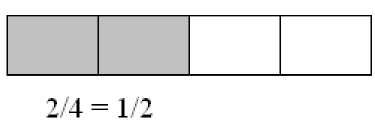
You need to make the corrections to Mutinta’s reasoning.

Addition of Fractions

Addition of fractions using diagrams

Mutinta and Bwalya bought a loaf of bread. Mutinta ate 1/4 and Bwalya ate another 1/4. How much bread did they eat altogether?
If you put together 1/4 and 1/4, what do you get?
This can be represented in a diagram as shown below.





Before you continue, do activity 4.10 below

Activity 4.9

Add the following by using diagrams

1) 1/5 + 2/5 2) 1/3 + 1/3 3) 1/3 + 3/4 4) ½ + 2/5 + 3/10

Note that when you find that the fractions have different denominators; change their diagrams using their common denominators.

Addition of fractions by Finding the Common Denominator

It is not possible to add goats to cows because goats will remain goats and cows will remain cows since they have different characteristics. However cows can be added to goats if you find a common name, for example, animals. The same is true for fractions with different denominators. Suppose you want to add 1/2 to 1/3, you have to find a common denominator.

Examples

1. 1/2 +1/6

   To add the given fractions look for the equivalent fraction that has denominator 6 so that both fractions have the same denominator. The equivalent fraction of 1/2 that has denominator 6 is 3/6. These can now be added because they have the same denominator 6

   1/2 +1/6 = 3/6 + 1/6 = 4/6

2. 1/2 + 3/7 = 7/14 + 6/14=13/14

Look at these examples of adding fractions

1/3 + 5/8 = ?

Notice that 1/3 can be written as 2/6 = 3/9 = 4/12 = 5/15 = 6/18 = 7/21 = 8/24

$=\cdots$

To find this set of equivalent fractions to 1/3, we have:

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{1 \times 3}{3 \times 3} = \frac{1 \times 4}{3 \times 4} = \text{etc.}$$

Similarly

$$\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32} = \frac{25}{40} = \text{etc.}$$

Therefore:

$$\frac{1}{3} + \frac{5}{8} \quad = \quad \frac{8}{24} + \frac{15}{24} \quad = \quad \frac{23}{24}$$

Another example

2/3+1/4

2/3 = 4/6 = 6/9 = 8/12
1/4 = 2/8 = 3/12
so 2/3 + 1/4 = 8/12 + 3/12 = 11/12

Now do activity 4.11 below before you continue.

Activity 4.10

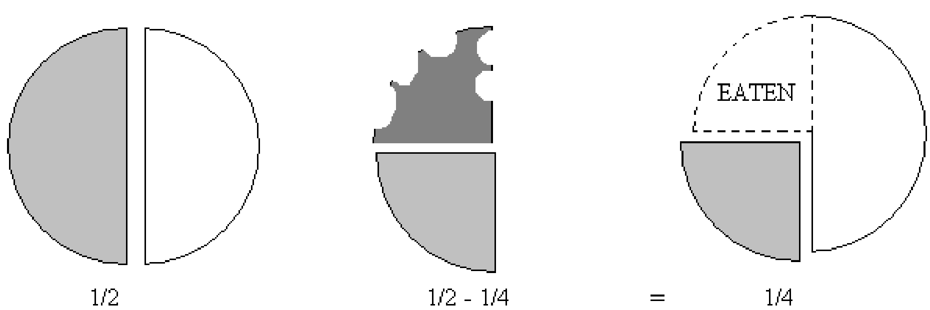
Add the following fractions by finding the common denominator

a) 1/4 +1/2 b) 1/6 + 1/3 c) 1/2 + 2/3 + ¼

Subtracting Fractions

The method of subtraction is similar to the method of addition. You have to find equivalent fractions with the same denominator.

My mother gave me 1/2 of a cake. I ate one quarter of the cake. How much of the cake is remaining? (1/2 – 1/4)

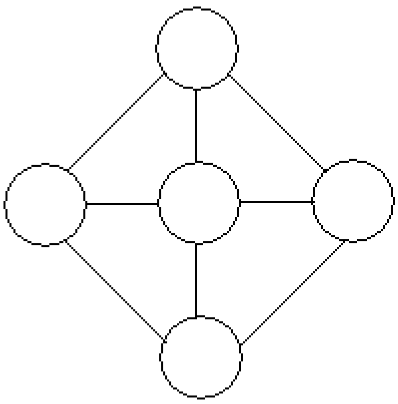


Now spend about one hour doing activity 4.12. You may consult a colleague where you are not sure. Keep your answers in your teaching file.

Activity 4.11 **1 hour**

1. Use both methods introduced under addition to solve the following.
   a) 3/5 - 1/5 b) 1/3 – 1/4 c) 1/2 – 1/6

2. (a) Put these fractions 1/6, 1/3, 1/4, 2/3**,** 5/6, in circles drawn below so that each diagonal adds up to the same number. Each fraction should be used once.

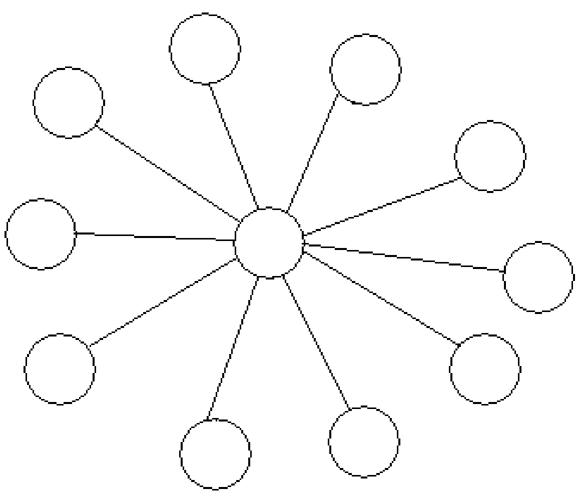


- What procedure did you use?
- What mathematical concepts are involved in this puzzle?
- Is there a relationship between the number of circles and the highest denominator?

(b) Make a similar and simple puzzle for your class.

3. Place these fractions in the circles so that each diagonal adds up to the same number.
   Each fraction should be used only once.

   1/2, 1/6,1/4,1/3, 5/12, 2/3, 3/4, 5/6, 11/12, 1/12, 7/12.



4. (a) A unit fraction can be written as a sum of two unit fractions. Study the following examples and try to discover a general pattern.

1/4 = 1/5 + 1/20
1/6 = 1/7 + 1/42
1/2 = 1/3 +1/6

(b) Formulate three similar problems suitable for the appropriate lower basic class.

5. You are given two measuring cups. One can hold ¼ of a litre and the other can hold 1/3 of a litre. How would you use these two cups to measure exactly

i) 5/6 litres ii) 5/12 litres

Multiplication of Fractions

i. Multiplying Fractions by Whole Numbers

To multiply a fraction by a whole number, write the whole number as an improper fraction with a denominator of 1, then multiply as fractions.

Example:

$$8 x \frac{5}{21} = ?$$

We can write the number 8 as $\frac{8}{1}$. Now we multiply the fractions.

$$\frac{8}{1} x \frac{5}{21} = \frac{40}{21}$$

You may have noticed when multiplying fractions, you multiply numerators on their own and denominators on their own as well.

Consider; $\frac{2}{15} x 10 = ?$

We can write the number 10 as $\frac{10}{1}$. Now we multiply the fractions.

$\frac{2}{15} x \frac{10}{1} = \frac{20}{15} = \frac{4}{3}$ after simplifying

ii. Multiplying Fractions by Fractions

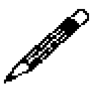
When two fractions are multiplied, the result is a fraction with a numerator that is the product of the fractions' numerators and a denominator that is the product of the fractions' denominators.
Example:

$\frac{4}{7} x \frac{5}{11} = ?$

The numerator will be the product of the numerators: 4 × 5, and the denominator will be the product of the denominators: 7 × 11.

The answer is $\frac{4}{7} x \frac{5}{11} = \frac{20}{77}$

Remember that like numbers in the numerator and denominator cancel out.



Activity 4.11

Work out the following:

1. $\frac{14}{15} x \frac{15}{17}$

2. $\frac{4}{11} x \frac{22}{36}$

iii. Multiplying Mixed Numbers

To multiply mixed numbers, convert them to improper fractions and multiply as before.

Example:

$4\frac{1}{5} x 2\frac{2}{3} = ?$

Converting to improper fractions, we get $4\frac{1}{5} = \frac{21}{5}$ and $2\frac{2}{3} = \frac{8}{3}$. So the answer is

$$\frac{21}{5} x \frac{8}{3} = \frac{168}{15} = 11\frac{1}{5}.$$

Activity 4.12

Work out the following:

$1\frac{3}{4} x\, 1\frac{1}{8}$

$3\frac{2}{5} x\, 2\frac{3}{25}$

iv. Reciprocal

The reciprocal of a fraction is obtained by interchanging its numerator and denominator. To find the reciprocal of a mixed number, first you need to convert the mixed number to an improper fraction, then interchange the numerator and denominator of the resulting improper fraction. Notice that when you multiply a fraction and its reciprocal, the product is always 1.

Example:

Find the reciprocal of $\frac{2}{5}$. We interchange the numerator and denominator to find the reciprocal which is $\frac{5}{2}$. You will apply the concept of reciprocals in the next section which deals with the division of fractions by fractions.

v. Dividing by fractions

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Example

Work out the following:

$\frac{3}{5} \div \frac{7}{12}$

Solution

To solve this problem you change the division sign to a multiplication sign. In effect, you multiply the dividend by the reciprocal of the divisor as follows:

$$\frac{3}{5} \text{ x } \frac{12}{7} = \frac{36}{35} = 1\frac{1}{35} .$$

vi. Dividing Mixed Numbers

To divide mixed numbers, you need to convert mixed numbers to improper fractions, then multiply the first number by the reciprocal of the second.

Examples

$$1\frac{3}{5} \div 2\frac{5}{6} = ?$$

Solution

First, convert both fractions to improper fractions:

$$\frac{8}{5} \div \frac{17}{6}$$

Now you change the division into multiplication and use the reciprocal of the divisor.

$$\frac{8}{5} \text{ x } \frac{6}{17} = \frac{48}{85}$$

vii. Comparing Fractions

To compare fractions, you must first change them so they have the same denominator.

To compare $\frac{2}{3}$ and $\frac{3}{5}$

- First look at the denominators (the bottom numbers).
- Change both numbers into the lowest common multiple for two denominators.
- Once the fractions have the same denominator, you simply compare their numerators. The bigger the numerator the bigger the fraction. This is illustrated below.

$$\frac{2}{3} \xrightarrow[\times 5]{\times 5} \frac{10}{15} \qquad \frac{2}{3} = \frac{10}{15}$$

$$\frac{3}{5} \xrightarrow[\times 3]{\times 3} \frac{9}{15} \qquad \frac{3}{5} = \frac{9}{15}$$

$\frac{10}{15}$ is bigger than $\frac{9}{15}$

So $\frac{2}{3}$ is bigger than $\frac{3}{5}$

Decimal fractions

If you have taught decimals, how have you been introducing them to your pupils?

Children have a lot of difficulties in grasping the concept of decimals unless they understand clearly the *place value system.*

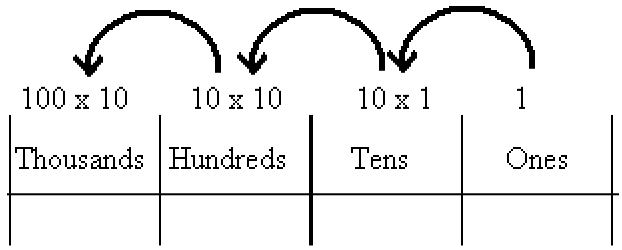
Revise activities of place value. Write a three-digit number like 435. What is the value of each digit? Draw the place value headings on the board as shown below.

?	Hundreds	Tens	Ones	?

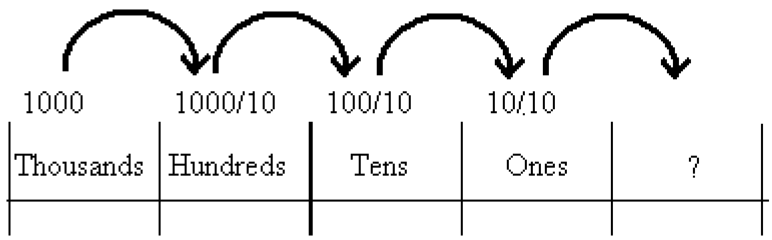
What are the values of the places to the left and right of the diagram?

To the left of Hundreds we have Thousands and to the right of Ones we have tenths.

Let us start from the place value for ones (sometimes called units). As you go each place to the left you have to multiply by 10.

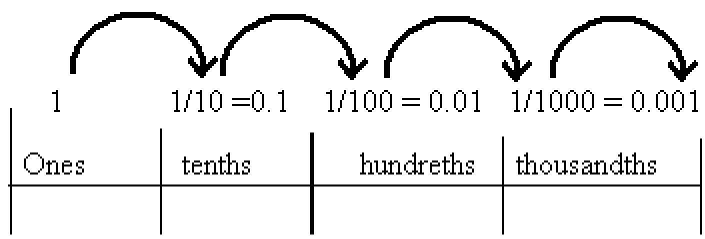


Now what happens if you start from the place value for thousands and move to the right? Here you notice that as you move each place to the right, you divide by 10.



Now what happens if you start from the place value for ones and continue to move to the right? You notice that we get fractions of tens, hundreds, thousands and so on.
These are known as tenths, hundredths and thousandths.

In other words, decimals are fractions with 10,100, 1,000 etc as denominators.

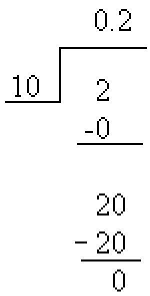


It is at this stage as we move from ones, the last place value on the whole numbers side, to the tenths, the first place value for the fractions side, that we introduce a new form of writing fractions using a *decimal point.* In the place value table above, the thick line separates the whole numbers side from the fractions side. It is this thick line that we change into a decimal point when writing the fractions in decimal form.

Here is an example of some decimals for fractions with denominator 10.

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	1

The other way to introduce decimal point is by converting (changing) common fractions to decimal fractions by using long division method. For example, if we want to convert 2/10 to decimal form, we divide 2 by 10 as follows:



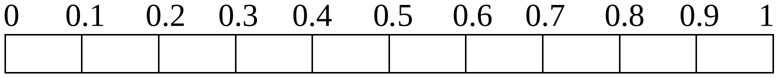
2 divide by 10 is 0. Write 0 above 2 and put a decimal point after it. 10 multiply by 0 is 0. Write 0 under 2 and subtract 0 from 2 which is 2.

Bring down 0 and write it next to 2, and you have 20. 20 divide by 10 is 2. Write 2 after the decimal point. 10 multiply by 2 is 20. Write down 20 under the other 20 and subtract. 20 minus 20 is 0.

Activities 4.13

1. (a) Ask pupils to draw 5 separate 10cm strips and graduate (or mark) each strip into ten equal parts, as shown below:

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

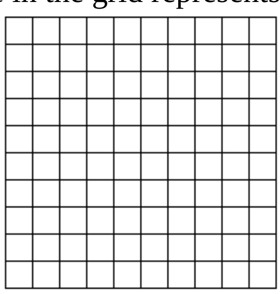


0 1/10 2/10 3/10 4/10 5/10 6/10 7/10 8/10 9/10 1

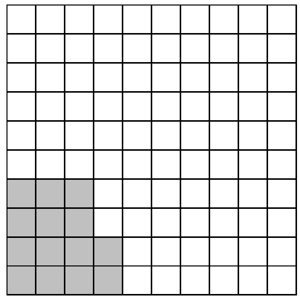
(b) Ask pupils to shade or colour fractions like 1/10, 3/10, 7/10,... Emphasise that decimals are written as 0.1 or 0.3 or 0.7 etc and not just .1

(c) Ask the pupils to read out aloud the decimals they have shaded.

2. (a) Introduce the hundredths in a sequence similar to that of the tenths. Pupils will need 10 x l0 squared paper diagrams like the one shown below. Each small box or square in the grid represents one hundredth (1/100)



(b) Discuss and let pupils distinguish between a tenths and a hundredth. Ask pupils questions like how many hundredths make 2, 3, 4,…,tenths?



14/100 = 0.14

(c) Draw on the board 10 x 10 grids and shade in parts of the grid as shown in the diagram above. Ask pupils to:

(i) Copy the diagrams in their exercise books

(ii) Identify and write down the fractions represented by the shaded parts in the grids. Ask them to write both the common fractions and decimals.

Addition and Subtraction of Decimals

Calculations involving decimals follow the rules as for whole numbers. The numbers in like place-value columns are added or subtracted.

For example, to add 0.5 + 1.26 + 32.06 + 155.361 you arrange into place value column form as shown below and add as you do in whole numbers.

Th	H	T	O	tenth	hundredth	thousandth
			0.	5		
			1.	2	6	
		3	2.	0	6	
	+1	5	5.	3	6	1
	1	8	9.	1	8	1

Therefore, the answer is 189.181

I hope you and your pupils find it is easy to add and subtract decimal numbers since it is done in the same way as you add and subtract whole numbers.

Addition and subtraction of decimals should be related to real life situations such as calculations involving shopping bills or measurements. For example, a box contains items weighing 0.26 kg, 2.71 kg and 0.85 kg. Find the total weight of the items.

Activity 4.14

Formulate 5 simple problems for an activity on mental calculations in each of the following:

(a) Converting fractions with denominators 10,100, or 1,000 into decimal numbers.
(b) Converting decimal numbers into fractions with denominators 10,100 or
1,000
(c) Addition of decimal numbers.
(d) Subtraction of decimal numbers.

Percentages

We have expressed common fractions as decimal fractions and vice versa. Percentages are numbers that can be expressed as common fractions, whose denominators is 100. The percent means out of 100. The other name for percent is hundredth. For example: If a pupil got 50 marks out of 100. This can be expressed as $\frac{50}{100}$, which is same as 50%.

You have noticed that sometimes the total marks are not 100. To get the percentage, the following example illustrates the procedure that you can follow. Bwalya got 26 marks in Mathematics out of a total of 50 marks. Express the marks as a percentage.

- Make a common fraction of the marks out of the total as $\frac{26}{50}$
- Make a common fraction of 1 as $\frac{100}{100}$
- Multiply 26 out of 50 by $\frac{100}{100}$
- $\frac{26}{50} = \frac{26}{50} \times \frac{100}{100} = \frac{52}{100} = 52\%$

Activity 4.15

You can use the following fractions to practice with your pupils on the conversion of common fractions to percentages:

(i) $\frac{4}{5}$ (ii) $\frac{6}{7}$ (iii) $\frac{11}{15}$ (iv) $\frac{17}{20}$

Converting Percentages to Common Fractions

In activity 4.15, you converted common fractions to percentages. Now express the percentages as common fractions.
To do this you think of percentage as per hundred or a common fraction with the denominator 100. The answer obtained can be simplified by dividing by a common factor.

For example:

20% can be written as $\frac{20}{100}$

Simplify by dividing the numerator and denominator by 20. The equivalent fraction is $\frac{1}{5}$.

Activity 4.16

Now practice the following in class, converting the percentages to fractions.

(i) 42% (ii) 67% (iii) 74% (iv) 58%

Converting Percentages to Decimals

The fractions and percentages can also be presented as decimals. Percentages like fractions can be converted to decimals. To do this, you convert the percent to hundredth. Then the hundredth to decimals. See the two examples below:

1. 30% is same as $\frac{30}{100}$ which is 0.3
2. 43% can be converted to decimal as $\frac{43}{100} = 0.43$

Here are word problems involving percentages. In class of 50 pupils, 30% of the class are girls. Find the number of boys.
You work out this problem as finding 30% of 50.

$$\frac{30}{100} \times 50 = 15$$

$\therefore$ there are 15 girls.

To find the number of boys you subtract the total number of pupils that's 50 minus the number of girls 15.

That's 50 – 15
= 35 boys.

Or

You can work out the percentage of boys since you know the number of girls as 30%.
Then subtract 30% from 100% to find the percentage of boys.
$\Rightarrow$ 100% - 30% = 70%
Then the percentage of boys is 70%

The number of boys $= 50 \times \frac{70}{100}$

$= 35$

Percentages and other Fractions

You may have noticed the relationship among common fractions, decimal fractions and percentages. To convert a percentage to a decimal fraction, you should express the percentage as a common fraction, then divide the numerator by a denominator. For example $30\% = \frac{30}{100} = \frac{3}{10} = 0.3$

Example

$7\% = \frac{7}{100} = 0.07$

Now work out the following:

	Percentage	Fraction	Decimal	Fraction
1	9%	$\frac{9}{100}$	0.09	
2	26%			
3	47%			
4	35%			

Activity 4.17.

1. Prepare a lesson plan on how you would teach the conversion of common fractions to percentage to a middle basic class.

2. Teach the lesson to the class.

Ratio and Proportion

The following activities will help you develop the ideas of ratio and proportion in your class. Ratio and proportion can be defined in terms of common fractions and relationships in terms of comparisons.

Ratio
A ratio is a comparison between two or more quantities.

Example

If Monde is given K20 and Bwalya is given K30, we express the amounts in ratios as 20:30 or 2:3.

Activity 4.18

Express the following as ratios

1. Labani weighs 54kg. Tabita weighs 69kg.
2. For each of these lengths, express the first line as a ratio of the second line.

(a) 2cm 3cm

(b) 8cm 5cm

Proportion

Sometimes we have to compare more than two quantities. For example, Mr. Mubiana, a farmer has 90 cattle and wishes to share them among his three children: Inonge, Miyanda and Phiri. Inonge is the eldest daughter and has to get three times what Phiri, the youngest son gets. Miyanda the second born son gets twice as Phiri. You can think of this situation as follows:

For every animal Phiri gets, Miyanda has 2 and Inonge 3. To find how many animals each child gets, Mr. Mubiana can think:

For every six animals (1+ 2 + 3), Phiri has l, Miyanda 2 and Inonge 3. So, Phiri gets $\frac{1}{6}$, Miyanda $\frac{2}{6}$ and Inonge $\frac{3}{6}$ of the total animals (90). To find the number of cattle each child gets, we have:

Phiri has $\frac{1}{6} \times 90$ animals = 15 animals

Miyanda has $\frac{2}{6} \times 90$ animals = 30 animals

Inonge has $\frac{3}{6} \times 90$ animals = 45 animals

Thus the children were given animals by their father in the proportion 1:2:3.

Activity 4.19

1. Mrs. Njekwa left a will to share K70 000 000 among twins and another son in the proportion 2:2:3. Apportion this money among the three children.

2 A family of 5 people need a 25kg bag of mealie-meal in a period of 24 days. How many days can 25kg bag last in a family of 8 people?

Direct Proportion

At times, we consider a situation in which when one quantity is changing the other is also changing. For example, if two pencils costK900 then three pencils will cost K1,350. This situation illustrates problems involving **direct proportion**. When one quantity increases or decreases the other quantity also increases or decreases respectively. The more pencils you buy, the more money you pay. The less the number of pencils you buy, the less money you pay. Here is another example of the same concept: Chilufya can walk 3km in 45 minutes. How many minutes can it take him to walk 7km.

3km $\rightarrow$ 45 minutes

1km takes: $\frac{45}{3}$ minutes = 15 minutes

7km takes 15 × 7 minutes = 105 minutes

Activity 4.20

1. Banda bought 5 tins of fish at K2 750. How much would Bupe pay if she buys 9 tins of the same kind of fish?

2. Make a lesson plan to teach concepts of ratio and proportion to your class. Put your experiences on the teaching file.

Inverse Proportion

Study the following situation. Two buses left Lusaka for Kabwe (140km away) at the same time. The first bus to arrive took two hours and the second took two and half hours. We can calculate the average speed of each bus:

Average speed of first bus: $\frac{140}{2}$ km/h = 70km/h

Average speed of second bus: $\frac{140}{5/2}$ km/h = 140 × $\frac{2}{5}$ km/h = 56km/h

You may have observed that as the bus takes less time to cover a certain distance the speed is higher and vice versa. This situation gives rise to the **inverse proportion**, where when one quantity increases the other quantity decreases; or when one quantity decreases, the other quantity increases.

Example:
If four men can do a piece of work in 6 days. How many days can 2 men do the same piece of work if they work at the same rate?

Solution

Man/men	Days
4	6
1	4×6
2	?

Therefore 2 men take $\frac{4 \times 6}{2}$ days = 12 days

Reflection

What have you been able to learn in this unit that you previously did not understand and found difficult to teach? Record this in your diary.

UNIT

5

INTEGERS

Very often when we are introducing children to number, we use a set of things e.g. 3 mangoes, 5 pencils. When adding, it is easy to see that 5 mangoes plus 3 mangoes gives 8 mangoes because we can count the mangoes. Similarly, when we have 8 mangoes and take-away 3 mangoes, we physically take away the 3 and count and see that we have remained with 5. Unfortunately, we are unable to do the same for negative numbers. We cannot have a set of negative 3 mangoes. Therefore, we need special care to put meaning when we work with negative numbers.

Learning outcomes

As you study and work through this unit, you are expected to:

- introduce the concept of negative numbers to pupils in a manner that makes sense to them.
- explain to pupils meaningfully about addition and subtraction of positive and negative numbers.
- give meaning (for your own benefit) to multiplication and division of positive and negative numbers.

Integers

When we buy a bottle of Fanta from a shop, there is a level of the drink in the bottle we expect. We will be surprised if the Fanta was completely full or if it is below the level we expect. We can introduce the concept of positive and negative numbers by comparing levels of Fanta in different bottles (or any other drink) to the level of the Fanta in a standard bottle.

Activity 5.1

Draw a large chart of bottles of drink showing the first bottle with the drink at the standard level (where we expect the level to be) and other bottles with various levels of the drink. Some above, some below and some at the same level as the standard bottle as shown below:

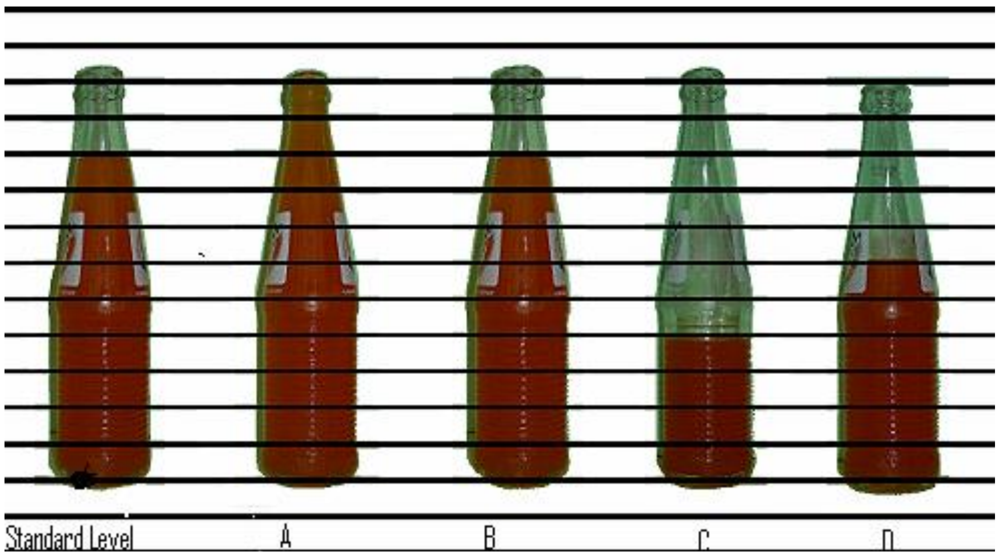


Figure 5.1: Levels of drink in a bottle

Display the chart Fig. 5.1 so that all children can see the levels of the drinks clearly. Draw pupils' attention to the level we expect and the line through it.

- For each bottle ask pupils to compare the level in the bottle to the level we expect. Ask them by how much the two levels differ:
  The level in bottle A is 2 lines
  above the standard level.
  Bottle B is 3 lines below the standard level.
  Bottle C is 0 lines away from the standard level.
  Bottle D is 1 line below the standard level.
- Explain to pupils that for easy reference, 2 lines above will be called **positive** 2 and written as $^{+}2$. When we have 3 lines below the standard level, we will call it **negative** 3 and written as $^{-}3$. When it does not differ from the standard level, we shall call it 0. Explain to pupils that 0 is neither positive nor negative.
- Call individual pupils to come and show on the chart where the following levels are: $^{+}1$, $^{-}2$, 0, $^{+}3$, $^{-}4$, $^{-}1$ $^{+}2$, $^{-}3$, etc.

Integers in Everyday Life

There are many other situations in which we can develop the concept of positive and negative numbers. Some of them are the following:

- Having money (positive). Having no money (0) and owing money (negative)
- The acceptable level of water in a well (0). The water above the acceptable level of water in a well (positive). Part of the well between the acceptable and the not acceptable level of water in a well (negative)
- A bag of mealie-meal (say 25kg) being standard. Bags weighing more than 25kg (positive). Weighing less than 25kg (negative)
- Temperature that is at freezing point of water (0^0C), above the freezing point of water (positive) and below freezing point (negative).

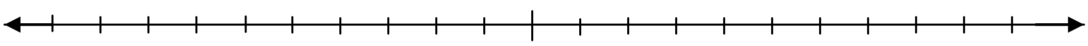
Activity 5.1

1. Jot down points giving what you consider to be advantages and disadvantages of each approach in relation to the pupils you teach.

2. Write down at least one other way of introducing positive and negative numbers.

Number Line

The scale we used to compare levels of drinks in Figure 5.1 is actually a number line If we drew it horizontally and extended it , it would appear as follows:



$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

Note that the spaces between the numbers must be equal. The arrows at each end of the line indicate that the number line is continuing.

Activity 5.2

Draw a big number line placed permanently in front of the classroom above the chalkboard. Ask pupils to come forward and point at a number that you ask for e.g. $^{+}2$, $^{-}3$, 0, $^{-}5$, $^{+}1$

Addition and Subtraction Using the Number Line.

The number line is a very important tool for teaching addition and subtraction of positive and negative numbers. But it works effectively only if you understand why you are doing what you are doing and you practice with pupils regularly.

The first step is to be clear with the difference between a **directed number** and an **operation.** When we say $^{-}2$ or $^{+}3$, we know where on the number line (which side of zero) the number should be written. Therefore, when we attach a sign to a number, $^{+}$ or $^{-}$, the numbers are called directed numbers. **Plus (+)** is an operation meaning add. **Minus (-)** is also an operation meaning (usually) take away.

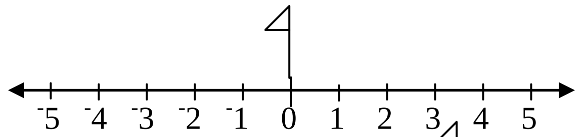
When adding or subtracting integers using the number line, the first step must always be to go to 0 and face the positive direction. Then the following rule should be used consistently.

Positive:	Move forward
Negative:	move backwards
Plus:	Followed by
Minus:	Find the difference between

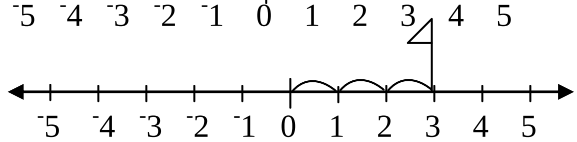
Let us look at examples of each case of addition and subtraction that we can expect to meet: (i) $^{+}3 + ^{+}2$ (ii) $^{+}3 + ^{-}2$ (iii) $^{-}3 + ^{+}2$ (iv) $^{-}3 + ^{-}2$ (v) $^{+}3 - ^{+}2$ (vi) $^{-}3 - ^{+}2$
($^{-}3 - ^{+}2$ (vii) $^{+}3 - ^{-}2$ (viii) $^{-}3 - ^{-}2$

(i) $^{+}3 + ^{+}2$

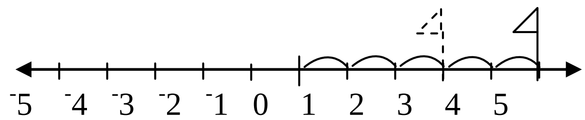
Go to the number line at 0 and face the + ve-direction



$^{+}3$ Move 3 steps forward
+ followed by

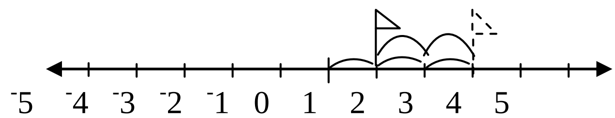


$^{+}2$ Move 2 steps forward



∴ $^{+}3 + ^{+}2 = ^{+}5$

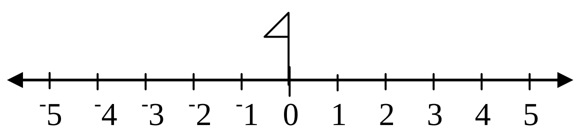
(ii) $^{+}3 + ^{-}2$



$^{-}2$ means move 2 steps backwards

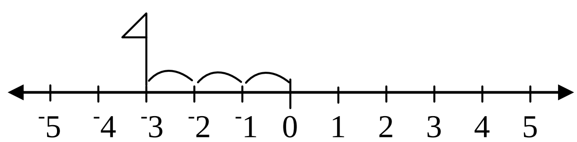
∴ $^{+}3 + ^{-}2 = ^{+}1$

(iii) $^{-}3 + ^{+}2$
Go to the number line at 0 and face the positive direction

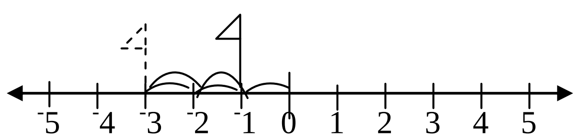


$^{-}3$ means that Move 3 steps back-

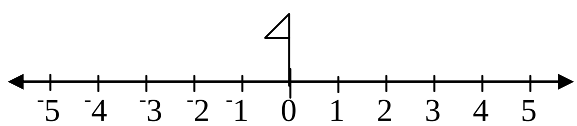
wards + means Followed by



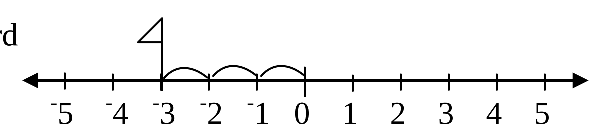
$^{+}2$ Means move 2 steps forward
Therefore $^{-}3 + ^{+}2 = -1$



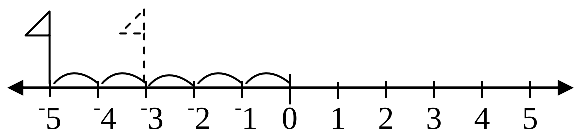
(iv) $^{-}3 + ^{-}2$



$^{-}3$ means that Move 3 steps backward
+ (Followed by) and $^{-}2$ means that
Move 2 steps backwards .



Therefore ${}^{-}3 + {}^{-}2 = {}^{-}5$

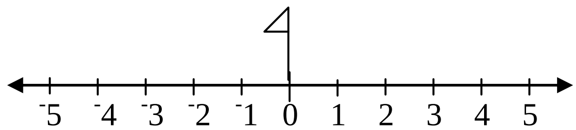


Subtraction Using a Number Line.

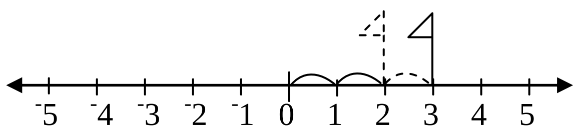
At primary level you it is important to give meaning to subtraction of integers. So, let us look at 'minus' as finding the difference between the two integers. This may be interpreted as what should be added to the subtrahend to get the minuend.

(v) ${}^{+}3 - {}^{+}2$

Go to the number line at 0 and face the positive direction



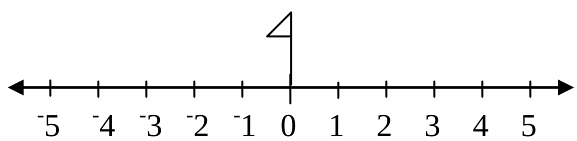
${}^{+}2$ (subtrahend) means move forward 2 steps. – means that find the difference. ${}^{+}3$ is 1 step away from ${}^{+}2$ in the positive direction



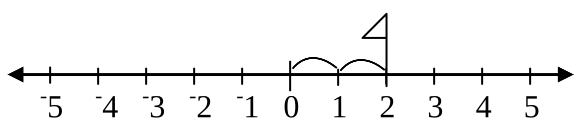
Therefore ${}^{+}3 - {}^{+}2 = {}^{+}1$

(vi) ${}^{-}3 - {}^{+}2$

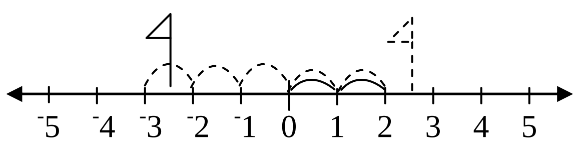
Go to the number line at 0 and face the positive direction.

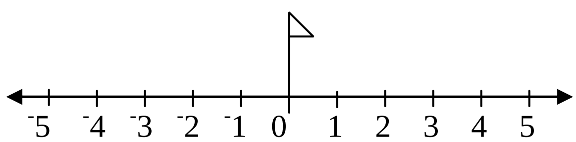


${}^{+}2$ move forward 2 steps

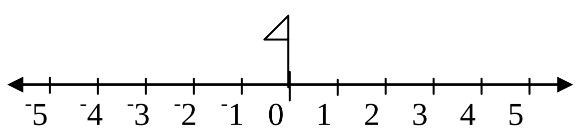


Now move backwards till you find ${}^{-}3$. Since you moved 5 steps backwards to find ${}^{-}3$, then ${}^{-}3 - {}^{+}2 = {}^{-}5$

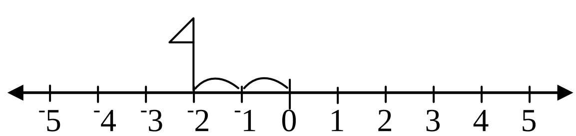




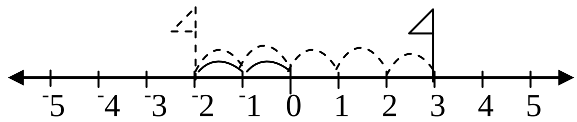
(vii) ${}^{+}3 - {}^{-}2$



${}^{-}2$ move 2 steps backward

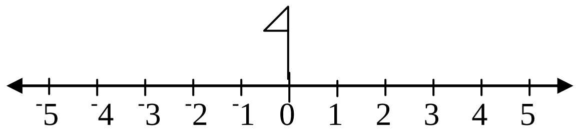


Find $^{+}3$. Move forwards till you find $^{+}3$. Since you moved 5 steps forwards to find $^{+}3$, then $^{+}3 - ^{+}2 = ^{+}5$

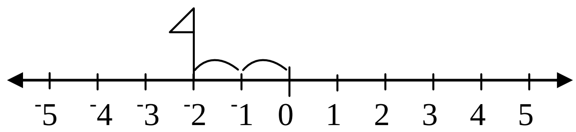


(viii) $^{-}3 - ^{-}2$

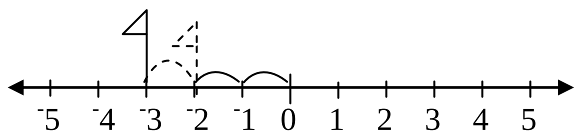
Go to the number line at 0 and face the positive direction



$^{-}2$ move 2 steps backwards



Where is $^{-}3$? It is 1 step behind. Therefore $^{-}3 - ^{-}2 = ^{-}1$



Activity 5.3

Take your class outdoors. Draw a big number line on the ground for all children to see. Do the following calculations by getting one pupil at a time to physically make the movements on the number line. Give instructions to the pupil so that there is one instruction at a time.

a.	$^{+}4 + ^{+}3$	b .	$^{+}5 - ^{+}1$	c.	$^{-}3 + 5$	d.	$^{-}2 + ^{-}6$
e.	$^{-}4 + ^{+}4$	f.	$^{+}5 - ^{-}3$	g.	$^{-}1 - ^{-}6$	h.	$^{-}7 + 0$
i.	$^{+}2 - ^{+}2$	j .	$^{-}3 + ^{-}4$	k.	$0 + ^{-}5$	l.	$^{+}4 + ^{-}1 - ^{+}5$

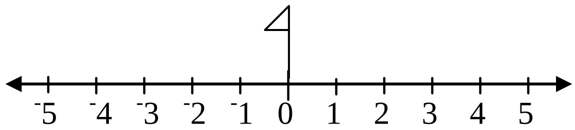
Add more tasks until children are able to move quickly on the number line and find correct answers.

Deriving the Rule of Subtracting Integers

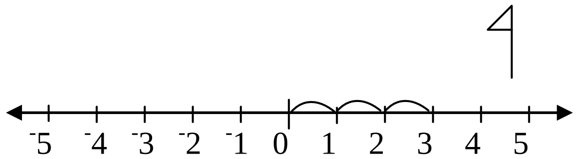
(vi) After the pupils have got the meaning of subtraction under integers, at a later stage, pupils can be exposed to the strategy which leads to deriving the rule of subtracting integers for instance ***subtraction as about turn (turn around)***. You enjoy yourself looking at the strategies in the following situations.

(i) $^{+}3 - {}^{+}2$

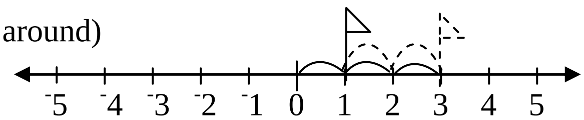
Go to the number line at 0
and face the positive direction



$^{+}3$ means move forward 3 steps

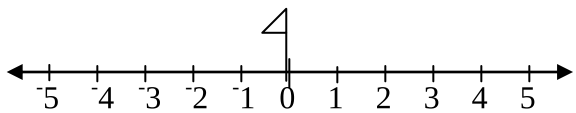


- means that about turn (turn around)
$^{+}2$ Move 2 steps forward

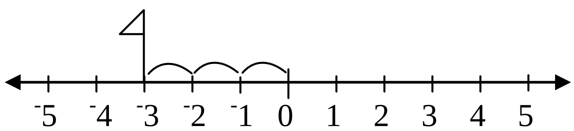


Therefore $^{+}3 - {}^{+}2 = {}^{+}1$

(ii) $-3 - {}^{+}2$

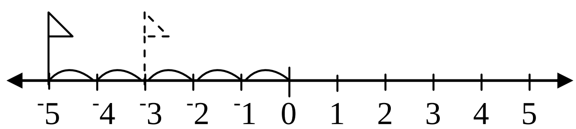


–3 Move backwards 3 steps



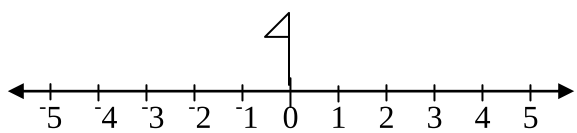
– About turn

$^{+}2$ Move forward 2 steps

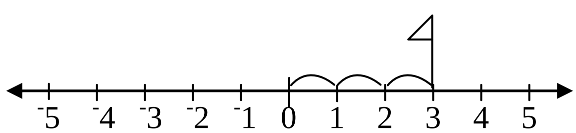


Therefore $^{-}3 - {}^{+}2 = {}^{-}5$

(iii) $^{+}3 - {}^{-}2$

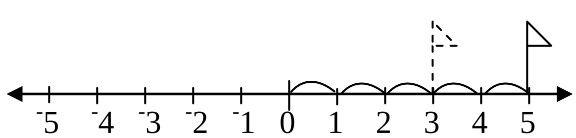


+ move 3 steps forward



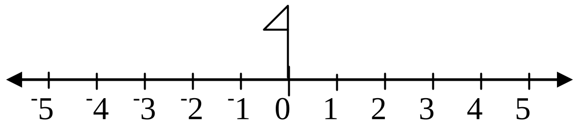
– turn about
$^{-}2$ move 2 steps backward

Therefore , $^{+}3 - {}^{-}2 = {}^{+}5$

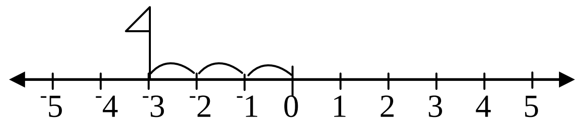


(viii) $^-3 - ^-2$

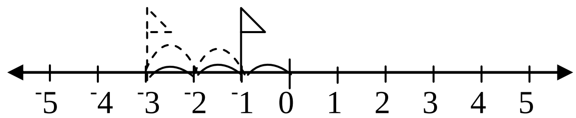
Go to the number line at 0
and face the positive direction



–3 Move 3 steps backwards



- About turn
–2 Move backwards
2 steps



Therefore $^-3 - ^-2 = ^-1$

Multiplication

Multiplication involving negative numbers is not done at Primary School. However, as a teacher, you still need to be armed with sound mathematical explanations for the rules we often use in multiplication involving negative numbers. Very often, pupils and teachers alike will just say the following:

$+ \times + \;=\; +$
$+ \times - \;=\; -$
$- \times + \;=\; -$
$- \times - \;=\; +$

Activity 5.4

Write down the strategies you have used before to explain the above rules for multiplication of integers. Justify the rules

Rules used When Multiplying Integers

We will consider each of the four rules of multiplication at a time.

+ × + = + Example 2 × 3 = 6

This can be explained using repeated addition. i.e. $2 \times 3 = 2 + 2 + 2 = 6$

+ × − = - Example $^-2 \times {}^+3 = {}^-6$

This too can be explained using repeated addition and the commutative property.

$^-2 \times {}^+3 = {}^-2 + {}^-2 + {}^-2 = {}^-6$

Note that we have already dealt with addition of integers using a number line. Therefore, it should be clear why $^{-}2 + {}^{-}2 + {}^{-}2 = {}^{-}6$.

+ × - =

Example $^{+}2$ x $^{-}3$ = $^{-}6$

Again, we can use repeated addition.
First, we know that $2 \times {}^{-}3 = {}^{-}3 \times 2$

Therefore
$2 \times {}^{-}3 = {}^{-}3 \times 2 = {}^{-}3 + {}^{-}3 = {}^{-}6$
– × – = +

Example $^{-}2 \times {}^{-}3 = 6$

We have shown so far how + × + = +;
+ × – = - and – × + = -. It is more challenging to show how – × – = +. Follow carefully.

$^{-}2$ is equal to $^{+}1 - {}^{+}3$
$^{-}3$ is equal to $^{+}1 - {}^{+}4$

Therefore, $^{-}2 \times {}^{-}3 = ({}^{+}1 - {}^{+}3)\,({}^{+}1 - {}^{+}4)$
$= {}^{+}1({}^{+}1 - {}^{+}4) - {}^{+}3({}^{+}1 - {}^{+}4)$
$= [({}^{+}1\ {}^{+}1) - ({}^{+}1 \times {}^{+}4)] - [({}^{+}3 \times {}^{+}1) - ({}^{+}3 \times {}^{+}4)]$
$= [{}^{+}1 - {}^{+}4] - [{}^{+}3 - {}^{+}12]$
$= ({}^{-}3) - ({}^{-}9)$

Using the number line as we did earlier,
$^{-}3 - {}^{-}9 = {}^{+}6$
Therefore $^{-}2 \times {}^{-}3 = {}^{+}6$
In general – × – = +

Note that a strategy like the one below is useful for remembering the rule but has no mathematical explanations.

Friend of my friend is my friend i.e. + × + = +
Friend of my enemy is my enemy i.e. + × – = –
Enemy of my friend is my enemy i.e. – × + = –
Enemy of my enemy is my friend i.e. – × – = +

Division of Integers

Rules that apply to multiplication of integers apply to division as well. But we need to justify the rules.

(i) + ÷ + = +

Example $^{+}12 \div {}^{+}3 = {}^{+}4$
We may consider division as repeated subtraction (in the way that

multiplication is repeated addition) i.e.

$^{+}12 \div {}^{+}3$	$^{+}12 - {}^{+}3 = {}^{+}9$	-	1
	$^{+}9 - {}^{+}3 = {}^{+}6$	-	2
	$^{+}6 - {}^{+}3 = {}^{+}3$	-	3
	$^{+}3 - {}^{+}3 = 0$	-	4

Therefore $^{+}12 \div {}^{+}3 = {}^{+}4$

(ii) - ÷ - = + Example $^{-}12 \div {}^{-}3 = {}^{-}4$

This can be shown by repeated subtraction.

$^{-}12 - {}^{-}3 = {}^{-}9$ ———————— 1
$^{-}9 - {}^{-}3 = {}^{-}6$ ———————— 2
$^{-}6 - {}^{-}3 = {}^{-}3$ ———————— 3
$^{-}3 - {}^{-}3 = 0$ ———————— 4

Therefore $^{-}12 \div {}^{-}3 = {}^{+}4$

(ii) + ÷ – = –

Here, we shall look at division as the inverse operation of multiplication.

Example $^{+}12 \div {}^{-}3 =$

If $^{+}12$ is the product and $^{-}3$ is a facto□f $^{+}12$. What is the missing factor? This is translated as $^{+}12 = {}^{-}3 \times$

Another situation is of - ÷ + = - . The process is the same as in the previous situation.

Example $^{-}12 \div {}^{+}3 = \square$

$^{-}12 = \square \times {}^{+}3$

= $^{-}$4 $\square$

Activity 5.5

(i) If you were to explain the rules of multiplication and of division to a learner using the strategies we have presented here, what difficulties do you anticipate,
   (a) on your part as the one explaining?
   (b) on the part of the learner ?

(ii) What would you change to make the explanation more convincing but remaining mathematically accurate?

Summary

In this unit, you have met ways of explaining the meaning of a negative number and you have also learned how to handle addition and subtraction of integers in a way that would help pupils. The number line is a vital tool and it should always be one of the teaching aids on your wall to be used regularly. Finally, you have seen how we can justify the rules on multiplication and division, which we often use

UNIT 6

SHAPES

In this unit we discuss shapes of various types. Children in your class will find these shapes exciting. You need to explore ways in which you may bring out mathematical concepts using these shapes.

Learning Outcomes

As you study and work through this unit you are expected to:

- identify polygons
- explain the properties of polygons
- perform calculations based on the right angled triangle

Reflection

As you start studying this module it is important that you note down in your diary those aspects of measures in which you have been experiencing difficulties to teach. Keep reflecting on these as you progress through the unit and see whether your difficulties will be answered

Shapes

Activity 6.1

2. Study the diagram in figure 6.1.

Fig. 6.1

What shapes do you see in the pictures shown in Fig. 6.1? On a separate piece of paper draw the outlines of the shapes you identified in the pictures above. You probably have drawn the four shapes; oval, triangle, rectangle and a prism.

From the above activity we may say that shape is an outline of an object. You may introduce the idea of shape to your pupils using similar activities. Shapes can either be geometrical or non-geometrical. Geometrical shapes are those that have the mathematical properties, measurements, and relationships of points, lines, angles, surfaces, and solids. Our world is full of geometrical or non-geometrical shapes.

Two - dimensional shapes

Two dimensional shapes are commonly known as flat shapes. They include circles and polygons. A flat closed shape made up entirely of straight lines is called a polygon.

You are familiar with the classifications of polygons using the aspect of number of sides such as triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons, nonagons, decagons and duo-decagons. Below are some of the aforementioned shapes.

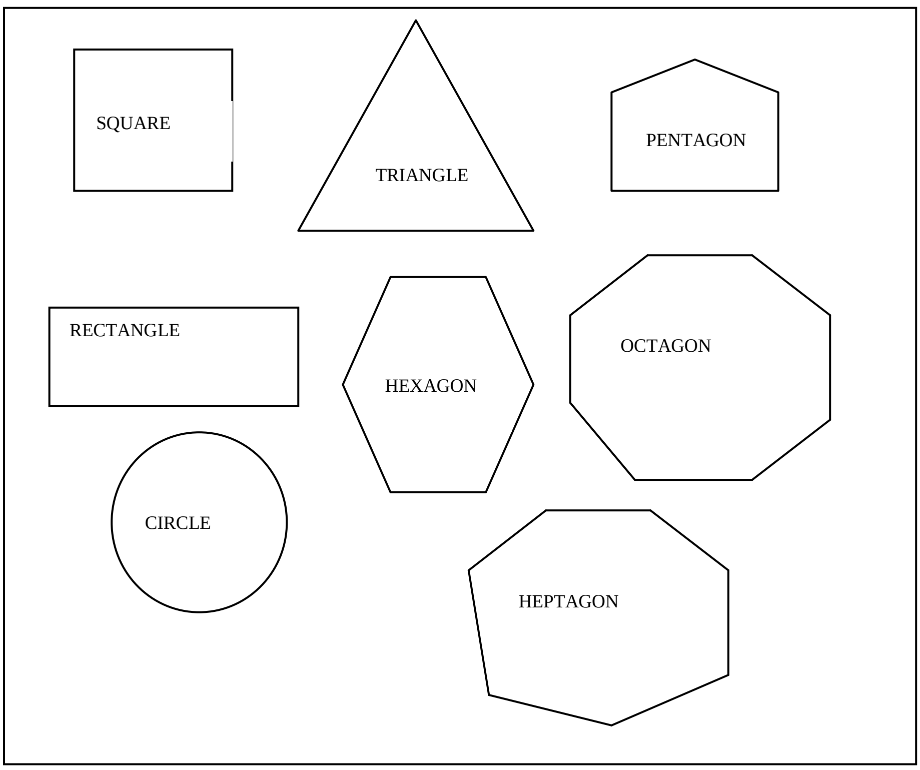
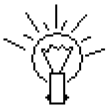


Fig. 6.2

The circle

You can introduce the shape of a circle through activity 6.2



Activity 6.2

1. Ask pupils to bring round objects like tins. You can bring coins if available. You engage pupils in drawing circles around round objects.

2. Prepare large cut-outs of the four shapes - circle, square, rectangle and triangle.
   - Write the name boldly on the back of each one. Hang these shapes.
   - Display different types of pictures of the above four shapes and other shapes.
   - Pick a circle. Ask pupils to identify shapes similar to a circle from the display.
   - Ask pupils to explain why the shape they picked is similar to a circle.
   - Tell the pupils the name 'circle'. Ask pupils to say the name, describe a circle and write the name.
   - Ask pupils to find real objects with circular shapes and trace the outline.

3. Pupils make jigsaw circle puzzles as follows;

   - They draw circles on plain sheets of paper by tracing around circular objects.
   - Tell them to cut out their circles in any number pieces of different shapes, as shown in the diagram below. Each pupil decides on how to cut his/her circle and into how many parts (three to six parts). Pieces should be kept in bags.

Fig 6.3

   - If the water colours or coloured-pencils are available, pupils may colour their cut-outs.
   - Ask the pupils to exchange their pieces and assemble them to form circles.

A circle is the collection of points in a plane that are all the same distance from a fixed point (or a collection of points equidistant from a fixed point). The fixed point is called the center. A line segment joining the center to any point on the circle is called a radius. This is illustrated in the figure below.

Fig 6.4

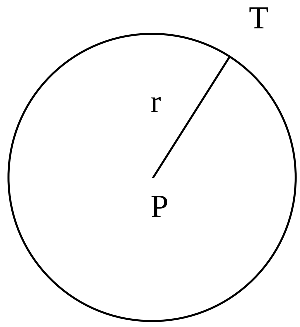
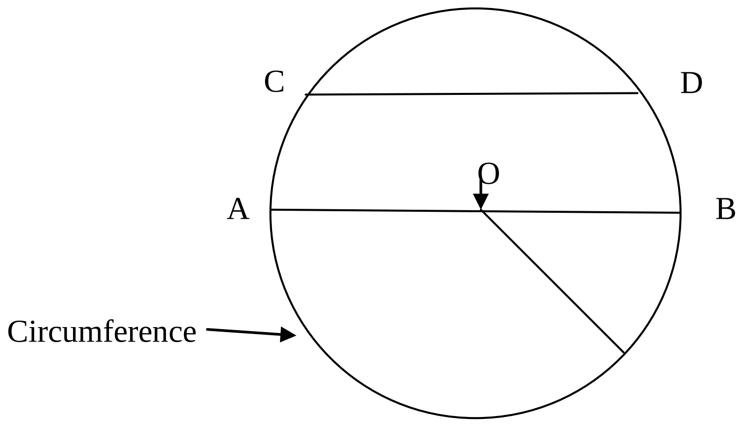


Fig. 6.5

OThe line PT is the radius r and the collection points around P is the circumference of the circle.



Rectangles and Squares

A grade-10 class was divided in pairs to discuss, for five minutes, whether a square is a rectangle and to record their resolutions. A pair of pupils, Siku and Mwale was now disagreeing about whether a square is a rectangle.

Siku: “A square is a rectangle because...”

Mwale(*Interjecting*): “Ha! How can a square be a rectangle? A square has all the four sides equal.

You may say to the pair: “A rectangle and a square are similar for they both have four sides and their opposite sides are equal. Their interior angles are at 90^o. If the four sides are equal, it means the opposite sides of a square are equal. Therefore a square satisfies the general definition of a rectangle. Hence a square is a rectangle. But a rectangle is not a square”. In a square **all** sides are equal and in a rectangle ***only sides opposite each other are equal.***

Activity 6.3

1. Pupils work in groups. Tell pupils to use available materials to make some squares and rectangles. These could be made from paper cuttings or models from wires or straws.

   - Observe the strategies they are using to come up with the two shapes.
   - Discuss with pupils the similarities and differences of squares and rectangles.

2. Tessellating rectangles

A tessellation is a repetition of a shape to cover completely a surface without gaps or overlaps, as shown in the diagram below. Some shapes are more suitable than others for this purpose. We see the idea of tessellation in floors made of tiles.

Ask pupils to make rectangles of different sizes from card boxes. Tell them to colour their rectangles. Tell the pupils to put the rectangles together to make patterns.

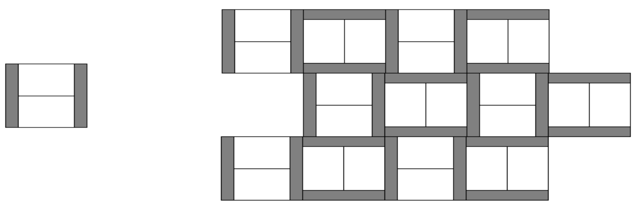


Fig 6.6 Rectangle tessellation

Triangles

Triangles may be classified in three main categories. These are scalene, isosceles and equilateral. Equally, quadrilaterals can further be classified in six main categories, that is, rectangles, squares, rhombus, kites, parallelograms and trapezium.

What makes a triangle?

A grade three class was discussing a triangle.

Teacher: ‘What is a triangle?”

Misozi*:* “It is a shape with three sides and three corners”.

Teacher: “*Good* Misozi*!* Now explain to us how you can make a triangle with three matchsticks?

Misozi*:* **“**Of course, there are three matchsticks representing three sides. I can make a triangle”. [makes a triangle with matchsticks as shown]

Misozi is right to say she can make a triangle with three matchsticks.

Fig. 6.7

Activity 6.4

1. Ask your pupils to find the number of triangles in the diagram below.

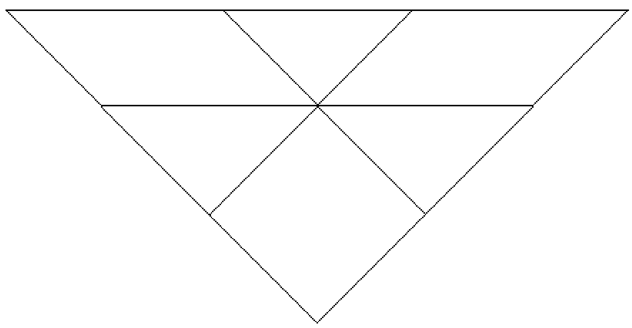


Fig. 6.8

What strategies did pupils use to come up with the triangles?

2. Take pupils for a geometry walk. Tell them to identify circles, squares, triangles and rectangles on objects, including clothing materials. Prepare questions that may promote the pupil's greater awareness of geometry in the real world. It is important to explain to the pupils that the shape of an object may be determined by both the desired beauty and the object's function.

Other Polygons

A polygon is a figure bounded by three or more sides.

Examples:

The following are examples of polygons:

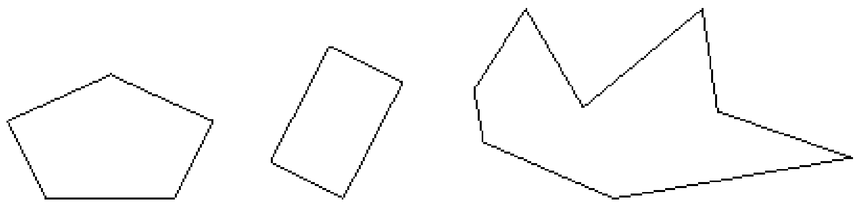


Fig. 6.9

The figure below is not a polygon, since it is not a closed figure:

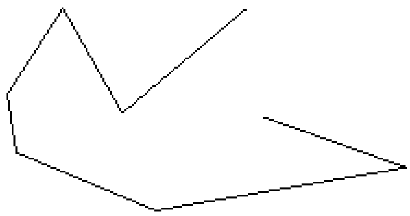


Fig 6.10

Polygons can also be classified into those that are regular and irregular.

Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same. The sum of the angles of a polygon with n sides, where n is 3 or more, is $180° \times (n - 2)$ degrees.

Examples:

The following are examples of regular polygons:

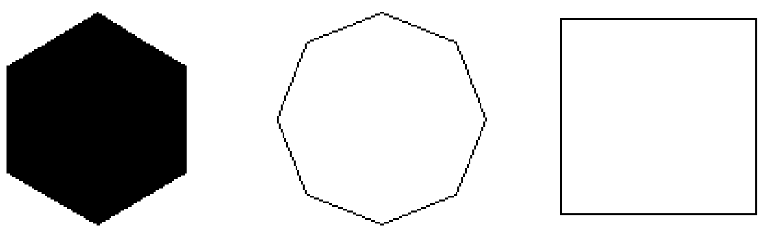


Fig. 6.11

Activity 6.6

1. By the time pupils learn about polygons, they would have already learnt about the three polygons that is the rectangle, square and triangle. Look at the shapes in Figure below

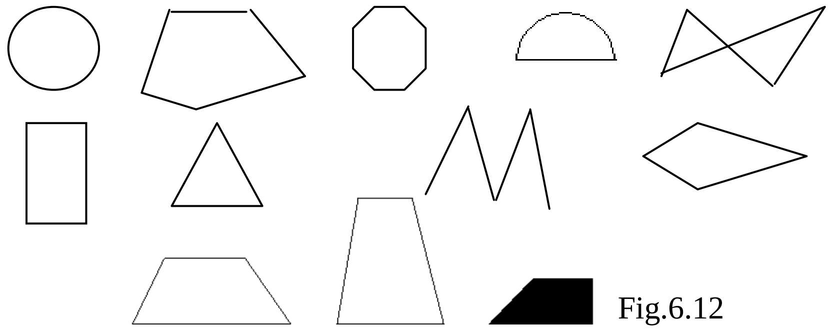
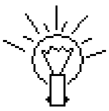


Fig.6.12

a) Identify the polygons in Fig. 9.17. How do you know that they are polygons?
b) Identify quadrilateral(s) (if any). Justify your answer.
c) Identify regular polygons (if any). Justify your answer.



Activity 6.7

1. Place pupils in groups of six. Tell them to use rubber bands to make as many triangles as possible on their geoboards (Fig 6.13) and then draw the made shapes on square pieces of paper card boards. Ask them to cut out the shapes. Let them sort the triangles into different categories and name the different attributes they used to categorize the triangles, such as all sides equal, two sides equal and one different or all different sides. They should then differentiate categories of triangles and then display them.

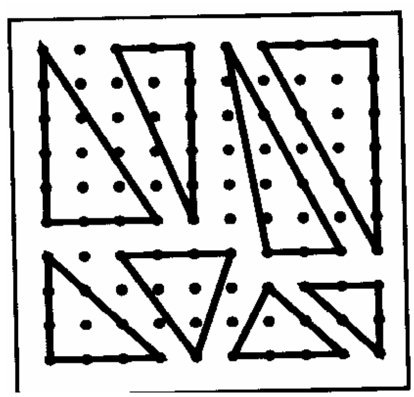


Fig. 6.13

Right Angled Triangle

A right angled triangle is a triangle in which one of the angles is 90^0. In a right angled triangle, the side opposite to the right angle is called the hypotenuse and the sides adjacent to the right angle are called adjacent sides. The Pythagoras Theorem states that in any right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the adjacent sides.

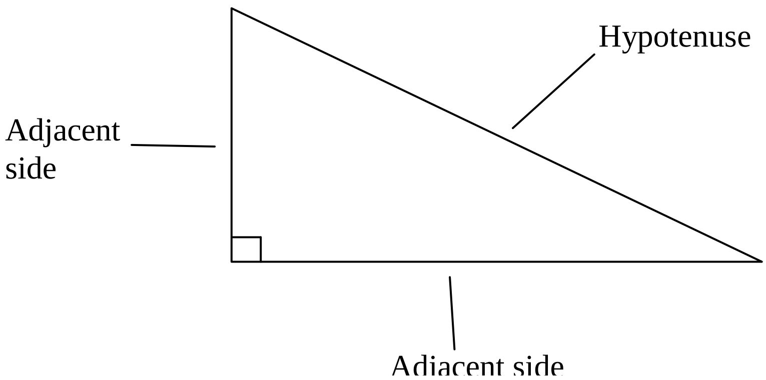


Fig. 6.14

For the right triangle above, the lengths of the legs are A and B, and the hypotenuse has length C. Using the Pythagorean Theorem, we know that $A^2 + B^2 = C^2$.

Example:

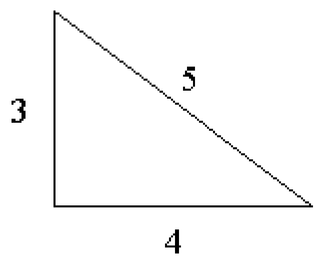
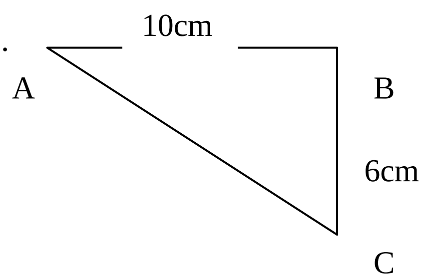


Fig. 6.17

In the right triangle above, the hypotenuse has length 5, and we see that $3^2 + 4^2 = 5^2$ according to the Pythagorean Theorem.

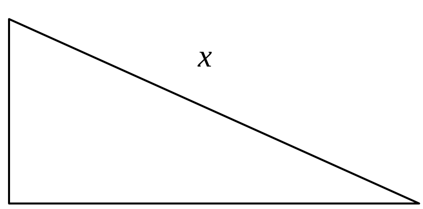
Activity 6.8

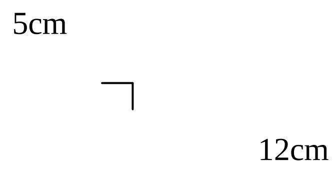
1.



Given that BC = 6cm and AC = 10cm, find the length of AB.

2. In the figure below,





Calculate the length of the side marked x .

Quadrilaterals
A quadrilateral is a four-sided polygon. The sum of the angles of a quadrilateral is 360 degrees.

Examples:

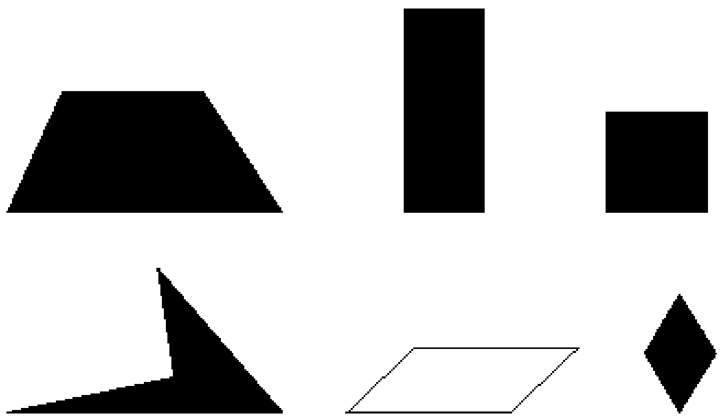


Figure 6.16

Rectangle
A rectangle is a four-sided polygon in which the opposite sides are equal and parallel and each interior angle is 90^0. The sum of the angles of a rectangle is 360 degrees.

Examples:

Fig. 6.17

Three-dimensional shape

These are classified into prisms, pyramids cones, spheres, cylinders and polyhedrons, or polyhedral.

A prism has two parallel bases and rectangular faces. All the faces are perpendicular to the base (Figure 6.18).

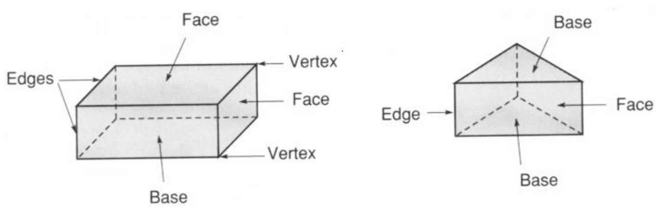
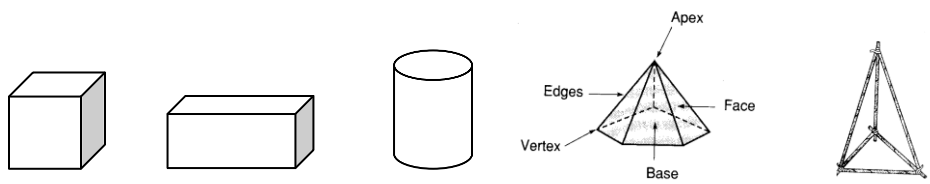


Fig. 6.18

Pyramids have only one base and triangular faces that meet at an apex (a point at the top) as in figure 6.19 Pyramids learnt in primary school have regular polygons as bases, despite that pyramids can have irregular polygonal bases. Both prisms and pyramid acquire their names from their bases, for example rectangular prism, triangular prism and pentagonal pyramid.

Solids which are entirely made up of polygonal faces are called polyhedrons (Figure 6.19) namely cubes, cuboids, circular prisms, pyramids and tetrahedron, just to name a few.



Pentagonal Pyramid tetrahedron

Fig.6.19

Pupils have a lot of experience inside and outside their homes with objects that have three-dimensions. You need to build on those experiences by using familiar three-dimensional models through activities. Your mathematics corner should be full of boxes with different shapes such as prisms, pyramids, cones, spheres and cylinders. Ensure that one from each type is displayed.

Your activities should be varied according to the age group of the class. The activities of young pupils may be simple such as finding examples of familiar shapes or forms while those of older pupils may be more challenging.

Many pupils cannot visualize a three-dimensional shape drawn in two-dimension space. Therefore you must ensure that you have models of solid figures. Also give them activities that will make them explore the characteristics of three-dimensional shapes, such as making solid shapes and sketching three-dimensional models (drawing the faces, vertices, apex and edges) and slicing (cross-section) of solids. When sketches are made ask pupils to figure out where the sketcher was positioned. You ask pupils questions, which will enable them to relate flat shapes with solid shapes. Most important of all remember to take pupils for a geometry walk which can be done inside or outside school ground. Young pupils can be given simple tasks such as identifying familiar shapes or forms. Older pupils can discuss whether a shape is natural or human-made and whether it has utilitarian or aesthetic value.

Line of symmetry

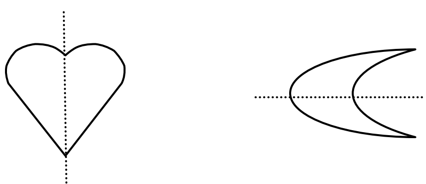
A line that separates a picture or design or a flat shape into identical halves is called a line of symmetry.

You can introduce line of symmetry using the following activities.

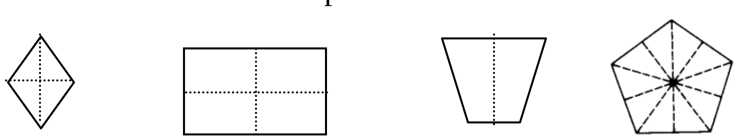
Activity 6.9

1. Prepare several lids containing various kinds of coloured paint. Tell pupils to fold their papers in half. On one side of the paper, pupils put drops of different colours of paint. Then they press the other half of the paper over the painted side and smooth it out. Pupils open the fold to see their designs. After the designs have dried up, display them. What do pupils notice? Do they consider the pictures on both sides of the fold to be the same? Let them justify their answers. Tell the pupils that the line along the fold is a line of symmetry. It divides the design into two like halves. Ask pupils to find flat shapes in the classroom or outside that are symmetrical (including numerals and, alphabetical letters).

2. Provide pictures like the ones below. Ask pupils to draw lines of symmetry on them as shown below.



3. Give groups of pupils paper cutouts of a square, rectangle, equilateral triangle, isosceles triangle, regular pentagon and other polygons. Let them determine the different ways each figure can be folded so that one half exactly covers the other half as shown in the shapes below.



Pupils record the number of lines of symmetry for each shape on a table like the one below.

Shape	**No. of sides**	**Lines of symmetry**
Square	4	4
Triangle		
Pentagon		

Encourage the pupils to find out if there is any relationship between the number of sides and the number of lines of symmetry in a regular polygon.

Does this relationship exist in a rectangle and an isosceles triangle? What about the other polygons? Are there any polygons that have no lines of symmetry? What can be said about non-symmetrical shapes?

Reflection

What have you been able to learn in this unit that you previously did not understand and found difficult to teach? Record this in your diary.

UNIT 7

MEASURES

In this unit we deal with measures. We deal with measures that contribute to size (length, area, mass and capacity) and those that have no physical quality (time, speed, money and temperature).

Learning Outcomes

As you study and work through this unit you are expected to:

- demonstrate skills in measurements of length, area, capacity, mass and time.
- design activities that enrich, and extend the learning experiences of your pupils in the above skill areas.

Measures

Given a broken ruler, Shawa fails to measure the length of the line segment in Figure 7.1. She asks for an unbroken ruler. Do you think she could measure correctly with the broken ruler?

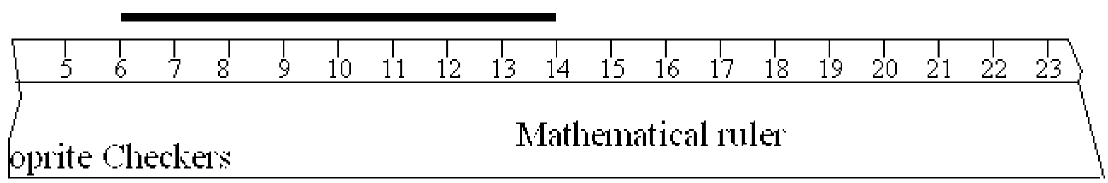


Fig. 7.1

Most teachers teach pupils "how to measure" instead of "what it means to measure". They insist on putting the starting point of a ruler "0" at the beginning of something being measured. We know that the measurement of the length of an object is the distance from one end to the other.
(Here: 14-6 = 8).

To develop the concept of measurement the following three types of activities will be used.

1. Comparisons to help pupils focus on the property of the object and to practice comparative (e.g. longer of two objects) and superlative (e.g. the longest of more than two objects) language of size. Besides direct comparison of two or more objects you as a teacher should find out if pupils have reached a stage of maturity called conservation. Conservation is a development stage of children when they can

differentiate the sizes of objects. If they have then they are ready for measuring and will be able to tell that the quantity or length of an object does not change even if the object's position changes.
2. Measuring using non-standard units of measure helps pupils understand what measurement is and make them recognize the need for a uniform set of measures, laying a foundation for pupils' understanding of standard units.
3. Making measuring instruments and linking them with non-standard units of measure makes the transition flow smoothly.

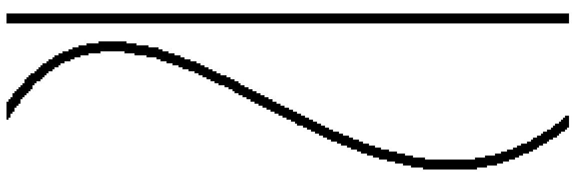
Besides the three activities the skill of estimation is very important and should be practiced in all measuring activities as it helps pupils to understand what it is they are measuring, and the measuring process. Estimation also helps pupils to develop familiarity with the unit of measure.

Measuring length

The meaning of length will be developed by activities. Do Activity 7.1

Activity 7.1

1. Comparing straight and curved lines. Which one of the following lines shown below is longer than other? How do you know?



2. Comparing parts of your body. Find parts of your body that have about the same lengths.

Did you find that when your fingers are closed together with your hand out stretched, the end of your thumb to the end of your little finger is about the same length as from end of your thumb to end of your middle figure?

3. Now prepare lessons based on the set of activities 6.10 given below. Teach the lessons. Take note of your experiences and keep them in your teaching file together with the lesson plan.

Teaching Tips

1. Conservation of length

a) Call two pupils to stand side by side in front of the class. Ask the class, "Who is taller"?

- Ask one to sit or stand on a desk. Ask, "Now, who is taller"?
- Make the difference clear between higher and taller by demonstrating using

sticks.

b) Take two shoelaces of the same length.

- Stretch them side-by-side and ask pupils which is longer?
- Then curve one. Ask pupils which shoe lace is longer than other?

2. Ordering lengths of objects. Ask pupils to arrange in order of length, more than two objects of different lengths.

3. Comparing parts of pupils' own bodies

Play the "if" Game.
Tell pupils to play the "if" game as follows:

- If you find two parts of your body that are about the same length, clap three times.
- If you are shorter than the teacher, smile.
- If you find a part of the body longer than the other, hop on one leg.

After the game, pupils report on their discoveries and the methods they used in comparing.

Are you aware that the length between your shoulders is about the same length as the circumference of the base of your neck?

Non-standard Units of Length

Activities with non-standard units bridge the gap between comparison work and the introduction of standard units. The non-standard units of measure for length are the natural units of measure, the parts of body - foot, span, stride as shown in figure 7.2 and unit models such as keys, paper clips and sticks.

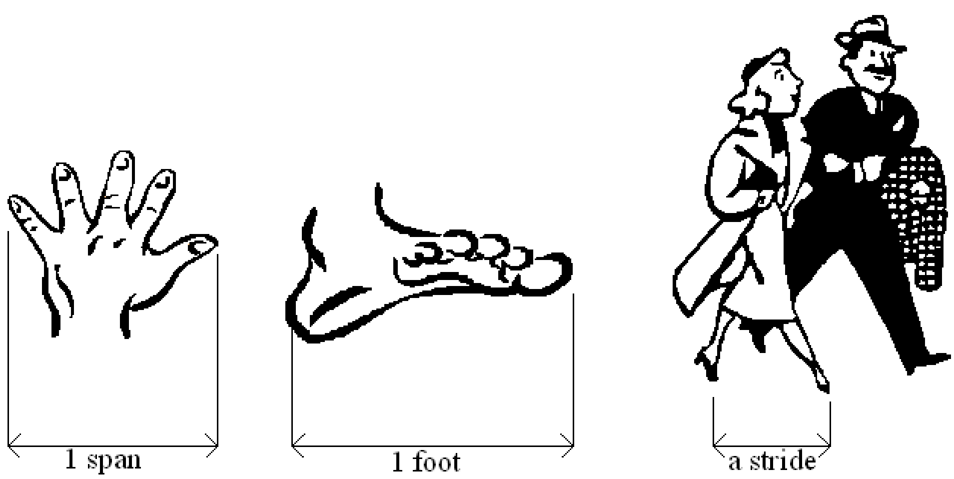


Fig 7.3
Consider any difficulties that might arise in the world if each nation had a local unit of measure different from the others.

Do the set of activities 7.2 in class.

Activities 7.2

1. Measuring using parts of the body. Demonstrate how to measure lengths using parts of the body e.g. foot, hand span and stride.
- Choose two lengths to be measured by pupils, one on measuring height inside the classroom and another on measuring length outside, using parts of the body.
- As a class the groups decide on two natural units of measure (part of the body) to measure the two lengths. Ask the group to estimate before they measure - use approximate language and state unit of measure (about ten paces or about six hand spans, etc.).
- Tell pupils to record their answers on a worksheet.
- Ask pupils to compare their estimates with actual measurements within their groups.
- Ask groups to compare their actual measurements. Are the groups finding any difficulty in communicating their measurements?
- Ask pupils to discuss any difficulties arising from choosing and using natural unit of measure.

Standard Units of Length

Pupils may have realised on their own the need to have a common unit of measure for easy communication through the series of activities you have done with them. This is what made nations come together and create standard units of measure. In lower primary, pupils are introduced to a metre (m), a centimetre (cm) and kilometre (km).

In unit 4 you learned about decimals. Here you will extend this knowledge to measuring length in decimals (metre). When we want to be precise we use smaller units of measure. Instead of saying about a metre, we find out what part of a metre the length is in decimals. Then read and record the measurement in decimals.

Have you ever seen the metric instrument used to measure length, shown in figure 7.3 below? It is called a trundle wheel.

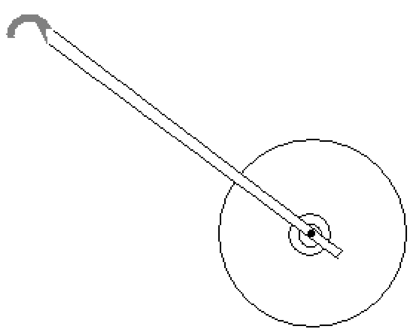


Fig. 7.3 Trundle wheel

Perimeter

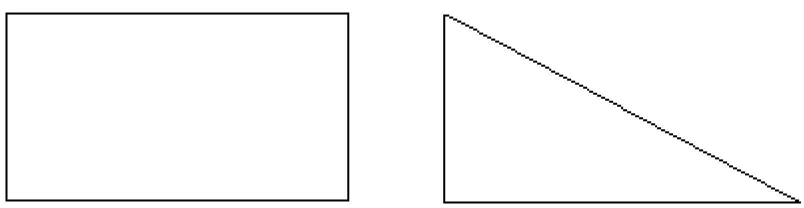
The idea of perimeter (distance around an object) will be developed using a set of activities 7.3. Prepare a lesson based on these activities, teach the lesson and remember to keep record of your experiences with pupils in class.
Keep the record in your teaching file.

Activity 7.3

1. Introducing perimeter. Discuss with pupils situations where it will be necessary to know distances around an object. For example, building a fence and making a belt. Pupils show perimeter of objects by walking or tracing around an object.

2. Finding Perimeter. Make a simple worksheet where pupils will compare perimeters of shapes.

Name:……………………………………Worksheet



Name the above shapes
Measure around each shape using a string
Draw a line that is just as long as the distance around the shape on the board in your classroom.

Area

You may use the following idea to introduce area.

- Collect several objects that are suitable for covering surfaces such as cloth material, books and rulers.
- Cover surfaces using two of the objects at a time. Ask pupils which object covers more or less surface.
- Pupils give explanations to their answers.

- Organise objects with different areas and ask pupils to arrange them in order of sizes.

Name the above shapes.

Non-standard Units of Area

Here you are going to use non-standard units of measure, such as books and sheets of newspaper as units of measuring area to build the idea that area is a measure of covering surface.

Do activities 7.4 before you proceed.

Activity 7.4

Measuring area with a unit model

- Choose a non-standard unit of measure to measure the area of an object of your own choice. For example, a rectangular sheet of paper to measure the surface area of the teacher’s desk.
- Measure in two ways. The first way is to make enough copies of your unit to cover the whole surface. The second way is iteration, (repeating) single unit. Which method did you find difficult? Why? Which method do you think, has a better effect on helping pupils remember the meaning of area? Explain.

Now do the set of activities 7.5 with pupils in class. Record your experiences in your teaching file.

Activity 7.5

1. divide your class into groups. You will need templates of circles, squares, triangles and rectangles.

- Tell each group to cover the area of an exercise book using each of the above four shapes, one after another.
- Discuss with pupils the possibility of one shape being more appropriate for covering surface than others. Which shape covers most area?

2. Measuring area with a square

Before you proceed, make sure pupils have made a decision that a square best covers the surface.

- Groups decide to measure areas of their desks using a square. Each group prepares a square of the same size as a uniform unit of measure.
- Tell pupils to estimate individually the areas of identical exercise books using the uniform unit. Pupils explain on the process they used to predict.
- Tell groups to measure in two ways using enough squares and iteration

process. Discuss with pupils on the method of covering which they prefer (prepare appropriate questions).

- Tell each pupil to compare his/her estimate with the group's actual measurement.
- Ask groups to compare their actual measurements.

Standard unit of Area

You can introduce a square metre as the first standard unit of area using activities 3 and 4 given below. Other standard units of area like the square centimetre and square kilometre can be introduced in a similar way.

3. Making a square metre. Tell pupils to measure a square metre. Pupils estimate and find out how many of their unit model can cover the square metre. Observe if their estimates are getting better.

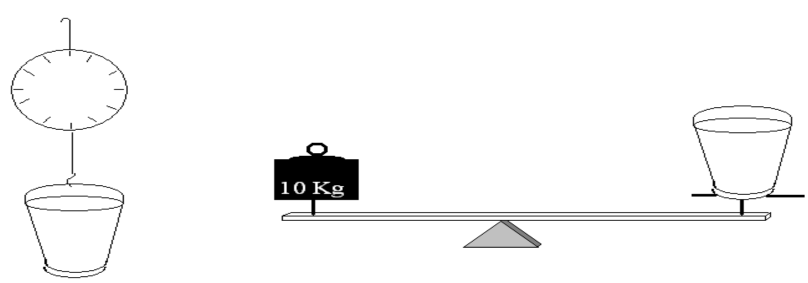
4. Measuring area in square metres.

- Tell pupils to find the area of the classroom by drawing square metres that cover the floor of their classrooms. Ask them how many square metres they have made. Tell them to approximate if there is a part or parts of a square metre.
- Now ask pupils to find the number of metres by adding. Ask them to find the number of rows and number of square metres in each row. Ask the pupils to find the total number of squares using addition. Tell them to make a multiplication sentence from their addition sentence. The multiplication is the origin of the formula for area.

At this level do not talk about the idea of formula, but encourage pupils to use multiplication as short cut to repeated addition.

Mass

The concept of measuring mass begins with finding out which of the two objects is heavier.
Figure 7.4 shows two instruments used for measuring mass **(**or weight).



Spring balance Beam balance

Fig. 7.4

Weight is a measure of the pull or force of gravity on an object. Mass is the

amount of matter in an object. On the moon, where gravity is much less than on earth, an object has a smaller weight but the same mass.
Gravity is the natural force acting on the earth's surface to pull down any object which is suspended in the air.

People use the terms weight and mass interchangeable in everyday life. However, mass and weight are not the same but the explanation of the distinction is beyond the pupils' scope at primary school level.

Measurement of Mass

Various scales are used to measure mass; the most common are the spring balance and the beam balance, shown in figure 7.5

You can introduce measurement of mass by using a variety of activities as given in the set of teaching tips below

Teaching Tips

1. Comparing masses of two objects. Prepare a variety of several pairs of objects, some pairs equal in mass, some small heavy objects and large light objects such as a sealed-empty carton box.
- Ask pupils to compare pairs of objects by holding one in each hand and estimating which is heavier or lighter. This procedure is called hefting.
- After hefting let pupils put the objects on a balance scale. Allow pupils to discuss the behaviour of lighter and heavier objects and those of equal mass on a balance scale. What object goes down or up why?

2. Making simple scales. Pupils make simple scales out of available materials such as a pair of identical two tins and rubber bands of same length as in the diagram below.

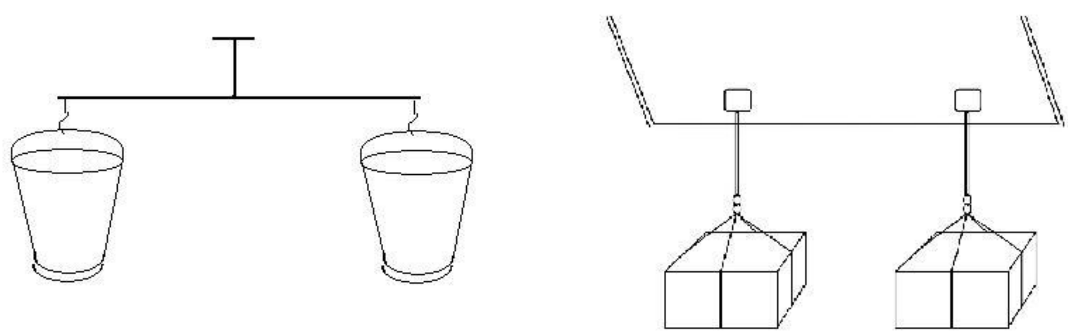


Fig. 7.5: Simple scales

- Ask pupils to compare and order three or four objects by hefting.
- Pupils check their estimates by weighing the objects on their simple scales.

Seesaw

3. Set up a seesaw in the playground. Mark some sitting positions equidistant from the pivot. Instruct groups of three pupils to use the seesaw to determine who is the heaviest, middle and lightest of the group. Ask pupils to record their findings on a chart such as this one.

GROUP	HEAVIEST	MIDDLE	LIGHTEST

Conversation of mass

4. Get two equal masses of clay. Shape them spherically. Ask pupils which one is heavier.

- Re-shape one of the spheres into sausage-like form. Ask which one is heavier now. Allow them to explain their answers.

Non-standard units of measure of mass

Any collection of uniform small unit objects such as sets of seeds, bottle tops and paper clips with the same mass can serve as weights.

5. Comparing masses. Prepare the objects whose masses are to be measured. Make sure pupils have sufficient units of mass to use when weighing.

- Let pupils discover how to balance a scale by putting an object in one tin and other objects in the second tin.
- Now tell them to play a guessing competition of mass. Pupils guess how many units balance an object? They find the right answer by weighing and then identify all those who got the correct answer.

Standard units of mass

The standard unit of mass that is introduced at lower primary level is a kilogram (kg).

Capacity

You observe a trader measure a bucket full of milk before she starts selling. She fills a cup with milk and pours in another bigger bucket. She looks up and sees you watching her. She smiles and continues measuring. At last she completes the task. Then she says, “I look crazy, what do you think I was finding out?’

It is likely you will answer she was finding out how many of that cup fill the bucket (the capacity of bucket).

Comparing capacity

Before pupils start comparing capacities of objects, give them enough practice of pouring from one container to another. Extend the skill of pouring without spilling to fill to the indicated levels of the containers.

The set of activities 7.6 have been given for you to do with your pupils in class. Prepare a lesson plan based on these activities, teach it and record your experiences. Keep the record of your experiences and the lesson plan on your teaching file.

Activity 7.6

1. Comparing capacities. Ask pupils to arrange sets of three containers in order of size from "holds most" to "holds least" by sight. Later they should check their order by pouring. Include some containers whose shapes make it difficult to tell by looking.

2. Conservation of capacity. You will need one different and two identical transparent containers.

- Tell pupils to fill identical containers with sand or water to the top. Ask which one holds more.
- Pour all the sand of one of the two identical containers into a transparent different shaped object. Hold up the two containers with sand (the container full of sand and the different shaped container with sand). Now, ask pupils which container holds more sand.

Non-standard units of measuring capacity

Here small containers such as small tins, spoons, cups and lids are used to fill

up larger containers.

Activity 7.7

Find a large container and 3 different small containers.

1. Fill the larger container with water. Estimate how many of each smaller container can be filled from the larger one. Check your estimation by actual measuring each smaller container full of water from the larger one. How good was your estimation for each smaller container?

2. Repeat this with your pupils. Tell pupils to bring or make measuring units such as tins and cups.

3. Tell the pupils to reverse the previous activity. How many smaller containers can fill a larger container?

Standard unit of capacity

The standard unit of measure under consideration here is a litre *(l.)*.

Do activities 10.6 before you continue.

Activity 7.8

Finding non-standard units that make a litre

(a) You will need a container that measures 1 litre and one or two cups of different sizes. You estimate and measure how many cups fill a litre. How good are your estimates?

Measuring in litres

(b) You will need two large containers of different capacities. Estimate their capacities in litres. Then measure the two capacities in litres.

The set of Teaching Tips below have been given for you to try out with pupils in your class.

Time

In the initial stages of learning about time, the emphasis is on the concept of time of day rather than the passage of time. Therefore, your task as a teacher is to make sure that the pupils understand the concept of time of day before you introduce them to measuring time. You may be aware that the concept of time involves an understanding of duration of time and the sequence of events in a given period of time.

Sequence of events activities

Do activities 7.9 with pupils in class. Keep record of your experiences.

Activity 7.9

1. Ask pupils how they can tell whether it is daytime or nighttime. The suitable answers are sun for daytime and moon or stars for nighttime. Don't accept answers such as "going to school" or "going to bed". Tell pupils to draw symbols for sun, moon and stars.
2. Ask pupils to tell you what they do at certain times of the day — morning, afternoon or night.

Conclude the lesson by teaching pupils the "sequence of events song". For example, the “early in the morning song”. The song is suitable for the morning session. It goes like this:

This is the way I wash my face, wash my face, wash my face. This is the way I wash my face early in the morning.
This is the way I brush my teeth, brush my teeth, brush my teeth. This is the way I brush my teeth early in the morning.
This is the way I eat breakfast....

This the way I walk to school…..

This is the way I greet my teacher, greet my teacher, greet my teacher. This is the way I greet my teacher early in the morning. Good morning teacher.

Acknowledge their greeting by a smile and tell them to sit down. You can make a variation to suit the session of your class or compose a new one.

3. After teaching the days of a week in order, demonstrate how to sing the days of the week song. It goes like this:

Sunday, Monday
Tuesday, Wednesday *TWICE*
Thursday, Friday, Saturday

There are seven days
There are seven days *TWICE*
There are seven days in a week

Duration of time

You introduce time by giving pupils a chance to experience “short term” time as it passes. This approach may help pupils to realize that time is something that makes it possible to do something.

Use the set of activities 7.10 to teach time.

Activity 7.10

1. Comparing time. Ask pairs of pupils to hop over a given distance. Discuss who took the longer time and who took the shorter time.

2. Conservation of time. Line up some pupils and tell them to hop forwards for a certain time. Stop the event and discuss which pupil hopped most in the time. Ask the pupils if they have each had the same amount of time. If necessary repeat this activity in later grades till pupils are able to understand the conservation of time.

3. Game, “Racing with sand time”
- Make and introduce a simple sand timer.
- Pupils make their own sand timers using plastic bottles of different sizes and holes, as shown below. Varying hole size and putting different amounts of sand will cater for a variety of time periods (4 seconds, 1 minutes etc. These units of measure are for your use only, not for pupils).

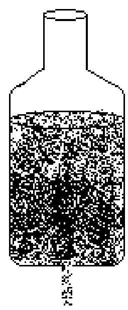
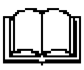


Fig. 7.6 A sand timer

- Pupils decide on the type of racing events. For instance hopping twenty times or running over a distance. Also tell them to decide on the amount of sand to put in the timer. If they complete the event before the sand runs out, they win the game. Otherwise, the sand timer wins the game and the timekeepers cheer: “sand timer, sand timer, sand timer”.



Non-standard unit of time

The non-standard units of measure of time might be the duration of a pendulum swing, or the movement of the sun’s shadow between two-fixed points (as the length of the shadow and its position on a sundial).

3. Measuring using a simple pendulum. You devise other timers such as a simple pendulum. Tell pupils to count the number of swings it takes them to complete an event.

4. Making a sundial. You and your class make a sundial using a stick stuck in clay. This is placed on a sheet of paper as shown below.

- Set the sundial outside when pupil’s class session begins. Tell pupils to mark the shadow’s position probably when the session starts, break time and time to go home. Write events against the shadows. Pupils observed the positions and lengths of the shadows for a week. Take note of the period (dates) you are doing this activity.
- You can make a variation by using pupils’ or a building’s shadows. Encourage pupils to take note of positions and lengths of their shadows at certain times of the day in relation to the position of the sun.

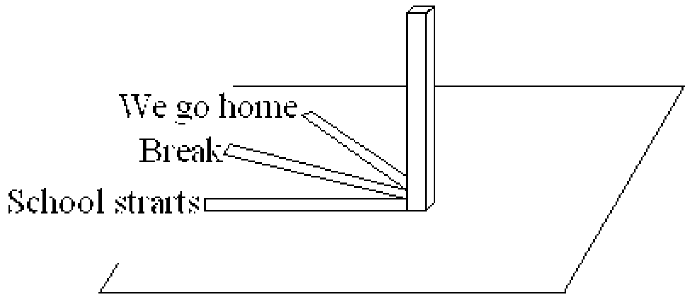


Fig. 7.7 A sundial

Standard unit of time

Natural measures of time are day, lunar (moon) month, seasons and year. Telling time is using a human rule for dividing up the day. The skill of telling the time in hours and minutes ("short term" time) is developed here and also activities on experiences with "long term" time. Since learning to tell the time is difficult, we have devised a lot of activities on telling time in hours and minutes. You should have real working models of both digital and analogue clocks in the classroom. Be prepared to spend a lot of time developing these ideas.

Activity 7.11

Making clock faces. You will need hard board and cutting tools.

- Make two demonstration clock faces, one for analogue (with hands) and the other one digital clock.
- Make a small analogue clock face template for pupils' practice. Later ask pupils to put hands on their models. Keep the three models in your teaching file.

You can use the set of activities 7.12 to teach more about time.

Activity 7.12

1. Discuss with pupils how they are able to tell whether it is early in the day or late and whether it is early in the night or late and explain that using a clock can solve the difficulties that came up here.

2. Reading time in hours. Introduce reading time in hours by using only the short hand (hour hand). Tell pupils to observe positions of numbers. Invite pupils to set the hand pointing at numbers or between numbers. Emphasize the use of approximate language. For instance "about nine hours" or "halfway between four hours and five hours" or" a little bit after eight hours".

3. Now put a long hand (minute hand) to your hour clock. Set time at different

to write time: 08:00,16:45; etc.

4. Duration of one hour. Give pupils experiences of duration of one hour. For example what activity takes duration of one hour? Ask pupils to set their small clock face models to the same time (in hours) as the analogue demonstration clock. Ask pupils to tell what time it is.

5. Duration of one minute. Get a stopwatch and models of 60 minutes clocks. Give pupils an experience of a minute. How much can the pupils do in a minute? For instance, how many hops can they make in a minute? Can they hold their breath for a minute? How many pendulum swings in a minute? (Pupils guess then measure).

6. Measuring time in minutes: Pupils read time in minute intervals from a 60 minutes clock (stopwatch). Tell pupils to set and read time in minutes on their 60 minutes clock model. Pupils find out how long it takes them in minutes to do certain events, for instance, walking or hopping to the head teacher's office. Make sure pupils understand the term "clockwise" and that 60 minutes = 1 hour.

Remember: guess, measure and record

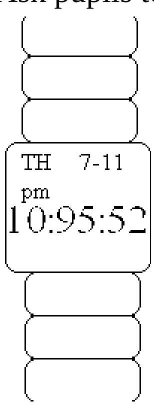
7. Reading time in hours and minutes. Now, get back and read time on models of analogue and digital clocks.

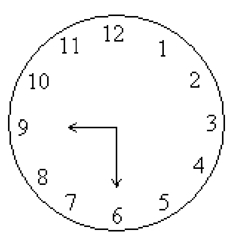
Help pupils link their knowledge of reading time in hours and reading time in minutes to enable them to read time in hours and minutes.

Ask pupils to show time in hours and minutes on an analogue clock then the same time on a digital clock. Ask them to read and record the times. Let them tell the meaning of the times. For instance 0715h as 15 minutes after 0700h or 45 minutes before 0800h.

Explain that a digital clock is a display of time in hours and minutes (minutes past the hour only.

Ask pupils to tell what is wrong with readings on clocks shown below:





8. Reading time on a real clock.

Throughout the year give pupils opportunity to read time on the clock in the classroom.

9. Months in a year

Ask the class what we use to tell what day it is. Look together at a calendar and review the idea that a calendar tells days, weeks, months and the year. Discuss the fact that clocks tell about hours in a day and that calendars help tell about longer periods of time.

- Keep a classroom calendar with removable tabs for months and recording dates. A pupil inserts a date for each new day.

Speed

In your school, there are times when sport competitions are organised. Pupils take part in various activities to compete for different prizes. Most of these sport activities are won by pupils who do their tasks faster than their friends.

In a 100m race, Bwalya took 20 seconds to reach finishing point while Phiri took 25 seconds. From this race it shows that Phiri took a longer time finish race than Bwalya. On average, Bwalya was covering 5m in a second while Phiri was covering 4m in a second. There the speed of Bwalya was higher than that of Phiri.
Therefore;

$$\text{Average speed (S)} = \frac{\text{Distance (D)}}{\text{Time (T)}}$$

or

$$S = \frac{D}{T}$$

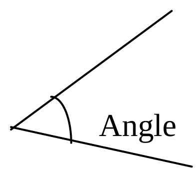
Angles

An angle is an amount of turning. At the beginning of your lesson, pupils must be given an opportunity to discover on their own what is meant by an angle. Your task as a teacher is to expose the pupils to some activities that show meaning of an angle before introducing the measurement of angles.

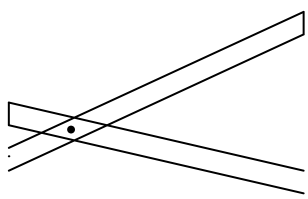
Activity 7.13

Do the following activities with your class. Note down all your experiences with pupils in the lesson and keep them together with your lesson plan on your teaching file.

1. With your pupils, open your mouths and examine how wide each mouth opens by looking from the side view. Make some drawings from the side views by using straight lines as in the two diagrams below.



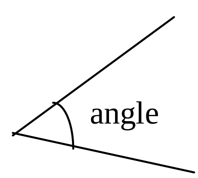
3. Ask pupils to make cardboard bars as shown in the diagram.



Cardboard bars

With the class turn the arms to different extents and ask your pupils to compare the gaps created between the cardboard arms.

The turning of cardboard bars and clock arms through some gap makes an angle.



Measuring angles

Angles are measured in degrees. A complete turn is said to contain 360 degrees. This makes one revolution. One revolution is made when a thing rotates in one direction and gets back in the same position. For example; an arm of a clock turns from 01 00 h through 02 00h, 06 00 h, 10 00hr until it gets back to 01 00h. This is said to be a complete revolution. One degree is written as 1°.

A protractor is used to measure the sizes of angles more accurately (Fig 7.8)

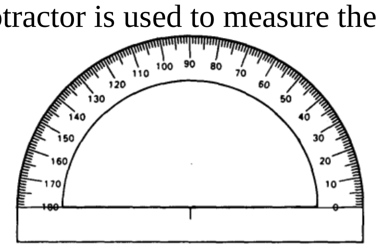


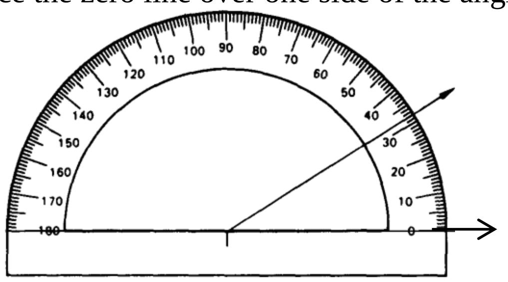
Fig 7.8

Activity 7.14

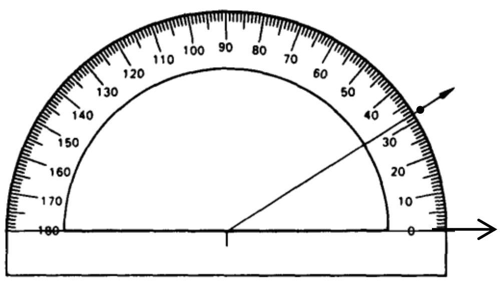
Demonstrate to the pupils by drawing a 50° angle with a protractor by following the steps below.

1. Draw a straight line from a point that will become a convergent point.

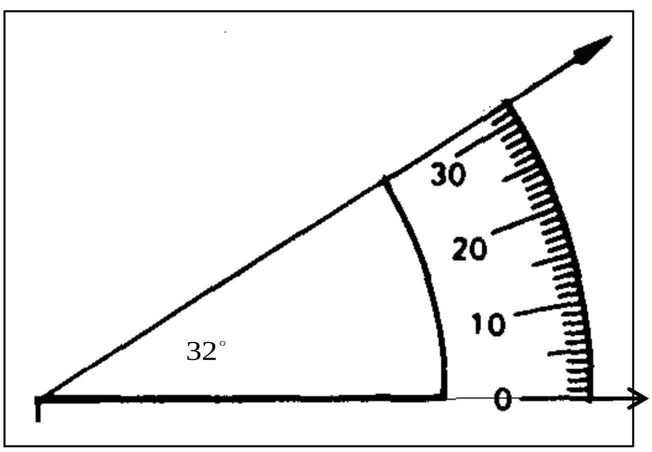
Place the centre of a protractor over the convergent point of the angle. Place the zero line over one side of the angle.



3. Write a point at the 32° (3)



4. Draw a line between the converged point and the point to make the other side of an angle. (4)



Now ask your class to draw the following angles by following steps done above and discuss afterwards.

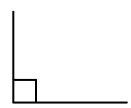
(a) 90˚, 45˚, 180˚, 270˚ 360˚.   (b) 21^0, 49^0, 108^0, 217^0, 325^0

Keep records of your experiences.

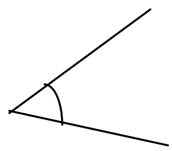
Types of angles

There are special names for some angles. We have the following types.

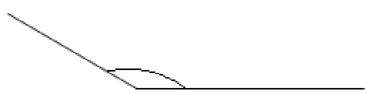
1. 90˚ angle is called right angle



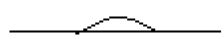
2. Angle less than 90˚ is called acute angle.



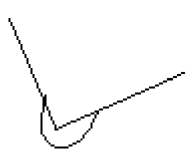
3. Angle greater than 90˚ but less than 180˚ is called Obtuse angle.



1. A 180˚ angle is called a straight angle.



5. An angle above 180˚ but less than 360˚ is called a reflex angle.



Summary

In this unit you have had the opportunity to improve on the knowledge of understanding of the topics of shapes and measurement and to enhance your skills in teaching them.

- You have been introduced to a range of teaching strategies of measures such as length, perimeter, area, mass, capacity and time.
- You have learned how to develop activities for your class to enrich their learning experiences, emphasizing the need for practical experience and creativity.

UNIT 8

SET THEORY

This unit introduces you to the concept of sets. You will be exposed to the basic ideas of sets as groups of objects. This should enable you teach basic ideas of sets at all levels of basic education level.

Learning Outcomes

As you study and work through this unit you are expected to:

- describe sets.
- sort objects into sets.
- compare sets.
- use membership symbols $\in$ and $\notin$
- state equal and equivalent sets.
- find union and intersection sets.
- represent sets on Venn diagram.

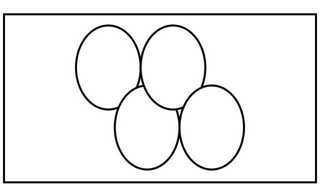
Think of all sets that you belong to and list them down.

Activity 8.1

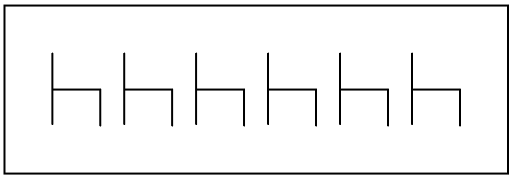
What do you understand by the term set?
Compare your definition with what is written below

Describing Sets

You may think of sets as “families”. For example, we may talk of the set of creatures with eight legs, the set of creatures able to live on land or in water; the set of tables. In lower grades (1-4) the early tasks given are those of sorting objects into their “families” and then move on to comparing and ordering sets. At middle basic level, pupils are introduced to the four operations on sets such as union and intersection sets. They are also introduced to drawing Venn diagrams.
Another way of looking at sets is to describe it in terms of its members. This allows children see the common and different features of a set.



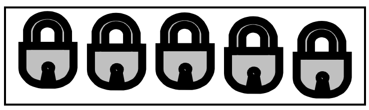
This is a set of eggs.



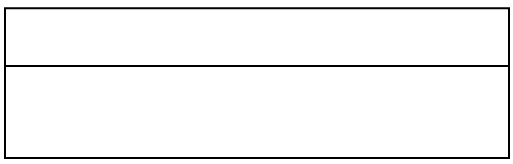
This is a set of chairs

Activity 8.2

1. Complete the following.

(i) 

This is a set of …..

(ii) 

This is the set of …

We may define a set as a collection of well defined elements or objects. A set is well-defined if we can tell whether or not a particular object is an element of that set. Each object in a set is called an element or a member of that set.

Set Notation

We use capital letters to denote a set. Elements of a set are enclosed in curly brackets.

Examples:

$A = \{a, e, i, o, u\}$

$B = \{1,2,3,4,5\}$

C = {Mutinta, Bwalya, Banda}

Membership of a Set

Consider the sets A, B and C above, for instance i is a member of set A. We can write this in short using the symbol $\in$ to denote membership.

$i \in A$

And is read as "i is a member of A" or "i belongs to A". Similarly, $2 \in B$ and Mutinta $\in C$.

We can clearly observe that b is not a member of A or b does not belong to set A. This is written in short as $b \notin A$

Ways of Presenting Sets

There are four common ways of presenting sets namely:

i. listing
ii. description
iii. set builder notation
iv. diagrammatic representation eg. Number line and use of Venn diagrams

Example
Consider the set, N, of the first five counting numbers. This set has been presented by description. Illustrate the set N by

a) listing,
b) set builder notation and
c) Venn Diagram

Solutions
a) Listing

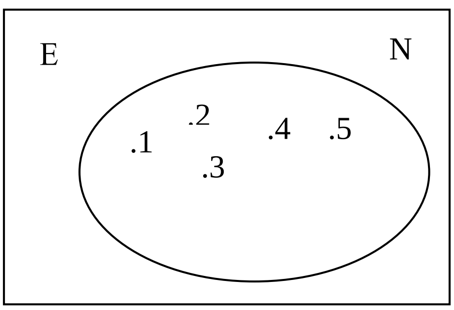
$N = \{1,2,3,4,5\}$

b) Set builder notation

$N = \{x : 1 \leq x \leq 5, x \in N\}$

Which is read as "N is the set x such that 1 is less than or equal to x and x is less than or equal to 5, where x is a natural number.

c) Venn Diagram



E denotes the universal set. The Universal set is the set that contains all the elements under discussion. It can also be denoted by **U.** The dot in front of each element indicates that each number is representing a distinct element and not number of elements (or cardinality)

Activity 8.3

Let C be the set of the first ten whole numbers. Illustrate C by

i. Listing
ii. Set builder notation
iii. Use of a Venn Diagram

Types of Sets

Finite and Infinite Sets

Let Set $A = \{1,3,5,7\}$, $B = \{1,2,3,4,5,...\}$ and $C = \{\ \}$

How many elements are in each of the sets A, B and C?

You may have noticed that it is possible to find the number of elements in set A and set C but it is not the case for set B. What reasons can you give for this observation?

It is evident that the number of elements set A is 4, denoted by $n(A) = 4$. Therefore set A is finite. Similarly, the number of elements in set C is zero denoted by $n(A) = 0$ and set C is also finite. It is not possible to state the number of elements in set B. Therefore, set B is an infinite set.

A set is finite if it is possible to list all the elements that belong to a given set and it is infinite if it is not possible to list all its elements. This implies that a set is finite if in the process of counting the different elements of that set, the counting process can come to an end, otherwise it is infinite.

Equal and Equivalent Sets

Equal Sets

What do you understand by equal sets? Two sets are equal if they have exactly the same elements.

Example

If $A = \{1,2,3\}$ and $B = \{3,1,2\}$, then set A is equal to set B, and is denoted $A = B$. Note that the order of elements does not matter. Similarly, if set $C = \{a.e.i.o.u\}$ and $D =$ { vowels in the English alphabet}, then $C = D$. Also, note that when two sets are equal, the elements are exactly the same and the number of elements in the two sets is also the same.

Equivalent Sets

Let set $A = \{a, b, c\}$ and $B = \{1,2,3\}$

What can you observe about the sets A and B?

It is clear that the elements in set A are different from those in set B, however, the number of elements in set A is the same as the number of elements in set B. This implies that the two sets, A and B are equivalent, denoted by $A \Leftrightarrow B$. Note that $n(A) = 3$ and $n(B)$=3, therefore $n(A) = n(B)$. You may have observed that equal sets are equivalent, but equivalents sets are not necessarily equal.

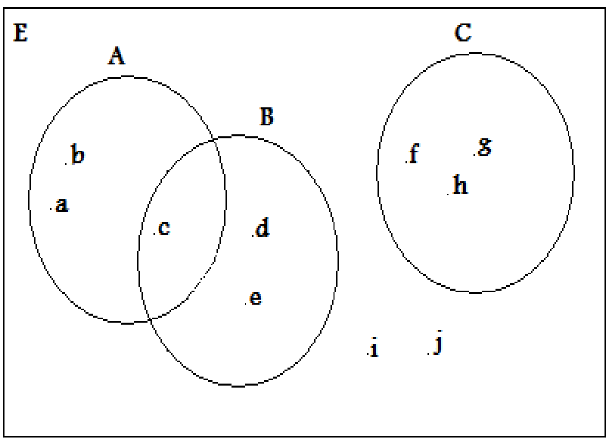
The Universal Set

The Universal, denoted by E or U, is the set of all elements under consideration in a given discussion.

For instance, E = {a, b, c, d, e, f, g, h, i, j}, where

A = {a,b,c}, B = {c,d.e} and C= {f,g,h}

The Venn diagram for this Universal set would be illustrated as



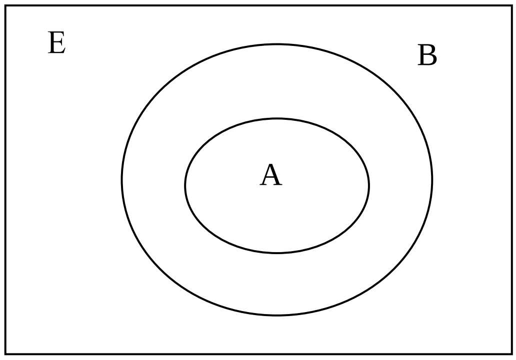
Subsets

Let set A = {3,4,5} and B = {2, 3, 4, 5, 6}

What can you observe about the two sets A and B? It is common knowledge that all the elements of set A are also found in set B.

Definition

Set A is a subset of set B, denoted by $A \subset$ B, if every member of set A is a member of set B. A subset can also be defined as a set that is contained in another set. Diagrammatically, this is shown as:



If two sets A and B are such $A \subset$ B but $A \neq$ B, then A is said to be a proper subset of B. Conversely, if two sets A and B are such $A \subset$ B but $A =$ B, then A is said to be an improper subset of B, denoted by $A \subseteq$ B.

Cardinality and the number of subsets in a given set

Let set A= {a,b}.
The number of elements in a set A, denoted by n(A), is called the cardinal number of the set A. The number of elements in set A is 2, therefore its cardinality is 2 or n(A) = 2.

Example 1
Given that set A= {a, b}. List all the subsets of set A.
The subsets of set A are {a}, {b}, {a,b} and { }
The number of subsets of set A is 4.
It should be noted that the empty set and the set itself are subsets of every set.

Example 2:
Given that set B = {1, 2, 3}. List all the subsets of set B.
The subsets of set B are {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} and { }.
The number of subsets of set B is 8.
POWER SET
The power set of the set A denoted by P(A) is the set whose elements are subsets of A. In the example above, P(A) = {{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}, { }}.

Activity 8.4

What relationship can you observe in Examples 1 and 2 (above) between the number of elements (cardinality) of sets A and B and their respective number of subsets? Can you think of a general way of finding the number of subsets in a given set if the number of elements is known?
Note down your observation and compare with the explanation below.

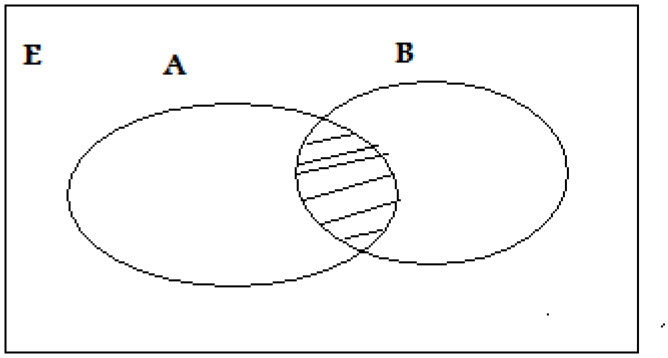
In set A, the number of elements is 2 and the number of subsets is 4. This can be expressed as $2^2 = 2 \times 2 = 4$
Similarly, the number of elements in set B is 3 and the number of subsets is 8. This can also be expressed as $2^3 = 2 \times 2 \times 2 = 8$.
It can therefore easily be observed that:
The number of subsets of a set with n elements is $\mathbf{2^n}$. This method of finding the number of subsets in a given set is very useful in sets with many elements.

Activity 8.5

1. Given that set C = {a,1, b, 2 }
   i. List all the subsets of set C
   ii. Find the number of subsets of set C
2. Find the number of subsets in each of the following sets:
   i. P = {1,2,3,4,5}
   ii. Q ={ }
   iii. R ={n,u,m,e,r,a,c,y}
3. Find the number of subsets of letters of the word "**Mathematics**"
4. i. If a set has 64 subsets, how many elements are in that set?
   ii. Given that set B has 128 subsets, find n(B).

Intersection of Sets

Let A and B be two sets then the intersection of sets A and B, denoted by $A \cap B$, is the set which contains elements which are in both set A and set B. This is the set of elements that are common to both sets A and B.
In a Venn diagram this is illustrated as:

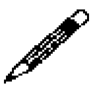


A $\cap$ B

Example:

Given that A= {a,b,c,d,e,f} and B = {c, e, g, h}.

A $\cap$ B = {c,e}



Activity 8.6

Given that the Universal set $E = \{x : 1 \le x \le 10, x \in N\}$, and

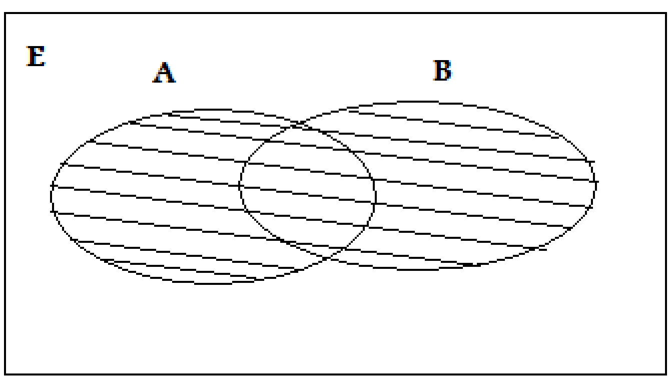
B = { $x : 1 < x < 10$, $\in$ Even Numbers}

C = { $x : 1 \le x < 10$, $x \in$ Prime Numbers}

Illustrate this information on a Venn diagram and hence find B $\cap$ C.

Union Sets

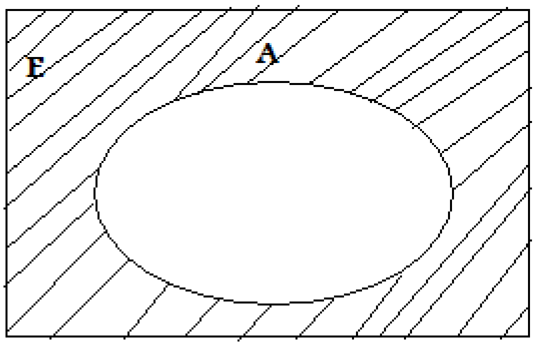
The union of two sets A and B, denoted by A $\cup$ B, is the set of elements that belong to either set A or set B or to both A and B. Diagrammatically, this is shown as:



A $\cup$ B

Complement of A Set

The complement of a set denoted by A' is the set of elements which are **not** in set A but are in the Universal set. This is illustrated as:



A'

Example

Given that $E = \{1,2,3,4,5\}$ and $C = \{1, 3,5\}$, find C'

The elements that are in the universal set but are not in set C are 2 and 4.

Therefore, $C' = \{2,4\}$

Activity 8.7

Use the information given in Activity 8.6 (above) to find:

i. $B \cup C$
ii. $(B \cup C)'$
iii. B'
iv. C'
v. $B' \cup C'$
vi. $(B \cap C)'$

Reflection

It is now time for you to reflect on what you have learned in this unit. Write down in your diary the things that you think have added to your knowledge in the teaching of sets to primary school children or those aspects in which you used to experience difficulties but have been made easier after studying through this unit.

UNIT 9

SOCIAL AND COMMERCIAL ARITHMETIC

This Unit is intended to introduce you to some of the sub-topics that characterize Arithmetic. You will learn the following aspects; money, profit and loss, simple interest, averages and graphs.

Learning Outcomes

As you study and work through this unit you are expected to:

- carry out calculations involving money.
- interpret water and electricity bills.
- explain the concept of profit and loss.
- calculate profit and loss percentages.
- explain the concept of simple interest.
- calculate simple interest given appropriate data.
- present data on pictograms bar charts, bar line graphs and pie charts.
- interpret data from graphs.

Money

Money is used as a means to obtain goods and services. It is a value determining means by which society uses for their business transactions. Each country has its own money type called the currency.

Activity 9.1

Explain the relationship between Kwacha and Ngwee. You may realize that the smallest denomination Ngwee is almost non-existent because of the low value of the Kwacha. Thus the denomination used currently is the Kwacha. Since the value of the Kwacha is low we see that lowest meanings denomination of the Kwacha starts at K50 and so we have K50, K100, K500, K1000, K5 000, K10 000, K20 000 and K50 000.

It is important for you to introduce these Kwacha denominations at an early age because this is what pupils deal with in their homes.

Activity 9.2

1. Show pupils the denominations of the Zambian Kwacha.
2. Give a grade VI or VII class a research on finding the currencies used by the neighbouring countries.
3. In class pupils will report and discuss their findings in number 2 above.

Money Transaction

You may wish to introduce your pupils to addition and subtraction of the money they use in their homes, in terms of what they can buy using say K10 000. The idea of change will bring out the concept of subtraction. While paying for more than one item will bring out the concept of addition or multiplication.

Example 1

. A ten year old girl was sent by her mother to the market to buy the following foodstuffs at given prices. She was given K5 000.to spend.

- 2kg cabbage	@	K500 per kg
- ½ kg tomatoes	@	K500 per kg
- ½ kg onions	@	K1 000 per kg
- 1 kg salt	@	K600 per kg
- 2kg potatoes	@	K1 200 per kg.

(a) How much did she spend?
(b) How much change did she remain with?

Solution

(a)	2kg cabbage @ K500 per kg gives	K1 000
	½ kg tomatoes @ K500 per kg gives	K250
	½ kg onions @ K1 000 per kg gives	K500
	1kg salt @ K600 per kg gives	K600
	2kg potatoes @ K1,200 per kg gives	K2 400
	Total cost	K4 750

Therefore the girl spent K4 750

(b) The change is obtained by subtracting the money spent from the money given to the girl.

Change = K5 000 – K4 750 = K250

Example 2
You may wish to introduce the idea of selective buying by priority to your class. This will give your pupils a sense of budgeting.

Activity 9.3

1. A grade V class is given K50 000 from which they are expected to purchase household goods that a home needs from the following list:

- 25kg bag of mealie meal at	K20 000
- A packet of washing detergent at	K2 500
- A tablet of soap at	K2 000
- Bottle of cooking oil	K6 000
- 2 torch cells at	K2 500 each
- Vegetables at	K3 000
- Potatoes at	K 2 500
- Audio Cassettes (Musical) at	K10 000 each
- Beef at	K12 000/kg
- Chicken (live) at	K20 000
- Salt at	K 600/kg
- Tray of eggs at	K15 000
- 6 bottles of soft drinks at	K 2 000 each
- 5 packets of sweets at	K 2 000 each
- 2kg tomatoes at	K 1 000/kg
- 1kg onion at	K 1 500/kg
- 10 kg bread flour at	K25 000

2. Divide your class in groups using the appropriate number. Let each group budget on K50 000 for the goods/foodstuffs needed in the home.

   From the above situation; develop a discussion with your class on making a priority list.
   - Work out a budget for the purchase of the prioritized commodities,
   - Ask each group why they opted to come up with such a list.

Multiplication and Division

Introduce the concept of multiplication and division involving money. Study the following two examples.

1. Your school has 500 pupils and the PTA has unanimously agreed that each pupil be charged K60 000 towards constructing pit latrines. How much will the school realize after levying the whole school?

Solution

Number of pupils in the school	500
Amount of money per pupil	K 60 000
Total amount the school has realized	500 x 60 000
	=K30 000 000.00

In another situation, a family has three children in one school; a grade VII, a grade IV and a grade I. Their mother has sent K120 000 for the Head to share it among them in the ratio 3:2:1 respectively.How much will each one of them get?

Solution

- You need to add the ratios 3 + 2 + 1 = 6
- Divide K120 000 by 6 = K20 000.

The amount K20 000 should now be multiplied by each of the proportions i.e. 3, 2 and 1.

For the grade I child, you have	1 × K20,000 = K20 000
For the grade 4 you have	2 × K20 000 = K40,000
And for the grade VII you have	3 × K20 000 = K60 000.

The total amount shared will be K20 000 + K40 000 + K60 000 = <u>K120 000</u>

Household Bills

Households spend money in many ways. Apart from paying for food, clothing and school fees, parents need to pay for services such as accommodation, water, electricity, cell phones calls and many others. In this section, we shall concentrate on water and electricity bills. Water and electricity charges are in two categories. We have the fixed charges and per unit charges. The unit used on water bills is cubic metre (m^3). The Unit for electricity is the kilowatt (kW).

Water Bills

If the fixed charge on water is K40 000 per month, the amount of money to be paid every month will remain K40 000 no matter how much water you use each month. This also applies for the fixed charge of electricity.

Let's now look at the cost of water billed in m^3 / Kwacha. If he cost is K500 per Unit and a family uses 6 cubic metres per day, how much will it spend in a month on water?

Solution

Amount of water used in 30 days = 30 × 6 cubic metres = 180 m^3
Amount spent = 180 × K500 = K90 000

It is important to remind your class that this value can only be got if the amount of water used per day remains consistent at $6m^3$. In many cases the number of cubic metres used will vary each day depending on how much water is used.

Electricity bills

A family uses, on average, 12 units of electricity per day. If the cost of electricity per unit is K200, how much electricity is used in 30 days?

Solution.
Number of units used in 30 days = 12 × 30units = 360 units.

Total cost of electricity = 360 × K2 000 = <u>K720 000</u>

Profit and Loss

You have heard of and seen people conducting businesses. When one commences a business, the obvious objective is profit making. What do you understand by profit? The implication is that of gaining more financially than you bought the item that is resold. Loss refers to getting less money than you spent.

Let us assume that you have purchased a bicycle and latter resell it. The amount of money spent on the purchase of the bicycle is called the **cost price,** whereas the amount at which it is resold is called the **selling price.**

Determination of whether a profit or loss has been incurred depends on the cost price and selling price. When the cost price is more than the selling price, the implication is a loss and when the cost price is less than the selling price, the implication is a profit.

Activity 9**.4**

In the table below, fill in the appropriate missing details.

Item	Cost Price	Selling Price	Profit	Loss
A pen		K1 000	K250	
A book	K2,500			K500
An egg	K750	K1 500		
A shirt	K15 000			K3 000

Remember that we can express the profit or loss in three ways, and these are;

(a) As an amount of money
(b) As a fraction of the cost price

(c) As a percentage of the cost price.

Activity 9.5

Refer to the information in the table above to answer the following questions:

1. Express the loss in the selling of a book as a fraction of the cost price in its lowest terms.

2. Express the loss in question 1, (above), as a percentage of the cost price. Based on what we have considered so far, you should be able to realize that;

   (i) A profit or gain of say x% implies that the selling price (S.P) is (100% + x%) of the cost price (C.P)

   (ii) A loss of say x% implies that the selling price is (100% - x%) of the cost price.

Simple Interest

In the olden days, people used to keep their money in several ways. One of them involved putting the money in a tin and then hide the tin in a place where only the owner could locate it .

The above method had its own advantages but certainly one disadvantage was that the money only grew when the owner added some more money.

In modern days, some people borrow money from friends and add some amount on what was borrowed when paying back (Kaloba). Equally, formal institutions like Banks lend out money. The borrowers are then expected to pay an additional amount when paying back.

However, there are instances when individuals or even organization keep their money with Banks, and then these Banks add some amount for using the individual's or organisation's money.

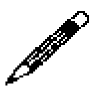
We call this amount of money that the Bank pays for using one's money interest. There are two types of interest that are common: the simple interest and the compound interest. At primary school level pupils learn about simple interest. The Banks mostly use compound interest which calculates interest on principal and previous interest. The amount a client deposits or borrows is called principal. Notice that the amount of money you pay back the Bank after borrowing is also called interest.

We can calculate the interest yearly or annually. It should be emphasized that this interest is written as a percentage of the principal. The percentage is the rate of interest per annum.

Example:

Suppose that John deposits one hundred thousand Kwacha (K100 000) into ZANACO Bank and is paid 10% interest per year. By the end of the year, John's account will have increased by K10 000. At the end of second year, John will have total interest of K21 000 if he does not withdraw. This type of interest is called compound interest. If his interest is calculated using simple interest, then his interest in amount at the end of the second year will be only K20 000. Why is this so? This is because simple interest is calculated only on the principal.

Computationally, Simple Interest = Principal × Rate × Time



Activity 9.6

1. What type of interest does the 'Kaloba System' use? How do you know?
2. What type of method does your savings account use to compute your interest? Justify your answer.
3. The following table, shows four columns; Principal, Time, Rate of interest per year and Simple interest. With the pupils, fill in the blank spaces below.

Principal	Time for which money is in the Bank	Rate of interest per annum	Simple interest
K200,000.00	1 year	2 ½ %	
	3 year	5%	K60,000.00
K1 000,000.00	6 months		K50,000.00
K5 000,000.00		20%	K750,000.00

From Activity 9.6 You should appreciate the skill of calculating simple interest, principal, rate and time.

Reflection

What have you been able to learn in this unit that you previously did not understand and found difficult to teach? Record this in your diary.

UNIT 10

BASIC PROCESSES OF ALGEBRA

You may have met the term 'Algebra' before. What do you understand by this term? In Algebra, letters are used as symbols to represent numbers. In this unit we discuss the algebraic expressions and how they combine according the basic arithmetical operations of addition, subtraction, multiplication and division.

Learning outcomes

As study and work through this unit you are expected to:

- use letters to represent symbols
- expand algebraic terms
- simplify algebraic expressions

Algebraic Expression

An algebraic expression is made up of the signs and symbols we use in algebra. These symbols include Arabic numerals, literal numbers and signs of operation. Such an expression represents one number or one quantity. Thus, just as the sum of 2 and 3 is one quantity, that is 5, the sum of a and b is one quantity, that is (a + b). Similarly, xy, x - y, and so forth, are algebraic expressions each of which represents one quantity or number.

So let us move on and see how we apply this concept.

Translating verbal phrases into algebraic expressions

By using letters, numbers and operation signs we can translate long verbal phrases into shorthand algebraic expressions. For instance, how can you write the expression "the sum of an unknown number and 5" in algebraic form?

Did you think of $x+5$? If you did, then you are correct. The letter x may be represented by another letter, so the algebraic expression could be $y+5$, m + 5, j + 5 and so on.

Activity 10.1

Translate the following words into mathematical expressions

i. 5 less than a number

ii. subtract 5 from a number

iii. multiply a number by 3

iv. A number divided by 4

v. Double a number and add negative 3

Variables

A variable is a symbol, usually a letter, that represents one or more numbers. An expression that contains one or more variables is an **Algebraic expression**. When you substitute numbers for these variables and follow the order of operations, you are **evaluating** the expressions. An expression is algebraic if at least one of its terms contains a letter. It is an arithmetic expression if all the terms are numeric.

Examples

i. Algebraic Expressions;

$a+2b, \qquad cx+dy, \qquad y^2-2y+7 \qquad \frac{1}{2x+5}$

ii. Arithmetic Expressions;

$2+4+7, \qquad \frac{1}{8}, \qquad 2 \text{ x } 5 \div 6$

Simplifying Algebraic Expressions

In an algebraic expression such as, $-3x+4$, the parts that are added are called terms. A term is a number, a variable or a product of a number and one or more variables. The numerical factor in a term is called the numerical coefficient or simply coefficient**.** For instance, in the term, $-3x$, -3 is the coefficient and x is a variable.

Similarly, in the term, $2y$, 2 is a coefficient and y is a variable.

Like and Unlike Terms

Like terms have the same variable raised to the same power. Like terms are added or subtracted by adding or subtracting the numerical coefficients and placing the result in front of the literal factor, as in the following examples:

$7x^2 - 5x^2 = (7 - 5)x^2 = 2x^2$
$5b^2x - 3ay^2 - 8b^2x + 10ay^2 = -3b^2x + 7ay^2$

The expression $5a+3b+c$ contains **unlike terms.** These cannot be added or subtracted to give a single term.

Ways of Simplifying Algebraic Expressions

Let us look at a few examples of how to simplify algebraic expressions.
Examples
Simplify by combining the like terms;
a) $3k - k$

$3k - k = 3k - 1k$

$= (3 - 1)k$
$= \mathbf{2k}$

Remember we said like terms have the same variable raised to the same power. So 3k and k are like terms. They have the same letter k raised to the same power 1. Therefore we may subtract k from 3k as we have done above.

b) $5q^2 - 10q - 8q^2 + q$
$5q^2 - 10q - 8q^2 + q = 5q^2 - 8q^2 + (-10q) + q$
$= (5 - 8)q^2 + (-10 - 1)q$
$= \mathbf{-3q^2 - 9q}$

Here, $5q^2$ and $8q^2$ are like terms (again look at the meaning of like terms). These may be added or subtracted. Similarly 10q and q are like terms. They, too, may be added or subtracted

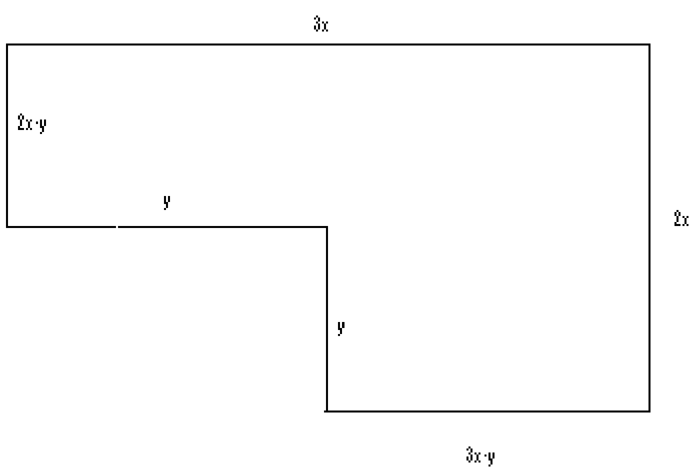
c) $-(m+n)+2(m-3n)$

$$-(m+n)+2(m-3n) = -m+(-n)+2m+(-6n)$$
$$= -m+2m+(-n)+(-6n)$$
$$= -1\times m+2m+(-1\times n)+(-6n)$$
$$= (-1+2)m+[-1+(-6)]n$$
$$= \boldsymbol{m-7n}$$

Activity 10.2

1. Simplify the following;
   i. $2a-3b+c-a+2b$
   ii. $xy-2x^3+x-3xy+x^3$
   iii. $2x^2+5x-4x^2+x-x^2$
   iv. $y(1+y)-3y^3-(y+1)$

2. Find the perimeter



Distributive Law

For every real number a, b and c,
$a(b+c) = ab+ac$

In order to find the value of $2(3+4)$, you may first add the numbers in the brackets together and then multiply the result by 2 or you may multiply 2 with each of the numbers in the brackets and add the two products.

2(3+4) =2 (7) =14 OR 2(3+4) = $(2\times 3)+(2\times 4)=6+8=14$

Examples

Expand and Simplify

a. 3(a + b)

b. 3(4x-2y) – 5(4y + x) + $y^2 - x^3$

c. $2x(x-3)+4(x^2+5)$

Solutions

a. $3(a+b)$

$\mathbf{3a+3b}$

b. 3(4x-2y) – 5(4y + x) + $y^2 - x^3$

$12x-6y-20y-5x-y^2-x^3$

$12x-5x-6y-20y+y^2-x^3$

$\mathbf{7x-26y+y^2-x^3}$

c. $2x(x-3)+4(x^2+5)$

$2x^2-6x+4x^2+20$

$2x^2+4x^2-6x+20$

$\mathbf{6x^2-6x+20}$

Activity 10.3

1. Simplify each of the following expressions

   i. $2x + 3y - 2 + 3x + 6y + 7$

   ii. $3b - (4b - 6b + 2) + b$

   iii. $4a^2(a^2-2ab^2)$

   iv. $xy(xy+x-y)+x^2y$

   v. x + 2xy +3y +4x + 5y

vi. $x^2 + 5x + 4y + 7x + y^2$

2. Find the total cost of 4 books at x Kwacha each and 3 rulers at y Kwacha each.

3. Find the coefficient of xy in the expansion of

   i. $(3x + y)(a + 2y)$

   ii. $(x - y)(3x - 2y)$

   iii. $(4x + 2y)(5x - 3y)$

   iv. $(x - 3y)^2$

Reflection

It is now time for you to reflect on what you have learned in this unit. Write down in your diary the things that you think have added to your knowledge in the teaching of basic processes of algebra to primary school children or those aspects in which you used to experience difficulties but have been made easier after studying through this unit.

UNIT 11

PROBLEM-SOLVING

In this unit you are going to learn some strategies used for promoting problem-solving in mathematics lessons. The unit is a bridge between all the other units in this module in that it focuses on a very important aspect of teaching mathematics. Cockcroft (1982) says, “Problem-solving is at the heart of mathematics”

Although problem-solving has been treated as a separate unit in this module, the teaching of problem solving should be integrated in the weekly lessons at appropriate stages of the development of the topics.

Objectives

As you study and work through this unit you are expected to:

- demonstrate the skills of problem-solving in mathematics
- teach problem solving skills to your pupils from grades 1 to 7.

Problems and problem solving

In life, we face different kinds of problems. You probably have experienced problems yourself. If so, were you able to solve any of them? How did you solve them?

Problems are of different kinds. They can be social, academic, spiritual, physical or financial.

Problem-solving is one of the skills that are being emphasised and encouraged in the teaching and learning of Mathematics today. Problem-solving plays a very important role in the mental development of the child. Problem-solving is the essence of learning Mathematics because it equips the child with techniques of solving problems in life

Before you learn the different strategies of solving mathematical problems, ask yourself the following questions:

- What is a problem?
- What does problem-solving imply?

Activity 11.1 will help you to find answers to these two questions.

Activity 11.1

(a) Consider problems 1, 2, 3 and 4 given below and try to solve them

(b) Try to identify the different techniques you use in solving these problems and note them down.

Problem 1
Bwalya sets off to school with ten mangoes in his bag. On the way, she meets two friends Habenzu and Monde. She gives 3 mangoes to Habenzu and 4 to Monde. How many mangoes does she remain with?

Problem 2
Find the product 27×6

Problem 3
In the multiplication below, a two-digit number is multiplied by a one-digit number to give a three-digit number. Find the missing digits to complete the boxes.

$3\square \times \square = 2\square 8$

Problem 4
You are given two empty containers, one of 3 litres capacity and the other of 5 litres capacity. You are required to use the two containers to come up with the amount of water of I litre required to make an orange drink. How do you measure the 1 litre of water?

How did you do?

Did you encounter any difficulties in solving the four problems? Most likely you found problems 1 and 2 simple to solve but problems 3 and 4 relatively difficult. Do you think a Grade 1 child who has just learned the operation of subtraction would find problem 1 simple to solve? What about a Grade 3 or 4 child who has just learned how to multiply a two digit number by a one-digit number, would she find it easy to solve problem 3?

From these questions you may understand what a problem is. In your teaching, for a given problem

- The children should be interested in finding a solution. Therefore, you as a teacher are required to present the problem in a relevant way
- It should be thought-provoking and challenging. There must be mathematical thinking required in solving the problem, not just routine calculations.

Which of the problems 1 to 4 given above are routine and which ones are challenging?

Problem Solving

As you were working out the problems 1 to 4, you may have asked yourself the following questions:

- Do I understand the problem?
- Have I met something like this before?
- Is it related to my real life situation?
- What techniques can I use to solve it?
- How do I write it out in mathematical language?

The process you went through to arrive at the solutions of the problems above is called problem-solving. For you to carry out the process of problem-solving, the following conditions must exist:

- You should have motivation and curiosity to solve the problem.
- You should have background knowledge
- You should understand what the problem is about.
- Your mind should be flexible enough to try out different techniques of finding the solution.
- You should be asking yourself questions about what information is given, what is not given and what is required to be found.
- You should be able to translate the problem into mathematical sentences and relationships.

Stages of problem solving

From what has been said so far, we can list the stages of problem solving as follows:

1. Read and understand the problem.

2. Identify the important facts.
3. Try many methods or techniques.

4. Provide an answer. Check your work and decide whether your solution makes sense. If it does then you have solved the problem. If it doesn't try again.

Activity 11. 2

1. In your own words, define the following terms

   (a) Problem (b) Problem-solving

2. Identify and list the factors which influence problem-solving

You should keep the answers to this activity in your activity file.

Strategies in Problem Solving

Have you ever played a game called nsolo? You probably know it by another name in your language. Nsolo is a game played by two people. They play it by moving pieces placed in four rows of holes dug in the ground or on large plank of wood or concrete block. They take turns in moving the pieces, trying to protect their own while capturing the opponent's.

Chess and drafts are common games which children as well as adults play. These games involve a lot of thinking to devise strategies to beat the opponent

In this section, you are going to study four strategies of problem-solving, namely:

- Guessing and checking
- •Eliminating
- Looking for a pattern
- Listing

As a general rule, you should try to use sketches and diagrams when solving problems.

The guess and check strategy

Some of the problems in which the guess and check strategy is applied are: operation grids; the 'super adder'; magic squares; arithmogons; 'clever' triangles; targets; missing numbers and cards.

Operation grids

	4	7
	8	3

	32	**21**
28	4	7
24	8	3

Fig. 11.1

You **may** use the grids of the type in figure11.1 on worksheets to give your pupils problems on operation grids.

The rows and columns are multiplied to produce the results at the top row and the left hand column.

The guess and check idea can be introduced by using a similar example. This time the numbers at the top and left are given, as shown in figure 10.2, and pupils are asked to complete the grid.

	20	**48**
24		
40		

Fig. 11.2

What two whole numbers can multiply to give 24? Guess 3 x 8 and check to see if this works. The discussion should show that 20 couldn't be found by either 3 or 8 multiplied by any whole number.

Therefore, the original guess is wrong, as shown in figure 11.3.

	20	**48**
24	3	8
40		

Fig. 11.3

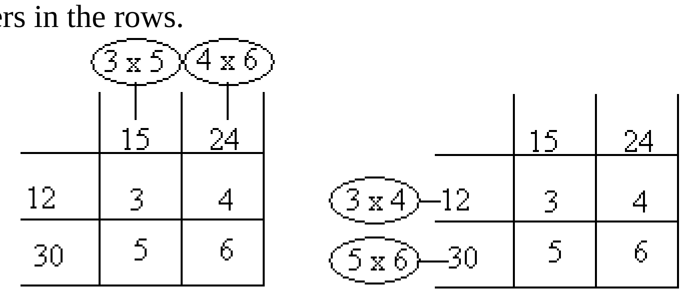
Further guessing and checking will produce the correct grid as shown in figure11.4.

	20	**48**
24	4	6
40	5	8

Fig 11.4

Activity 11.3

1. In the two grids below, the numbers at the top are found by multiplying the numbers in the columns and the numbers at the left are found by multiplying the numbers in the rows.



Now complete the grids (a) and (b) given below:

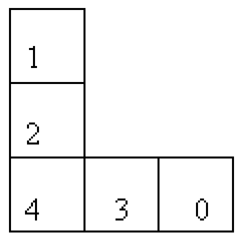
	6	15
10		
9		

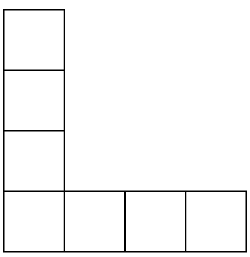
(a)

	84	72
63		
96		

(b)

2. The numbers 0 to 4 have been fitted into the 'L' shape so that the total of the row (7) is the same as the total of the column





Now fit the numbers 0 to 6 into this 'L' shape so that the row and the column have the same total.

3. Formulate 5 problems on operation grids suitable for your class and try them out with your pupils. (Remember you can use any of the operations addition, subtraction or multiplication). Keep record of your experiences in your teaching file.

The 'Super Adder'

There is a snake that jumps. In Lozi it is called Sibili. In English it is called Adder. What do you call it in your language?

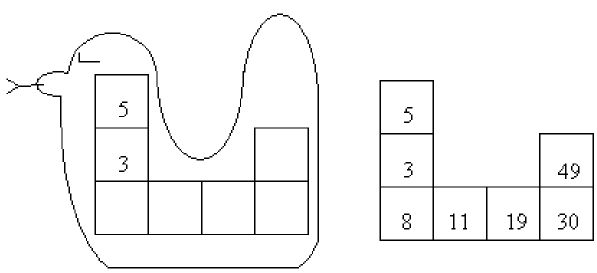
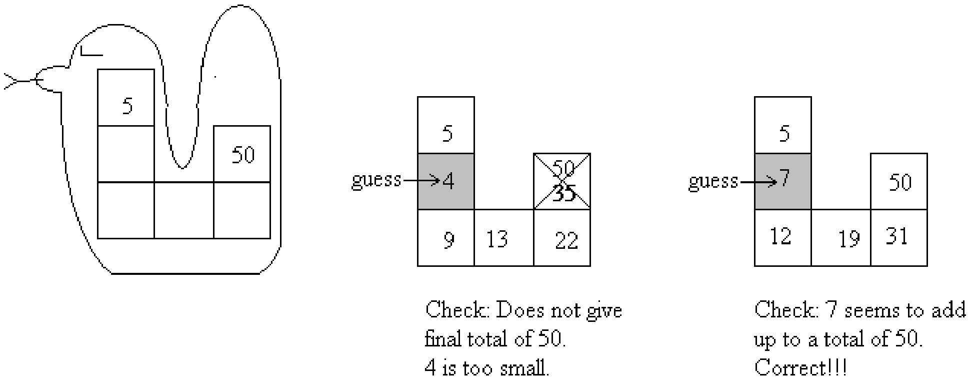


Fig.11.5

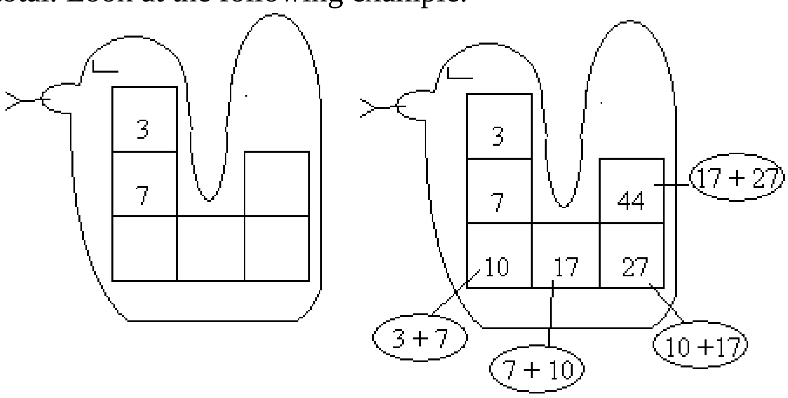
The super adder idea can be introduced in a similar way to the operation grid. The problems are written on work cards as shown in figure11.5.

Starting from the head, each pair of numbers adds together to give the next number on the snake.

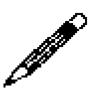
The guess and check strategy is introduced through the example illustrated in the figure below.



A number can be guessed for the second box and then checked to see if it gives the final total. Look at the following example.

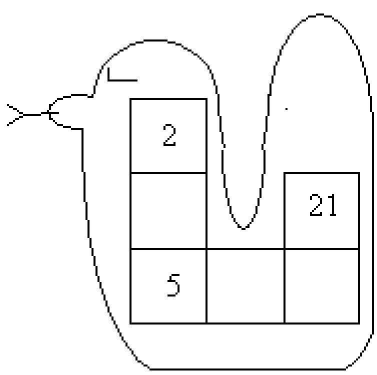


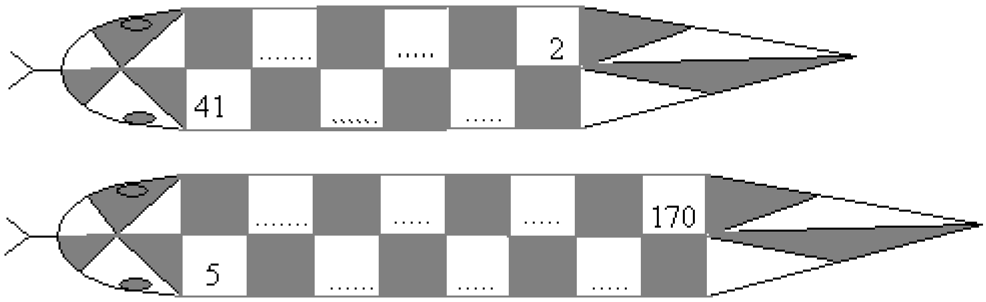
Pairs of numbers are added to give the next number in the snake.



Activity 11.4

Now, complete these super adders:





Magic Squares

You can introduce magic squares in your class in the following way:

Draw on the board a square as shown in figure 11.6. Tell the pupils that the trick in this magic square is that the rows, columns and diagonal totals are equal, in this case 15.

4	9	2
3	5	7
8	1	6

Fig. 11.6

Then draw another square, now with several numbers missing as shown in figure 10.8. You write the missing numbers on separate cards and give the cards to 7 pupils. Invite the pupils to complete the square as a group.

	7	
	11	

Fig. 11.7

If the process takes too long, then give the number 3 for the top middle box which still leaves them to do quite some bit of guessing and checking to give a completed correct grid.

Arithmogons

Arithmogons are similar to magic squares. Figure 11.9 shows an example of arithmogon.

Place the numbers 1 to 6 in figure 11.9(a) so that each line of three along a side adds to 12, and that the three corners should add to 15.

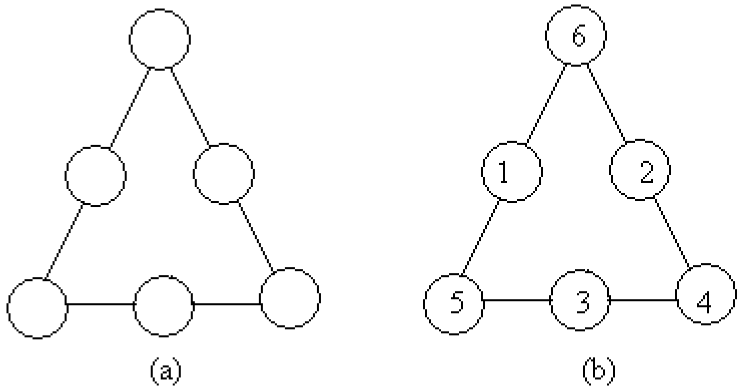


Fig. 11.9

Clever triangles

Figure 11.10 shows an example of clever triangles. The numbers in the circles at each end of a side add up to the number in the middle of that side.

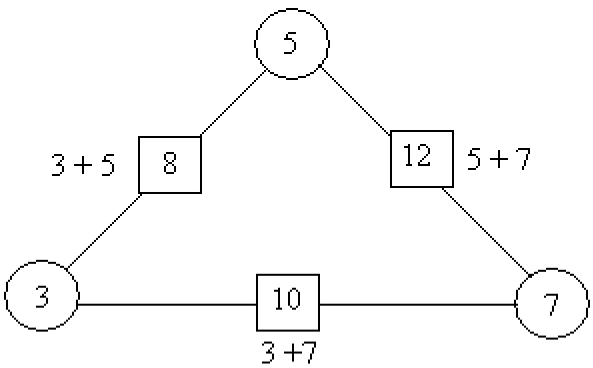


Fig.11.10

Activity11.5

1. Work out the solutions to the problems in worksheet 1 in your Teachers' Group.

2. Formulate 5 problems, one on each of the super adders, arithmogons, magic squares and triangles. Try them out in your class using the circus method.

3. Keep your results to this activity in your activity file.

Worksheet 1

1. Magic squares

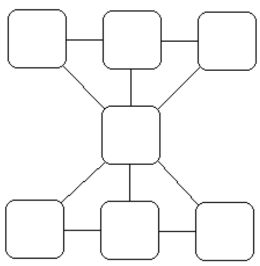
   Fill in the missing numbers in this magic square

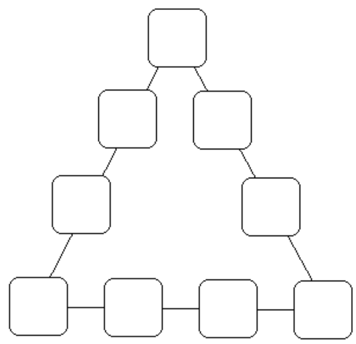
41	113	
89	71	
	29	101

2. Arithmogons (same total)

(a) Fit the numbers 1 to 7 in the boxes so that each row, column and diagonal of the three boxes adds up to 12

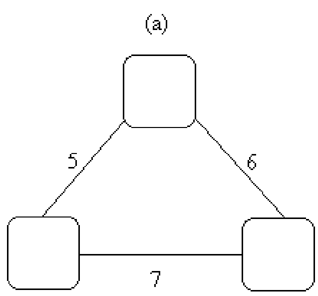
(b) Fit the numbers 1 to 9 into the diagram so that each side adds up to 20

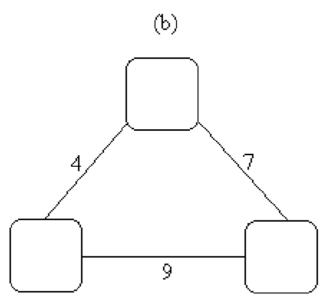




3. Clever triangles

Complete the boxes so that the numbers at each end of a side add up to the number in the middle of that side.





Missing Numbers

This requires finding missing numbers in an expression. The use of a calculator may be desirable but not necessary.

Start by asking pupils to find answers to the following multiplications:

19×19 =……
24×24 =……
71×71 =…….

Then introduce an example like the one given below:

$\square \times \square = 2209.$

Explain that the numbers in the two boxes should be the same. Discuss what should be done. The number must be greater than 21 but less than 71. Make a guess; say 63.

Checking 63 × 63 = 3969 gives a number too large. 63 is too large, so continue guessing and checking until they narrow down to the solution.

$$47 \times \boxed{47} = 2209$$

This idea can then be extended to consecutive whole numbers.

$$15 \times 16 = \boxed{\ldots\ldots} \quad \text{and} \quad 122 \times 123 = \boxed{\ldots\ldots}$$

Which two consecutive whole numbers will fit on the dotted lines?

□ × □ = 1482   and …….×……..= 8372

The idea can be further extended to the missing digits:

73 × □6 = 4088

Activity 11.6

You should take this activity to your Teachers' Group and work out the solutions to the following problems with your group members.

1. Missing numbers

(a) The numbers in each pair of boxes must be the same. Which numbers multiplied by themselves give these answers? Complete:

(i) □ × □ = 625   (ii) □ × □ = 5041

(b) Which two consecutive numbers, multiplied together, give these answers?

(i) □ × □ = 240   (ii) □ × □ = 380

(c) Fill in the missing digits:

(i) 3□ × □2 = 3198   (ii) □2 × 2□ = 1456

(d) Find the missing digits by completing the boxes:

(i)
```
  27□
+ 1□3
-----
  □20
```

(ii)
```
 46
× □
---
□□2
```

(iii)
```
     3□
   ----
4 | □52
```

Looking for a Pattern Strategy

Looking for a pattern is another strategy of problem-solving. The following are examples in which this strategy is applied.

Number patterns I

Start by showing pupils how patterns develop.

Write on the board pairs of numbers with a rule (an operation) given.

Example 1	*Example 2*
Rule	Rule
Add 8	Multiply by 4
4-------------12	3-------------12
6-------------14	6-------------24
3-------------11	7-------------…..
18------------….	

The next step is to introduce the idea of working at the pattern of the numbers to see what the rule is.

Example 3

Find the rule

Rule

………………

10----------5

18---------13

40---------35

The first result may tempt you to guess the rule as 'half it', but it requires disproving or confirming this through several examples. Indeed, after going through several examples, you find that the rule is 'subtract 5'.

After pupils have been given a lot of practice on finding rules involving one operation, they can now be introduced to two operations, one following another

Example 4

$\boxed{\text{Add 3}}$ $\boxed{\text{multiply by 4}}$

3-------------------6------------------------24
8------------------…..----------------------…..
4------------------…..----------------------…..

Give examples for other combinations of operations following one after the other.

The next step is to identify the operations that follow one after the other:

$\boxed{\text{…………………..}}$ $\boxed{\text{………………….}}$

13-------------------------------- 9 -------------------------------------24
7--------------------------------- 3 -------------------------------------1
16-------------------------------12 ------------------------------------- 4
19--------------------------------….-------------------------------------…..

Through a discussion of the pattern, the operations will be identified as subtract 4 followed by divide by 3. Give pupils lots of practice on identifying the combined operations problems.

The next step is to try examples in which the operations are combined in a single rule.

Example 5

Find the rule in the following pattern

$\boxed{\text{……………………….}}$

3-------------------------------------7
8-------------------------------------*17*
4-------------------------------------9

This is the most difficult stage where only some of the more able pupils will see the pattern. Discuss with pupils to show that the rule is 'double and add 1' or' multiply by 2 and add 1'.

Example 6 Find the rule

$\boxed{\text{……………..}}$

3 ------------------**6**
8------------------21

4------------------- **9**
5------------------ 12
10-------------------27

A discussion might reveal the fact that all of the numbers on the right hand side are multiples of 3. This may make it easier to see what has to be done to the first number to produce the second one.

This could then lead to the combined rule of 'subtract 1 then multiply by 3' or 'multiply by 3 and subtract 3'.

Activity 11.7

(a) Do the problems in Worksheet 2 in Appendix 2 in your Teachers' Group or with a colleague.

(b) Formulate some problems on patterns suitable for your class.

Number Patterns II

Begin by building a pattern with pupils.

1	11	111
× 1	× 11	× 111
1	121	12321

Then ask pupils to find the next two in the sequence

1111	11111	111111
×1111	×11111	× 111111
1234321	123454321	…………

Then ask pupils to describe and predict the answer to the blank (sixth) above.

Now ask them for the answer to the multiplication below, which is not the next in the sequence.

111111111
× 111111111
……………………

This requires a little bit more thought. There are 9 ones, so the middle digit of the answer is 9. Then go right and left reducing your numbers by 1 until you get the last number on either side, which is 1.

111111111
× 111111111
…………9…………
middle digit
so
111111111
× 111111111
12345678987654321

You can then follow the above multiplication sequences with the following addition sequences, which you can leave to pupils to do on their own:

Find the following additions

1	1	1
+ 1	11	11
	+111	111
		+1111

After supplying the answers to the above additions, ask pupils to predict the answer to the following addition:

1
11
111
1111
11111
111111
1111111
+ 11111111
……………

The 8 ones in the right column give 8 as the last digit of the answer 12345678.

Activity 11.8

Work out the problems in Work sheet 3 in Appendix 2. You may discuss them with a colleague.

Listing Strategy

It is sports season and you have been given the responsibility to draw up a netball tournament competition for all the schools in your Zone. How would you do it?

Timetables

This strategy involves combining a number of choices to see how many different ways there are of choosing. Use a simple timetable to introduce the idea. Four subjects have to be timetabled such that two are in the morning and two are in the afternoon:

Mathematics and Language are in the morning while Games and Drama are in the afternoon.

Timetable

Morning	**Afternoon**
Maths and languages	Games and Drama

Pupils are required to choose two subjects, one in the morning and the other in the afternoon. How many possible choices are there?

You do this in a systematic way by making a list so that you do not miss any. There are four ways altogether.

Possible choices: Maths and Games
Maths and Drama
Language and Games
Language and Drama

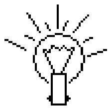
If a third subject is introduced into the morning timetable, how many possible choices of two subjects are there?

Morning	**Afternoon**
Maths, Languages, Art	Games and Drama

The possible choices now are:

Maths and Games
Maths and Drama
Language and Games
Language and Drama
Arts and Games
Arts and Drama

There are 6 possible choices.



Activity 11.9

(a) Work out problems 1 and 2 in Worksheet 4 in Appendix 3. Which of the grade levels 1 to 4 do you think the listing strategy is suitable for?

(b) Choose an appropriate class from grades 1 to 4 and teach them what you'velearned in this section. Then ask them to do the same problems you did (a).

Reflection

Probably this was one the most difficult units in this module for you to study, was it. Write down in your diary how much you understand problem-solving now as a result of studying through this unit. Will you be able to teach problem-solving to lower basic school children or not? If not, what is it that you still need to learn more in order for you to understand the subject better?

UNIT 12

INDEX NOTATION

In this unit we discuss index notation. Index notation involves writing numbers in short form. You will find this process interesting. Let's have a go at it.

Learning outcomes

As you study and work through this unit you are expected to:

- write numbers in index form
- write numbers in expanded form
- identify applications of index notation

Indices

You may have come across the word 'index'. What do you think this word means? Some words assume different meanings depending on the context in which they are used. The word 'index' is one of them. When used in Mathematics the word index assumes a meaning which we discuss here.

We know that:

$5 \times 5 = 25$

The product 5×5 can be written as 5^2.

$\therefore\ 25 = 5 \times 5 = 5^2$

5×5 is known as the expanded form (or factor form) of 25 and 5^2 is known as the index form of 25.
Generally when a number is multiplied by itself any number of times, the expression is simplified by using the index notation.

For example, $2^3 = 2 \times 2 \times 2$

$\therefore\ 2^3 = 8$

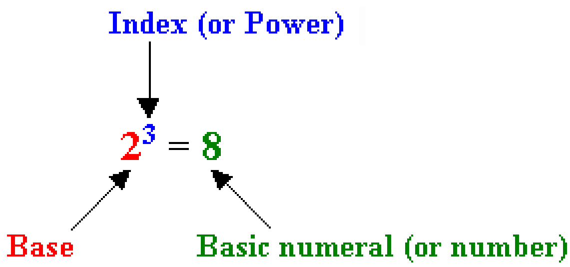
In the expression 2^3 = 2 x 2 x 2, can you identify the expanded form and the index form?

Base and Power

Every index notation has a base and a power. For example in the expression $2^3 = 8$;

- 2 is called the **base**.
- 3 is called the **index** or **power** (or **exponent**) because it indicates the power to which the base, 2, is raised.
- 8 is the **basic numeral** (or **number**).
- 2^3 is read as '2 to the power 3' or simply '2 cubed'.

That is:



Example

Write 4^3 as a number.

Solution:

$$
\begin{aligned}
4^3 &= 4\times4\times4 \\
&= (4\times4)\times4 \\
&= 16\times4 \\
&= 64
\end{aligned}
$$

As we said before, in the expression 64 = 43

- 3 is the power (or index or exponent)
- 4 is the base number
- 64 is a basic numeral or number
- 4^3 is the index form (or power form) of 64
- $4 \times 4 \times 4$ is the expanded form of 64
- For $64 = 4 \times 4 \times 4$ = 43, the base number 4 appears three times as a factor of the basic numeral (or number) 64
- 4^3 is read as '4 to the power 3' or simply '4 cubed'

Activity 12.1

1. Write each of the following expanded forms in index form:

a. $2\times2\times2\times2$

b. $3\times3\times3$

c. $6\times6\times6\times6\times6$

d. $8\times8\times8\times8\times8\times8$

2. Write each of the following in expanded form:

a. 10^3

b. 7^4

c. 9^5

3. Find the value of the following:

a. 3^2

b. 5^3

c. 7^2

4. Write 16 in index form using base 2.

Use of index notation

Index notation may be used in several situations some of which are:

i. When a digit keeps repeating itself in a mathematical expression.

Example

Write the following numbers as a product of prime factors:

a. 40
b. 56

Solution:

a. $40 = 2\times20$

$= 2\times2\times10$

$= 2\times2\times2\times5$

$= 2^3\times5$

b. $56 = 2\times28$

$= 2\times2\times14$

$= 2\times2\times2\times7$

$= 2^3\times7$

ii. When we want to write very huge numbers in a short form.

Example

The speed of light in space is 3×10^8 m/s. In ordinary number form this may be written as 300, 000,000 m/s. Because very large numbers in Science are usually expressed in this form, index form is also known as Scientific notation. The base is less than 10 and is equal to or greater than 1.

Activity 12.2

Find out the distances from the earth to the Sun, Moon, and Mars and state this distance in index form.

Compare your answers to those of fellow students. What challenges did you face in finding the distance of these bodies?

Summary

In this unit we have discussed how to write numbers in both index and expanded forms. Also, we have been able to identify various applications of index notation in the Science world

UNIT 13

STATISTICS

Statistics is the science of collecting, organizing, presenting, analyzing and interpreting numerical data to assist in making more effective decisions. Statistics makes it possible to predict the likelihood of events.

A few examples of statistical information we can calculate are average value (mean), most frequently occurring value (mode) and midpoint between the lowest and highest value of the set (median)

Learning outcomes

As study and work through this unit you are expected to:

- explore different ways of analyzing data
- interpret graphical data
- use statistical data to make to evaluate your classroom practice

Activity 13.1

You can collect statistical data in a number of ways. Ask each pupil in your class to write his or her age on a piece of paper. How many of your pupils are:

i. 7 years old?

ii. 8 years old?

iii. Between 9 and 15 years old?

iv. Above 15 years old?

Make a suitable table to show the data that you have collected.

Presenting Statistical Data

It is important to present data clearly. Good presentation makes statistical data easy to read and understand. Statistical data can be presented in various ways that include Frequency Tables, Pictograms, Pie Charts, Bar Charts and Line Graphs.

Frequency Tables.

The **frequency** of a particular data value is the number of times the data value occurs.

For example, if five students have a score of 75 in mathematics, and then the score of 75 is said to have a frequency of 5. The frequency of a data value is often represented by *f*. A frequency table is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies.

Example 1

The marks awarded in a science test in a Grade 7 class at Titandizane Basic School were as follows:

6	7	5	7	7	8	7	6	9	7
4	10	6	8	8	9	5	6	4	8

Construct a frequency table and from the table answer the following: In general: We use the following steps to construct a frequency table:

Step 1: Construct a table with three columns. In the first column, write down all of the data values in ascending order of magnitude.

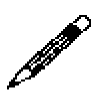
Step 2: To complete the second column, go through the list of data values and place one tally mark at the appropriate place in the second column for every data value. When the fifth tally is reached for a mark, draw a horizontal line through the first four tally marks as shown for 7 in the above frequency table. We continue this process until all data values in the list are tallied.

Step 3: Count the number of tally marks for each data value and write it in the third column.

Following the above steps, construct a frequency for the data given above.

Did your frequency table look like the one below?

Mark	Tally Marks	Frequency
4	||	2
5	||	2
6	||||	4
7	卌	5
8	||||	4
9	||	2
10	|	1
	TOTAL	20



Activity 13.2

Pupils in Mrs. Gondwe's class took a Mathematics test and their scores out of 10, are listed below:

3	7	6	2	5	9	10	8	7	1
8	4	3	5	6	7	8	7	6	5
3	6	9	8	7	5	9	6	7	8

Construct a frequency table, and from the table, find the following:

(a) How many pupils wrote the test?

(b) What was the most common mark?

(c) How many pupils obtained more than 6 marks?

Pictograms

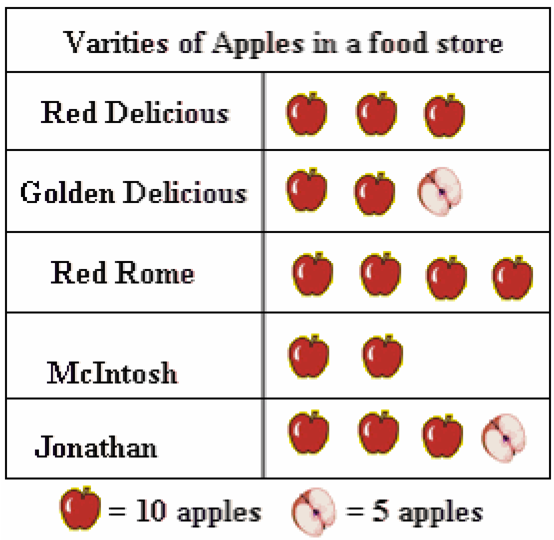
A pictogram or (pictotograph) uses pictures or drawings to give a quick and easy meaning to statistical data.

Example.

A group of pupils at a basic school were taken to a food store and asked to pick their favourite apple variety. The number of varieties of apples they picked were as follows:

VARIETY	NUMBER OF APPLES
Red Delicious	30
Golden Delicious	25
Red Rome	40
McIntosh	20
Jonathan	35

The data above can be shown in a pictogram as follows:



Activity 13.3

Pupils in a grade 7 class were asked the mode of transport they used when going to school. Their responses are shown in the table below.

Mode of Transport	Number of Pupils
Bus	2
Bicycle	4
Walking	5
Motorbike	1

Illustrate this information in a pictogram.

Bar Charts

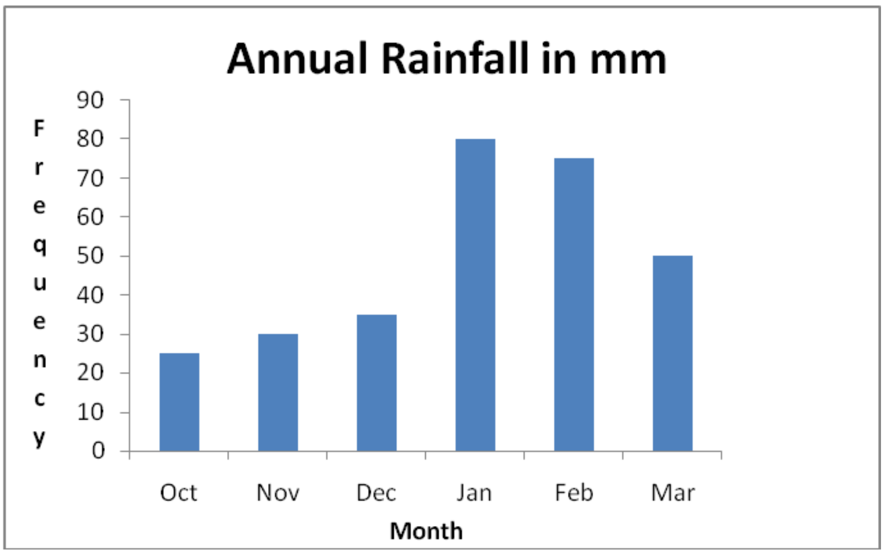
A bar chart or bar graph is a chart with rectangular bars with lengths proportional to the values that they represent. The bars can be plotted vertically or horizontally. The height of each bar represents frequency. In a bar chart, each bar has the same width.

Example

The average rainfall recorded by meteorological department in Zambia in certain year was as follows:

October:	25mm
November:	30mm
December:	35mm
January:	80mm
February:	75mm
March:	50mm

The information above can be shown on a bar chart as follows:



Activity 13.4

In a Grade 5 class, pupils were asked their shoe sizes. The following sizes were recorded:

4	2	3	5	6	6	7	8	4	5
3	6	4	2	7	4	3	5	7	6
7	8	6	5	4	3	4	2	3	5
5	5	3	2	6	3	5	4	4	6

(a) Make a bar chart for the above data.

(b) Use your bar chart to find the most common size.

Line Graphs

A line graph is most useful in displaying data or information that changes continuously over time.

Some of the strengths of bar graphs are that:

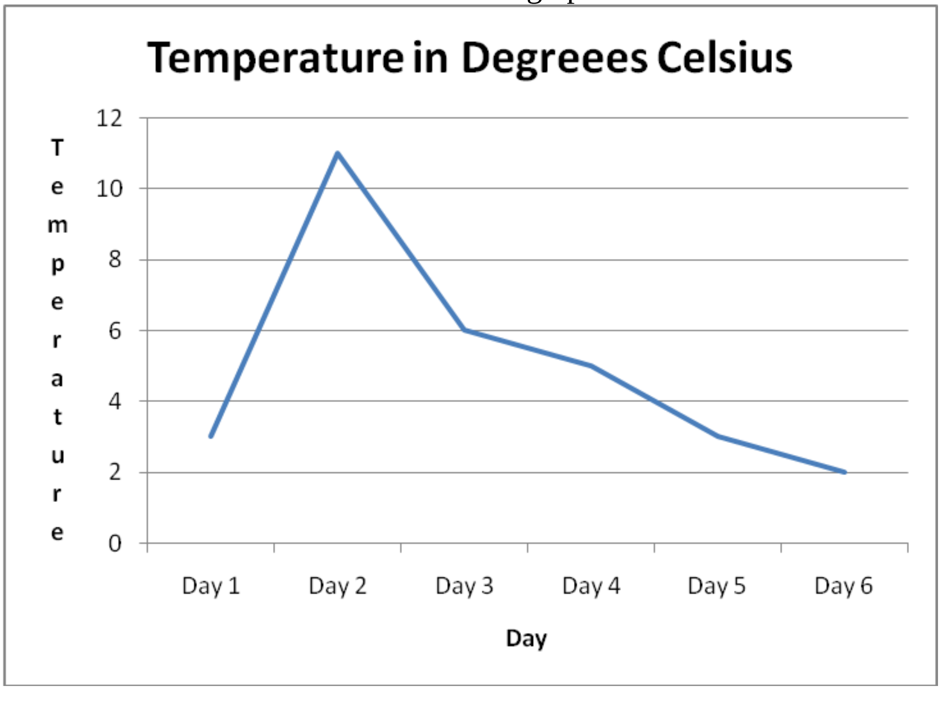
They are good at showing specific values of data, meaning that given one variable the other can easily be determined. They show trends in data clearly, meaning that they visibly show how one variable is affected by the other as it increases or decreases. They enable the viewer to make predictions about the results of data not yet recorded.

The example below shows the changes in the temperature over a week in January.

The table below shows daily temperatures recorded for Lusaka in June, for 6 days, in degrees Celsius

Temperatures in Lusaka City in ^{0}C	
Day	Temperature
1	3
2	11
3	6
4	5
5	3
6	2

This information can be shown on a line graph as:



Activity 13.5

The table below shows a teacher's weight in kilograms for five months.

Teacher's Weight	
Month	**Weight in Kilogram**
January	47
February	52
March	59
April	67
May	71

Show this information on a line graph.

Pie Charts

What do you understand by the term pie chart? You have definitely met this term before.
A **pie chart** (or a circle graph) is a **circular chart** divided into sectors, illustrating proportion. In a pie chart, the arc length of each sector (and consequently its angle and area), is proportional to the quantity it represents.
Pie charts are useful to compare different parts of a whole amount, for instance displaying different values of a given variable or distribution

Constructing Pie Charts

Study the following steps of constructing a pie chart:

Step 1: Calculate the angle of each sector, using the formula

$$\text{Angle of sector} = \frac{\text{Frequency of data}}{\text{Total frequency}} \times 360°$$

Step 2: Draw a circle using a pair of compasses

Step 3: Use a protractor to draw the angle for each sector.

Step 4: Label the pie chart and all its sectors.

Example

At Chimulenga Basic School, there are 375 pupils in the lower basic, 315 in the middle basic and 210 in the upper basic

(a) Calculate the angle which represents each of the following on a pie chart.

i. Lower Basic
ii. Middle Basic
iii. Upper Basic

(b) . Draw a pie chart to represent the numbers of pupils at this school

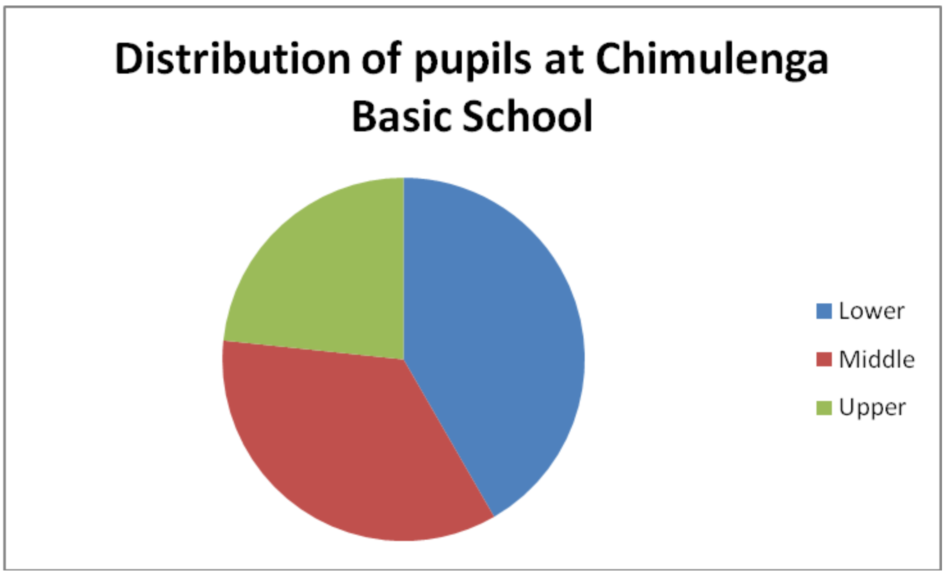
Solutions:

Lower Basic = $\frac{375}{900} \times 360^0 = 150^0$

Middle Basic = $\frac{315}{900} \times 360^0 = 120^0$

Upper Basic = $\frac{210}{900} \times 360^0 = 84^0$

Note that the sum of your angles should add up to 360^0



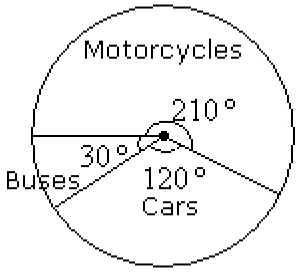
Activity 13.6

The following table shows the results of the friendly matches played by the Chakanga Basic School Girls' football team.

Result	Win	Lose	Draw	Abandoned
Frequency	7	10	5	2

Illustrate the information given above on a pie chart.

The following pie chart shows a survey of the numbers of cars, buses and motorcycles that passes a particular junction. There were 150 buses in the survey.



a) What fraction of the vehicles were motorcycles?

b) What percentage of vehicles passing by the junction were cars?

c) Calculate the total number of vehicles in the survey.

d) How many cars were in the survey?

Grouped Data

Grouped data is a statistical term used in data analysis. Raw data can be organized by constructing a table showing the frequency distribution of the variable (whose values are given in the raw dataset). Such a frequency table is often referred to as a grouped data

Data which have been arranged in groups or classes rather than showing all the original figures, for example, the data in a population pyramid. When the range, (the difference between the highest and the lowest measure), is large, it is useful to group the data into classes and choose a suitable class interval.

Example

Time taken (in seconds) by a group of pupils in a Grade 7 class to answer a simple mathematics question is shown in the table below.

20	25	24	33	13
26	8	19	31	11
16	21	17	11	34
14	15	21	18	17

(a) Using a class interval of 5 starting at 5, make a frequency table
(b) Which class interval had the highest frequency?
(c) How many pupils were in this class?

Solutions

(a)

Mark	**Tally**	**Frequency**
5-9	|	1
10-14	||||	4
15-19	卌 |	6
20-24	||||	4
25-29	||	2
30-34	|||	3
	Total	20

(b) 15-19
(c) 20

Activity 13.7

The heights in centimeters of Grade 5 pupils were:

119	145	133	130	147	148	146	130	143
152	149	150	147	126	138	123	127	151
153	143	152	153	145	154	157	137	139
130	133	139	124	148	141	132	145	150
144	153	150	149	127	142	136	155	138

(a) Using a class interval of 10 starting at 110, construct a frequency.
(b) Which class interval had the highest frequency?
(c) How many pupils were in this class?

Solutions
(a)

Mark	Tally Marks	Frequency
110-119	|	1
120-129	卌	5
130-139	卌 卌 ||	12
140-149	卌 卌 卌	15
150-159	卌 卌 ||	12
	Total	45

(b) 140-149
(c) 45

Measures of Central Tendency (Statistical Averages)

A measure of central tendency is the number that represents a data set and gives an idea of the middle quantity of the data set.
There are a number of measures of central tendency, however the most common ones used for different situations are the mean, the mode and the median

The Mean

The mean is a measure of central tendency found by adding all the numbers in a set of data and dividing the sum by the number of measures in the set of data.
Therefore

Mean = $\frac{Sumofmessures}{Numberofmeasures}$

The mean is also known as the arithmetic average.

Example

Find the mean of the following set of numbers: 10, 14, 86, 2, 68, 99, 1
Solution

$$Mean = \frac{10+14+86+2+68+99+1}{7}$$

$$= \frac{280}{7}$$

$$= 40$$

Activity 13.7

Find the mean of each of the following sets of numbers:
i. 3, 9, 2, 5, 6, 7, 8, 6, 9, 3,6
ii. 0.3, 0.2, 1.7, 0.2, 0.8, 1.3

The Median
The median is the midpoint of a distribution. That is, the median is the number in the center of a data set that has been ordered sequentially.
Find the median of

i. 43, 7, 30, 12, 16, 21, 26, 33, 12

ii. 4, 6, 7, 10, 11,13

Solution
(i) Arrange the numbers in order of size (either ascending or descending)
7, 12, 12, 16, 21, 26, 30, 33, 43

There are 9 numbers and 9 is an odd number, therefore the 5th number is in the middle. The fifth number is 21. This implies that

Median = 2

(ii) 4, 6, 7, 10, 11, 13

As the numbers of data is even and the numbers are arranged in order, the median will be halfway between the two middle numbers

The median is the mean of $\frac{7+10}{2}$

$$\text{Median} = \frac{17}{2}$$

$$= 8.5$$

Activity 13.8

Find the median of the following sets of numbers:

i. 17, 14, 10, 11, 13, 18, 12

ii. 8.4, 11.4, 9.5. 13.9, 13.0, 10.6

Mode

The mode is the most frequently occurring value in a set.

Consider the set $A = \{1,\ 2,\ 3,\ \mathbf{4},\ \mathbf{4},\ \mathbf{4},\ 5,\ 6,\ 7,\ 8,\ 8,\ 9\}$. Which number appears most often? The mode would be 4 as it occurs a total of three times in the set, more frequently than any other value in the set.

A data set can have more than one mode: for example, in the set {1, 2, 2, 3, 3}, both 2 and 3 are modes. If all points in a data set occur with equal frequency, it is equally accurate to describe the data set as having many modes or no mode.

Activity 13.9

Find the mode in each of the following sets of numbers:

(a) 3, 5,2,1 ,3,4

(b) 11, 9 ,7, 9, 12, 8, 11

Mean for a Frequency Table

Consider the following numbers 2, 4, 3, 5, 4, 5, 5. Think of all possible ways of finding the mean of this set of numbers. By now you are familiar with the method of adding all the measures and dividing by the total number of measures.

Thus: $\text{Mean} = \frac{2+3+4+4+5+5+5}{7}$

$$= \frac{28}{7}$$

$= 4$

You can also find the mean for this set of numbers by multiplying each measure by its frequency, adding the products and dividing the sum by the total number of measures.

Thus: Mean $= \dfrac{(1\times 2)+(1\times 3)+(2\times 4)+(3\times 5)}{7}$

$$= \frac{2+3+8+15}{7}$$

$$= \frac{28}{7}$$

$$= 4$$

The second method can be used to calculate the mean of data presented in a frequency table .

Example

The table below gives the number of dolls owned by each pupil in a Grade 4 class:

1	0	1	2	0	0	1	2	4	0
0	1	1	0	3	1	0	1	2	3
0	1	2	1	1	0	2	1	1	2
2	1	0	1	0	1	1	0	2	1

(a) Make a frequency table and calculate the mean number of dolls owned by each pupil

(b) State the modal mark

Mark	Tally Mark	Frequency	Mark x Frequency
0	卌 卌 ||	12	0 x 12 = 0
1	卌 卌 卌 |	16	1 x 16 = 16
2	卌 ||||	9	2 x 9 = 18
3	||	2	3 x 2= 6
4	|	1	4 x 1= 4
	TOTALS	40	44

(a) Mean mark $= \dfrac{\text{Sum of (measures x frequency)}}{\text{Total frequency}}$

$$= \frac{44}{20}$$

$$= 2.2$$

(b) From the frequency table, we can see that the mark with the highest frequency is 1. $\therefore$ Modal mark = 1

Activity 13.10

A Grade 7 class obtained the following marks in a science test marked out of 20:

11	12	14	13	16	15	13	13	12	13
15	16	16	15	13	15	12	13	14	17
11	13	17	16	13	12	13	17	16	16
11	12	13	15	16					

(a) Make a frequency table and calculate the mean mark

(b) State the modal mark

Reflection

It is now time for you to reflect on what you have learned in this unit. Write down in your diary the things that you think have added to your knowledge in the teaching of statistics to primary school children or those aspects in which you used to experience difficulties but have been made easier after studying through this unit.